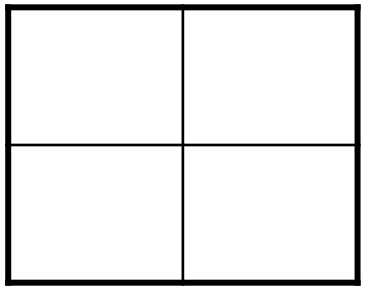
جامعة
المنارة
MANARA UNIVERSITY

Karnaugh Map

- Adjacent Squares
  - Number of squares = number of combinations
    - Each square represents a minterm
    - 2 Variables ⇨ 4 squares
    - 3 Variables ⇨ 8 squares
    - 4 Variables ⇨ 16 squares
  - Each two adjacent squares differ in one variable
    - Two adjacent minterms can be combined together

Example:

$$F = x\,y + x\,y'$$
$$= x\,(\,y + y'\,)$$
$$= x$$



Two-variable Map

	x y	Minterm	
0	0 0	m_0	$\bar{x}\,\bar{y}$
1	0 1	m_1	$\bar{x}\,y$
2	1 0	m_2	$x\,\bar{y}$
3	1 1	m_3	$x\,y$

m_0	m_1
m_2	m_3

Note: adjacent squares horizontally and vertically *NOT* diagonally

x \ y	0	1
0	$\bar{x}\,\bar{y}$	$\bar{x}\,y$
1	$x\,\bar{y}$	$x\,y$

Two-variable Map

- Example

	x y	F	Minterm	
0	0 0	0	m_0	$\bar{x}\,\bar{y}$
1	0 1	0	m_1	$\bar{x}\,y$
2	1 0	0	m_2	$x\,\bar{y}$
3	1 1	1	m_3	$x\,y$

m_0	m_1
m_2	m_3

	y	
	0	0
x	0	1

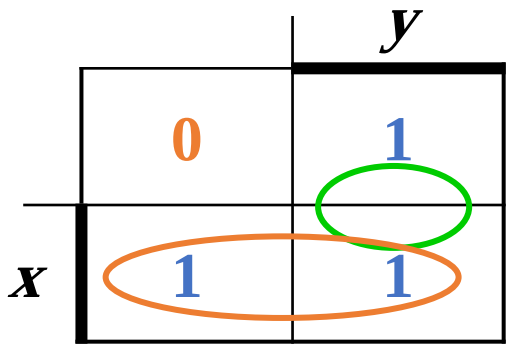
x \ y	0	1
0	$\bar{x}\,\bar{y}$	$\bar{x}\,y$
1	$x\,\bar{y}$	$x\,y$

Two-variable Map

- Example

	x y	F	Minterm	
0	0 0	0	m_0	$\bar{x}\,\bar{y}$
1	0 1	1	m_1	$\bar{x}\,y$
2	1 0	1	m_2	$x\,\bar{y}$
3	1 1	1	m_3	$x\,y$

m_0	m_1
m_2	m_3



$$F = \underbrace{\bar{x}\,y + x\,y}_{(\bar{x}+x)\,y} + \underbrace{x\,\bar{y}}_{x(y+\bar{y})}$$

$$F = y + x$$

x \ y	0	1
0	$\bar{x}\,\bar{y}$	$\bar{x}\,y$
1	$x\,\bar{y}$	$x\,y$

Three-variable Map

	x y z			Minterm	
0	0	0	0	m_0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0	0	1	m_1	$\bar{x}\,\bar{y}\,z$
2	0	1	0	m_2	$\bar{x}\,y\,\bar{z}$
3	0	1	1	m_3	$\bar{x}\,y\,z$
4	1	0	0	m_4	$x\,\bar{y}\,\bar{z}$
5	1	0	1	m_5	$x\,\bar{y}\,z$
6	1	1	0	m_6	$x\,y\,\bar{z}$
7	1	1	1	m_7	$x\,y\,z$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

x \ $y\,z$	00	01	11	10
0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{y}\,z$	$\bar{x}\,y\,z$	$\bar{x}\,y\,\bar{z}$
1	$x\,\bar{y}\,\bar{z}$	$x\,\bar{y}\,z$	$x\,y\,z$	$x\,y\,\bar{z}$

Three-variable Map

- Example

	$x\ y\ z$	F	Minterm	
0	0 0 0	0	m_0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0 0 1	0	m_1	$\bar{x}\,\bar{y}\,z$
2	0 1 0	1	m_2	$\bar{x}\,y\,\bar{z}$
3	0 1 1	1	m_3	$\bar{x}\,y\,z$
4	1 0 0	1	m_4	$x\,\bar{y}\,\bar{z}$
5	1 0 1	1	m_5	$x\,\bar{y}\,z$
6	1 1 0	0	m_6	$x\,y\,\bar{z}$
7	1 1 1	0	m_7	$x\,y\,z$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

$x \backslash yz$	00	01	11	10
0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{y}\,z$	$\bar{x}\,y\,z$	$\bar{x}\,y\,\bar{z}$
1	$x\,\bar{y}\,\bar{z}$	$x\,\bar{y}\,z$	$x\,y\,z$	$x\,y\,\bar{z}$

			y	y
	0	0	1	1
x	1	1	0	0
		z	z	

$$F = x\,\bar{y} + \bar{x}\,y$$

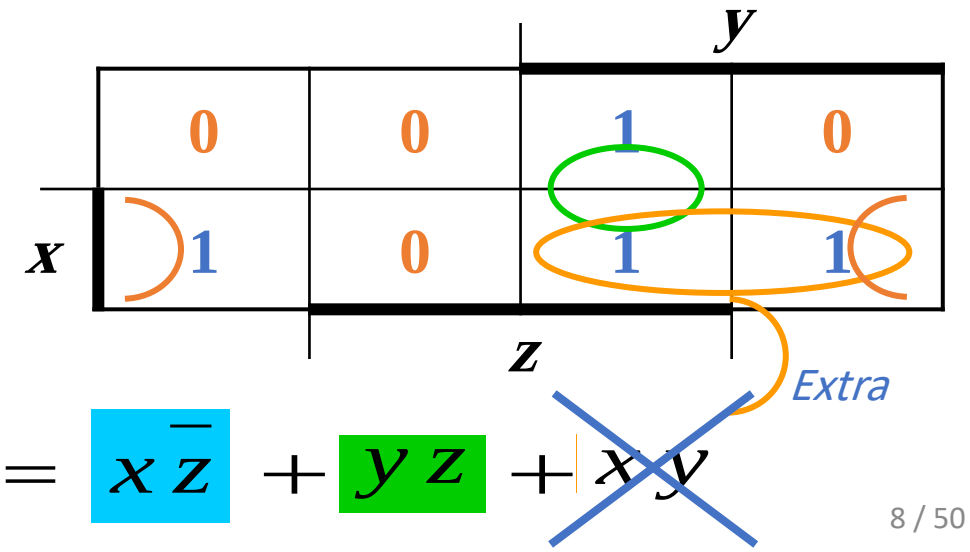
Three-variable Map

- Example

	$x\ y\ z$	F	Minterm	
0	0 0 0	0	m_0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0 0 1	0	m_1	$\bar{x}\,\bar{y}\,z$
2	0 1 0	0	m_2	$\bar{x}\,y\,\bar{z}$
3	0 1 1	1	m_3	$\bar{x}\,y\,z$
4	1 0 0	1	m_4	$x\,\bar{y}\,\bar{z}$
5	1 0 1	0	m_5	$x\,\bar{y}\,z$
6	1 1 0	1	m_6	$x\,y\,\bar{z}$
7	1 1 1	1	m_7	$x\,y\,z$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

x \ yz	00	01	11	10
0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{y}\,z$	$\bar{x}\,y\,z$	$\bar{x}\,y\,\bar{z}$
1	$x\,\bar{y}\,\bar{z}$	$x\,\bar{y}\,z$	$x\,y\,z$	$x\,y\,\bar{z}$

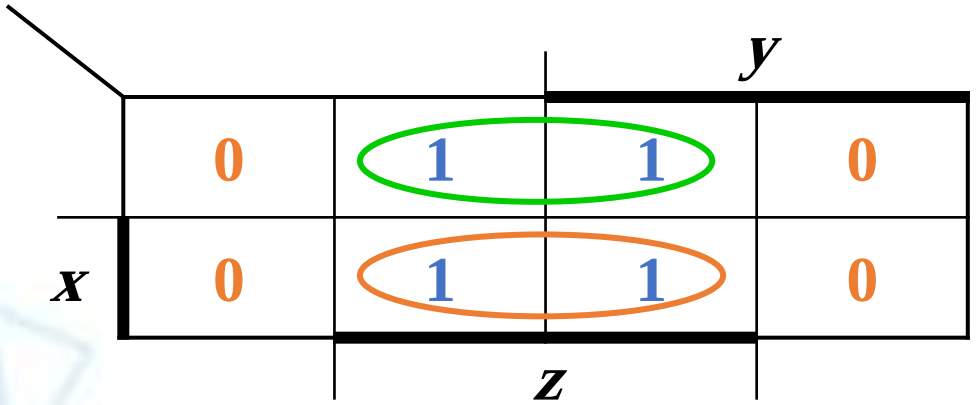


$$F = \bar{x}\,\bar{z} + y\,z + x\,y$$

Three-variable Map

- Example

	x y z	F	Minterm	
0	0 0 0	0	m_0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0 0 1	1	m_1	$\bar{x}\,\bar{y}\,z$
2	0 1 0	0	m_2	$\bar{x}\,y\,\bar{z}$
3	0 1 1	1	m_3	$\bar{x}\,y\,z$
4	1 0 0	0	m_4	$x\,\bar{y}\,\bar{z}$
5	1 0 1	1	m_5	$x\,\bar{y}\,z$
6	1 1 0	0	m_6	$x\,y\,\bar{z}$
7	1 1 1	1	m_7	$x\,y\,z$

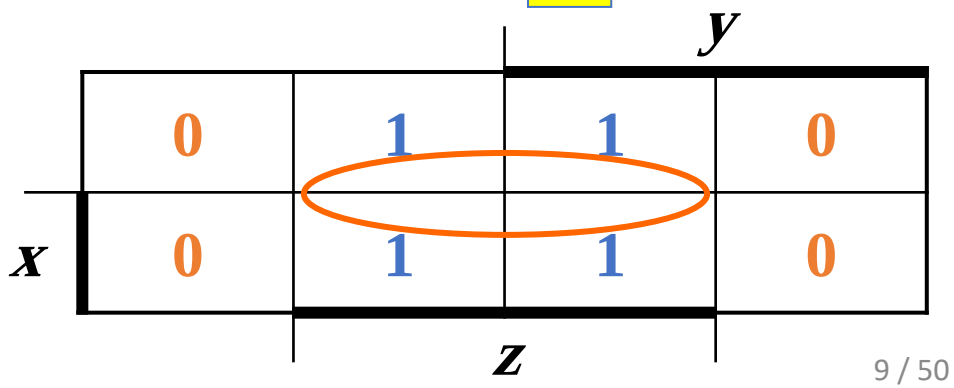


$$F = \bar{x}\,\bar{y}\,z + \bar{x}\,y\,z + x\,\bar{y}\,z + x\,y\,z$$

$\bar{x}\,z\,(\bar{y}+y)$ $\quad$ $x\,z\,(\bar{y}+y)$

$\bar{x}\,z$ $\quad$ $x\,z$

z



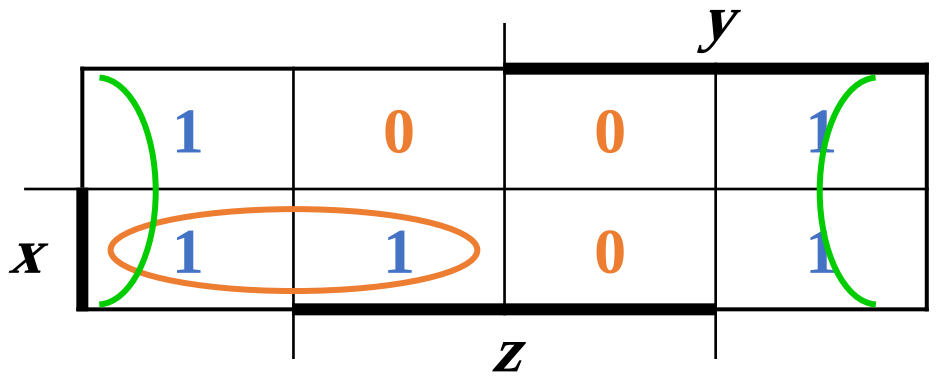
Three-variable Map

- Example

	$x\ y\ z$	F	Minterm	
0	0 0 0	1	m_0	$\bar{x}\,\bar{y}\,\bar{z}$
1	0 0 1	0	m_1	$\bar{x}\,\bar{y}\,z$
2	0 1 0	1	m_2	$\bar{x}\,y\,\bar{z}$
3	0 1 1	0	m_3	$\bar{x}\,y\,z$
4	1 0 0	1	m_4	$x\,\bar{y}\,\bar{z}$
5	1 0 1	1	m_5	$x\,\bar{y}\,z$
6	1 1 0	1	m_6	$x\,y\,\bar{z}$
7	1 1 1	0	m_7	$x\,y\,z$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

x \ $y\,z$	00	01	11	10
0	$\bar{x}\,\bar{y}\,\bar{z}$	$\bar{x}\,\bar{y}\,z$	$\bar{x}\,y\,z$	$\bar{x}\,y\,\bar{z}$
1	$x\,\bar{y}\,\bar{z}$	$x\,\bar{y}\,z$	$x\,y\,z$	$x\,y\,\bar{z}$



$$F = \bar{z} + x\,\bar{y}$$

Four-variable Map

	w x y z	Minterm	
0	0 0 0 0	m_0	$\overline{w}\,\overline{x}\,\overline{y}\,\overline{z}$
1	0 0 0 1	m_1	$\overline{w}\,\overline{x}\,\overline{y}\,z$
2	0 0 1 0	m_2	$\overline{w}\,\overline{x}\,y\,\overline{z}$
3	0 0 1 1	m_3	$\overline{w}\,\overline{x}\,y\,z$
4	0 1 0 0	m_4	$\overline{w}\,x\,\overline{y}\,\overline{z}$
5	0 1 0 1	m_5	$\overline{w}\,x\,\overline{y}\,z$
6	0 1 1 0	m_6	$\overline{w}\,x\,y\,\overline{z}$
7	0 1 1 1	m_7	$\overline{w}\,x\,y\,z$
8	1 0 0 0	m_8	$w\,\overline{x}\,\overline{y}\,\overline{z}$
9	1 0 0 1	m_9	$w\,\overline{x}\,\overline{y}\,z$
10	1 0 1 0	m_{10}	$w\,\overline{x}\,y\,\overline{z}$
11	1 0 1 1	m_{11}	$w\,\overline{x}\,y\,z$
12	1 1 0 0	m_{12}	$w\,x\,\overline{y}\,\overline{z}$
13	1 1 0 1	m_{13}	$w\,x\,\overline{y}\,z$
14	1 1 1 0	m_{14}	$w\,x\,y\,\overline{z}$
15	1 1 1 1	m_{15}	$w\,x\,y\,z$

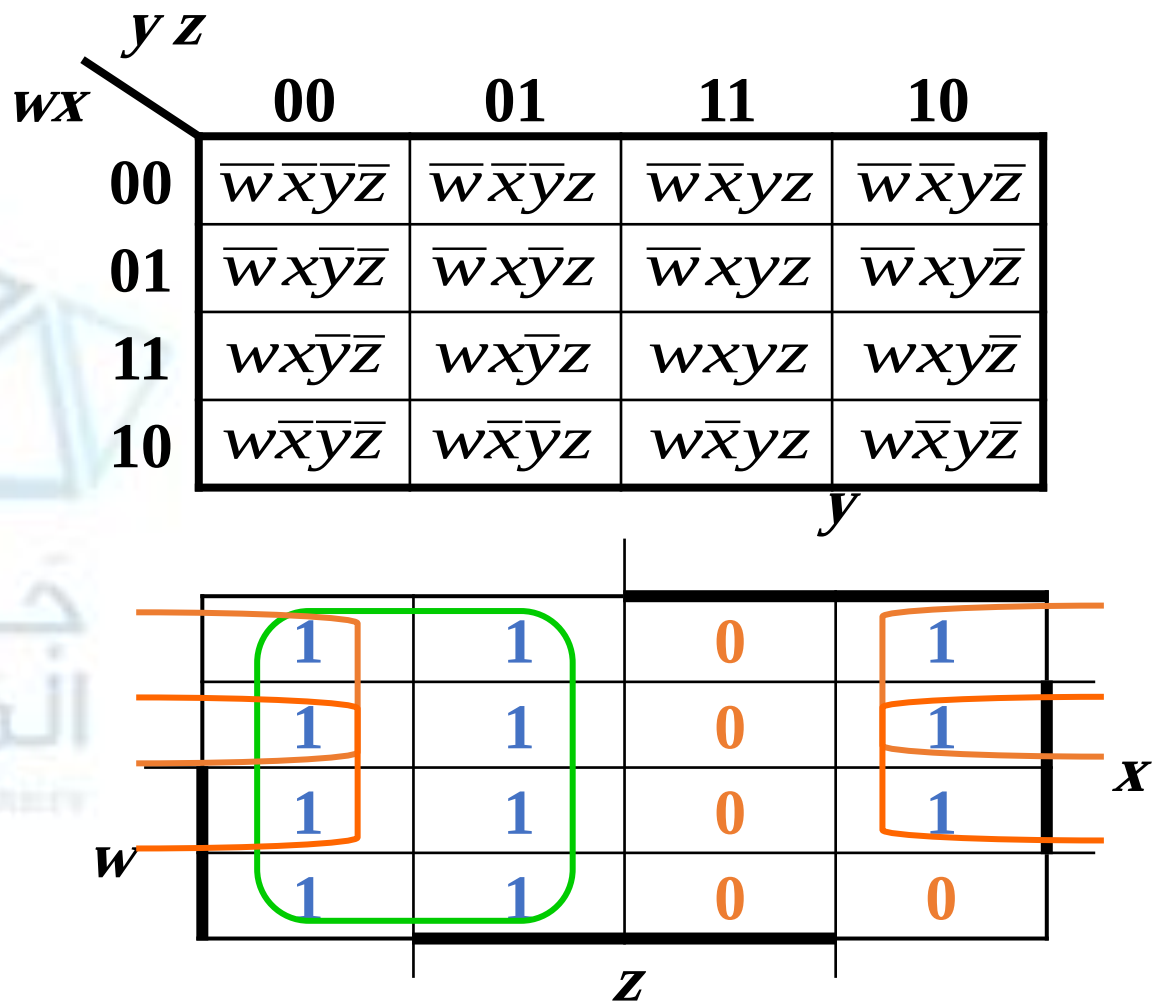
m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

wx \ *yz*	00	01	11	10
00	$\overline{w}\overline{x}\overline{y}\overline{z}$	$\overline{w}\overline{x}\overline{y}z$	$\overline{w}\overline{x}yz$	$\overline{w}\overline{x}y\overline{z}$
01	$\overline{w}x\overline{y}\overline{z}$	$\overline{w}x\overline{y}z$	$\overline{w}xyz$	$\overline{w}xy\overline{z}$
11	$wx\overline{y}\overline{z}$	$wx\overline{y}z$	$wxyz$	$wxy\overline{z}$
10	$w\overline{x}\overline{y}\overline{z}$	$w\overline{x}\overline{y}z$	$w\overline{x}yz$	$w\overline{x}y\overline{z}$

Four-variable Map

- Example

	w x y z	F	Minterm	
0	0 0 0 0	1	m_0	$\bar{w}\,\bar{x}\,\bar{y}\,\bar{z}$
1	0 0 0 1	1	m_1	$\bar{w}\,\bar{x}\,\bar{y}\,z$
2	0 0 1 0	1	m_2	$\bar{w}\,\bar{x}\,y\,\bar{z}$
3	0 0 1 1	0	m_3	$\bar{w}\,\bar{x}\,y\,z$
4	0 1 0 0	1	m_4	$\bar{w}\,x\,\bar{y}\,\bar{z}$
5	0 1 0 1	1	m_5	$\bar{w}\,x\,\bar{y}\,z$
6	0 1 1 0	1	m_6	$\bar{w}\,x\,y\,\bar{z}$
7	0 1 1 1	0	m_7	$\bar{w}\,x\,y\,z$
8	1 0 0 0	1	m_8	$w\,\bar{x}\,\bar{y}\,\bar{z}$
9	1 0 0 1	1	m_9	$w\,\bar{x}\,\bar{y}\,z$
10	1 0 1 0	0	m_{10}	$w\,\bar{x}\,y\,\bar{z}$
11	1 0 1 1	0	m_{11}	$w\,\bar{x}\,y\,z$
12	1 1 0 0	1	m_{12}	$w\,x\,\bar{y}\,\bar{z}$
13	1 1 0 1	1	m_{13}	$w\,x\,\bar{y}\,z$
14	1 1 1 0	1	m_{14}	$w\,x\,y\,\bar{z}$
15	1 1 1 1	0	m_{15}	$w\,x\,y\,z$

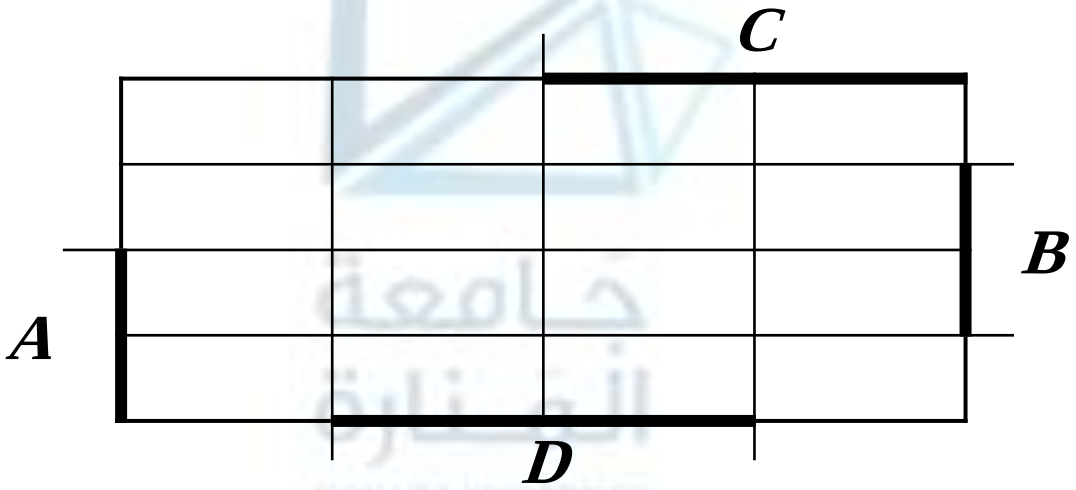


$$F = \bar{y} + \bar{w}\,\bar{z} + x\,\bar{z}$$

Four-variable Map

- Example

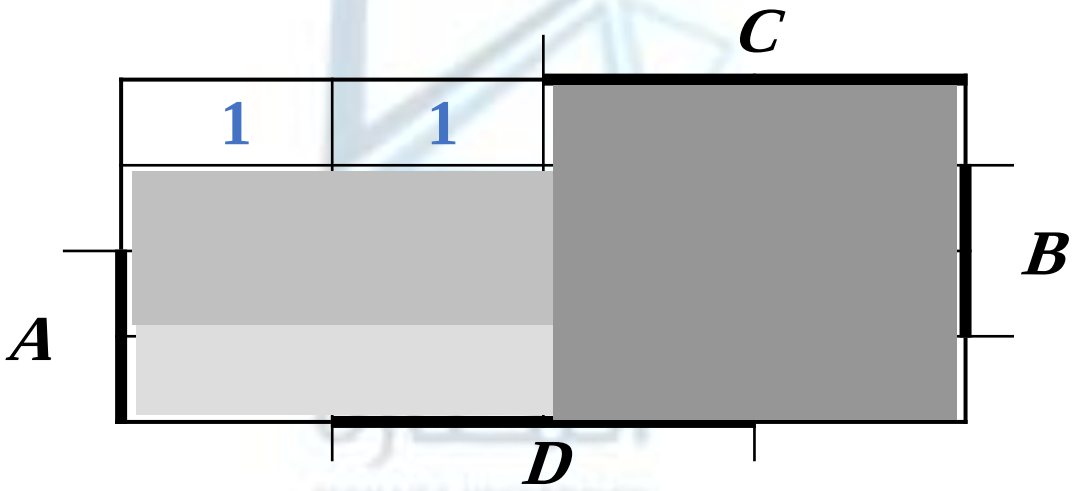
  Simplify: $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



Four-variable Map

- Example

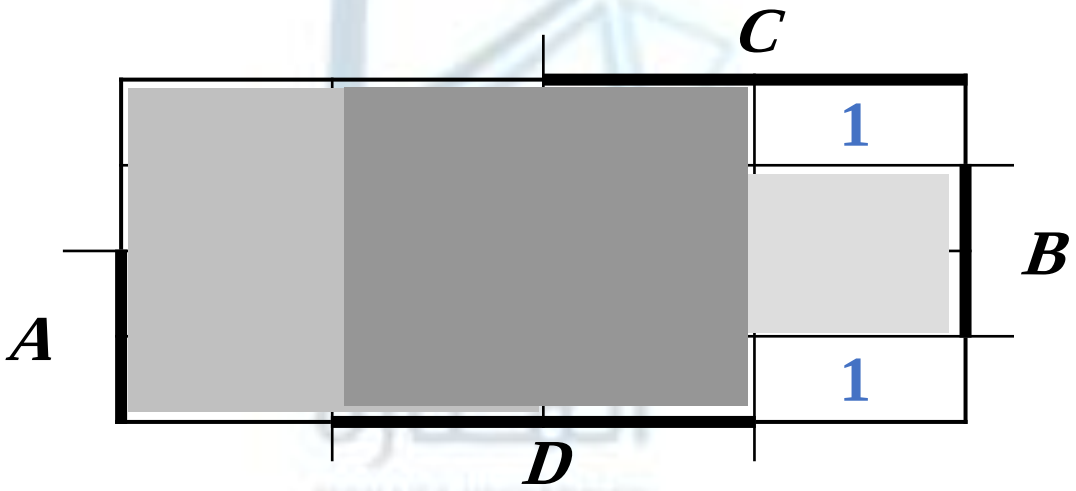
  Simplify: $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



Four-variable Map

- Example

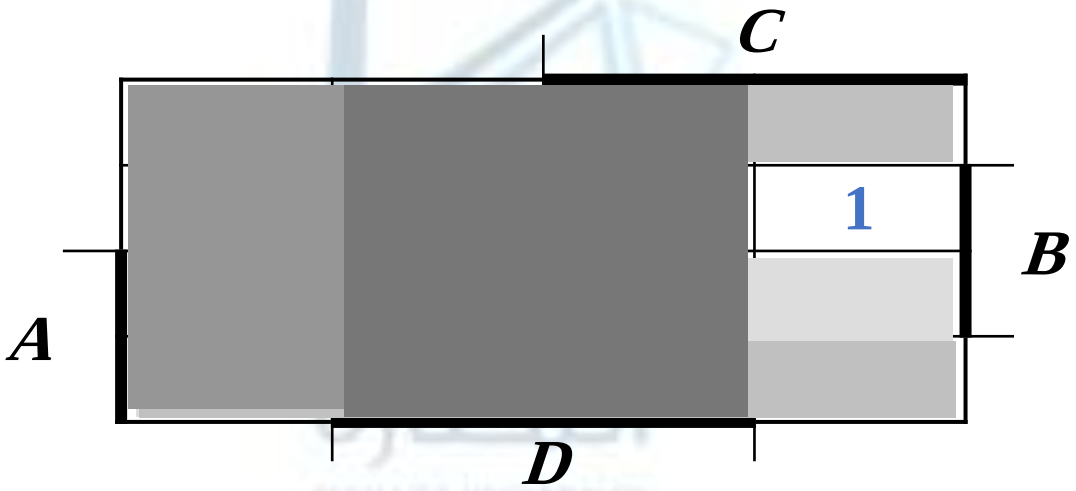
  Simplify: $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



Four-variable Map

- Example

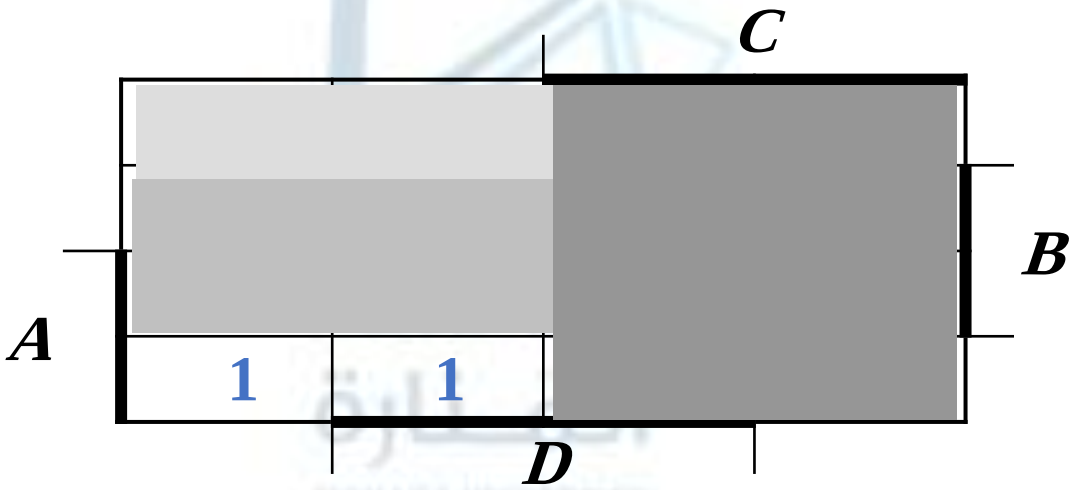
  Simplify: $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



Four-variable Map

- Example

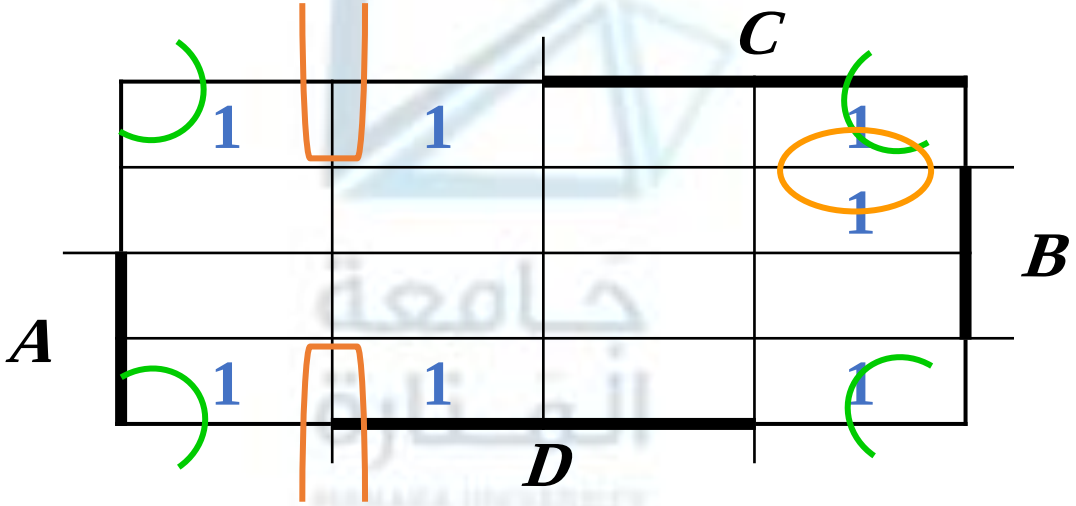
  Simplify: $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



Four-variable Map

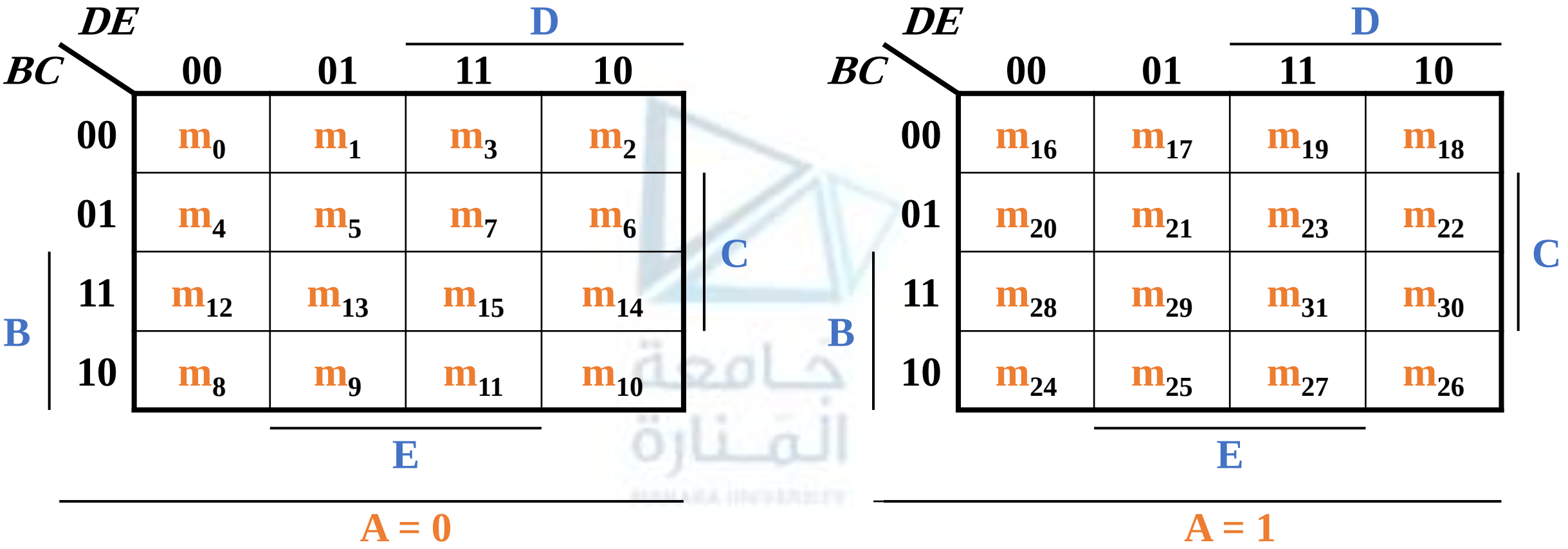
- Example

  Simplify: $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



$$F = \overline{B}\,\overline{D} + \overline{B}\,\overline{C} + \overline{A}C\overline{D}$$

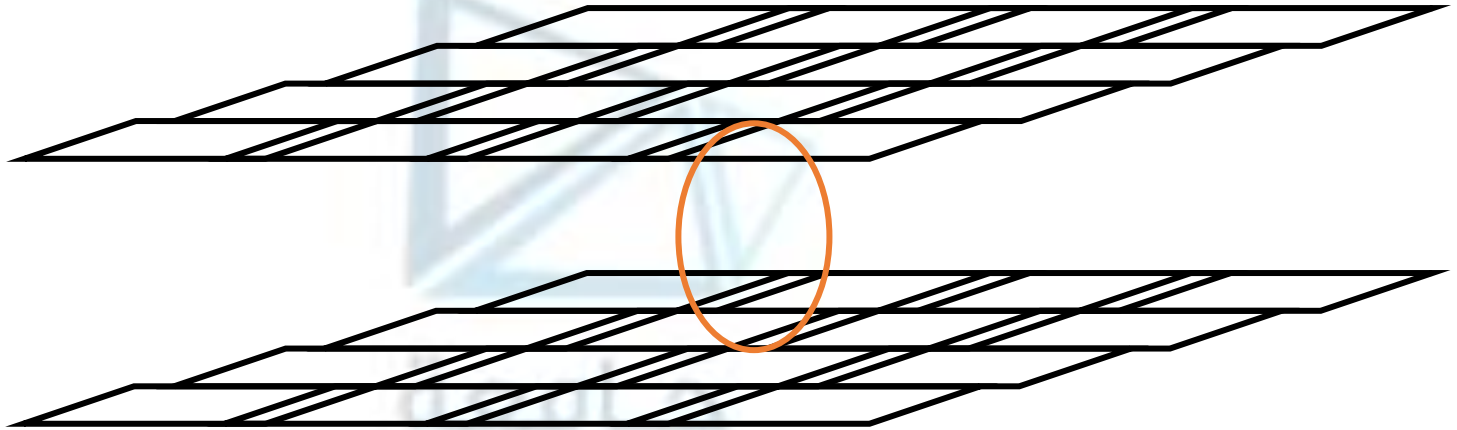
Five-variable Map



Five-variable Map

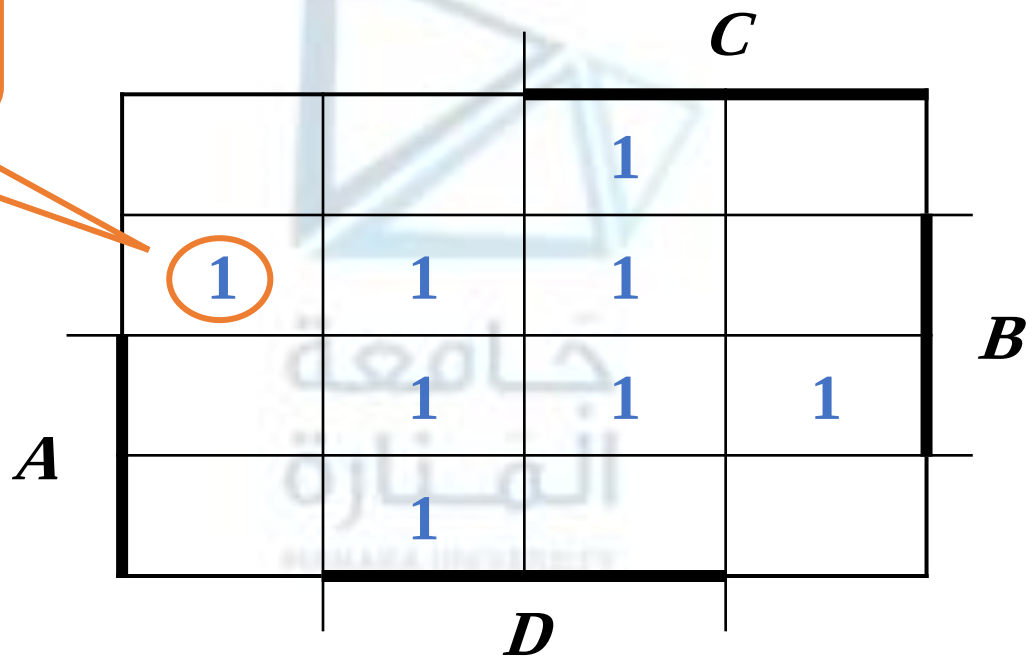
$A = 0$

$A = 1$



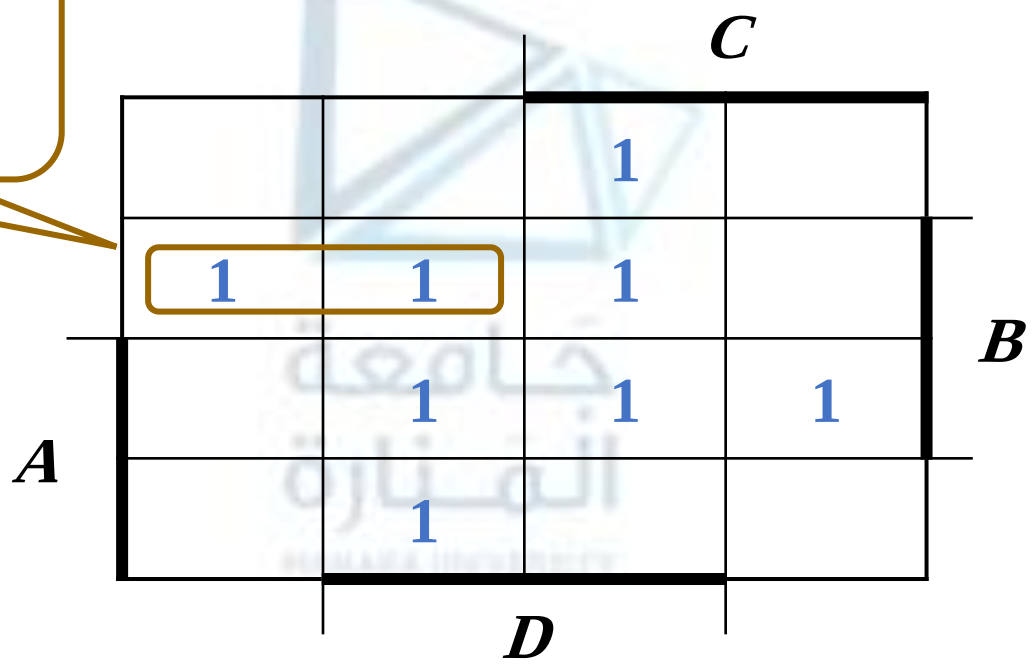
Implicants

Implicant:
Gives $F = 1$



Prime Implicants

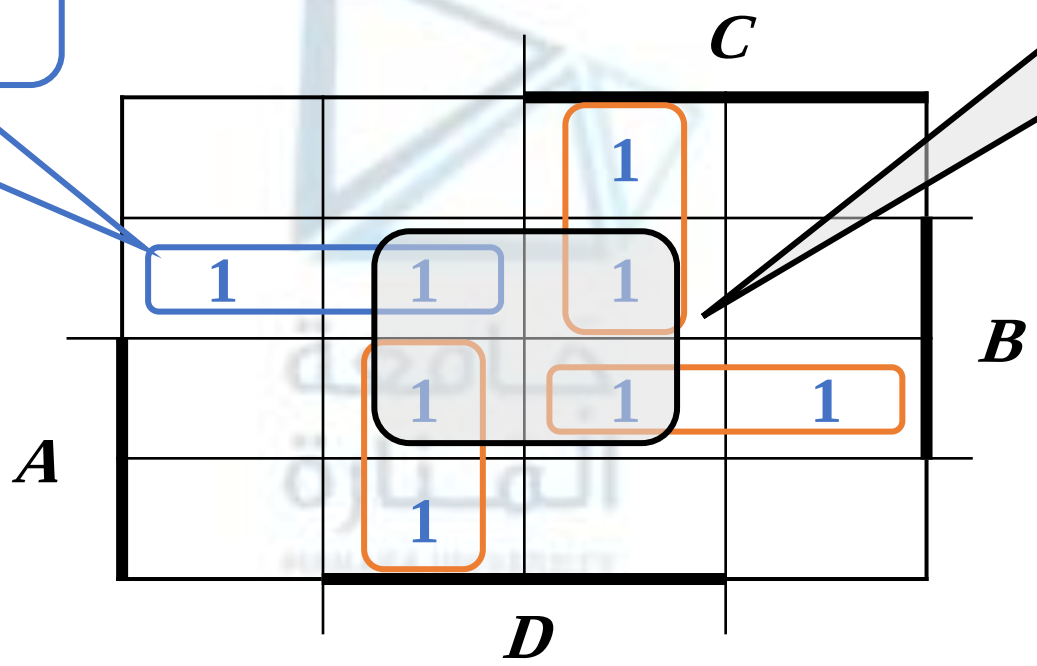
Prime Implicant:
Can’t grow beyond this size



Essential Prime Implicants

Essential Prime Implicant:
No other choice

Not essential



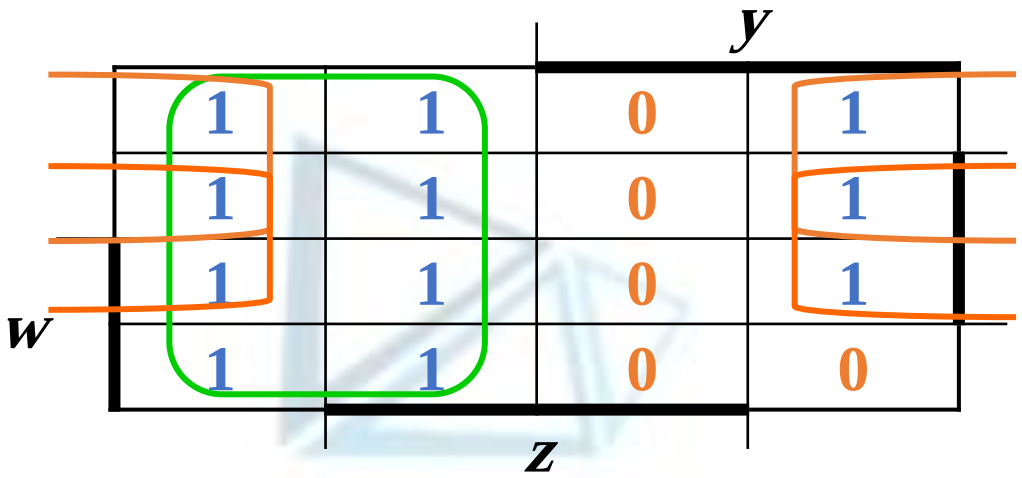
8 Implicants

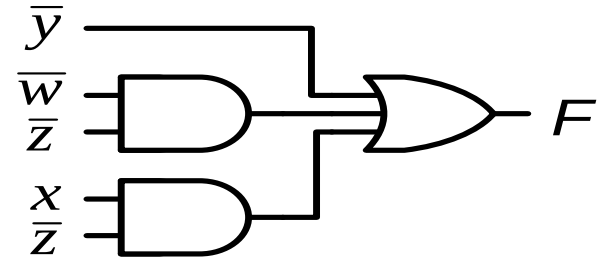
5 Prime implicants

4 Essential prime implicants

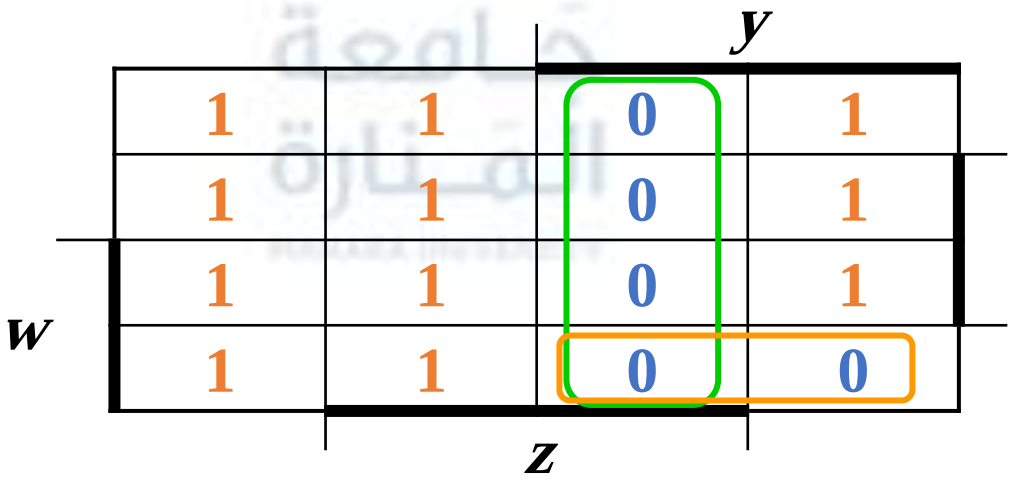
Product of Sums Simplification

	w x y z	F	$\overline{F}$
0	0 0 0 0	1	0
1	0 0 0 1	1	0
2	0 0 1 0	1	0
3	0 0 1 1	0	1
4	0 1 0 0	1	0
5	0 1 0 1	1	0
6	0 1 1 0	1	0
7	0 1 1 1	0	1
8	1 0 0 0	1	0
9	1 0 0 1	1	0
10	1 0 1 0	0	1
11	1 0 1 1	0	1
12	1 1 0 0	1	0
13	1 1 0 1	1	0
14	1 1 1 0	1	0
15	1 1 1 1	0	1



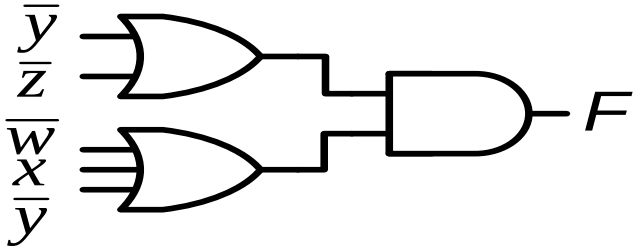


$$F = \bar{y} + \bar{w}\,\bar{z} + x\,\bar{z}$$



$$\overline{F} = y\,z + w\,\bar{x}\,y$$

$$\overline{\overline{F}} = \overline{y\,z + w\,\bar{x}\,y}$$



$$F = (\bar{y} + \bar{z}) \bullet (\bar{w} + x + \bar{y})$$

Don’t-Care Condition

- Example

$$A = \begin{cases} 1 & \text{if a quarter is deposited} \\ 0 & \text{otherwise} \end{cases}$$

$$B = \begin{cases} 1 & \text{if a dime is deposited} \\ 0 & \text{otherwise} \end{cases}$$

$$C = \begin{cases} 1 & \text{if a nicle is deposited} \\ 0 & \text{otherwise} \end{cases}$$

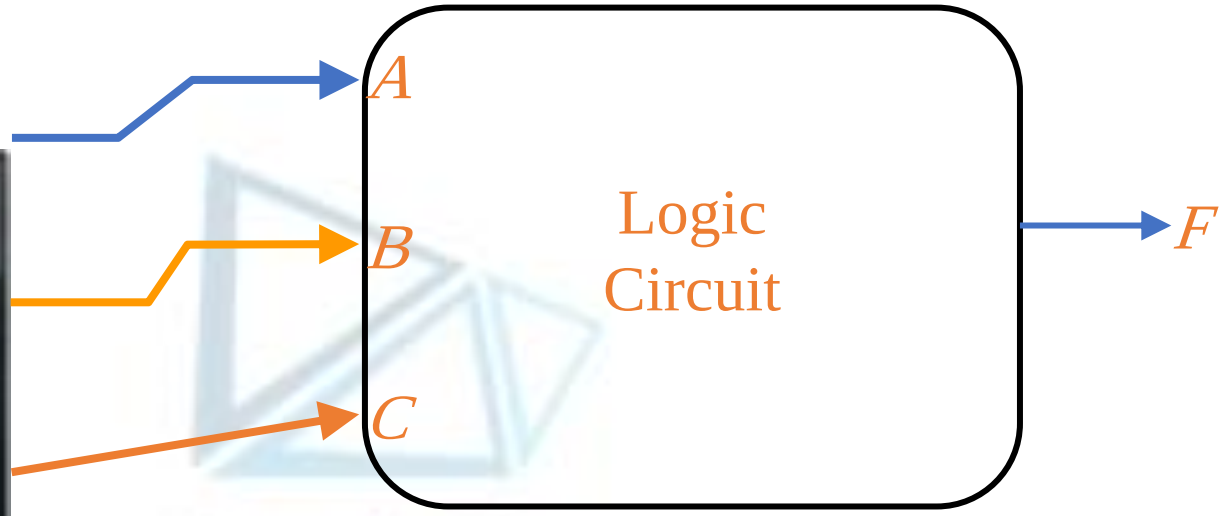
A	B	C	$ Value
0	0	0	$ 0.00
0	0	1	$ 0.05
0	1	0	$ 0.10
0	1	1	Not possible
1	0	0	$ 0.25
1	0	1	Not possible
1	1	0	Not possible
1	1	1	Not possible

You can only drop one coin at a time.

Used as “don’t care”

Don’t-Care Condition

- Example



A B C	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	x
1 0 0	1
1 0 1	x
1 1 0	x
1 1 1	x

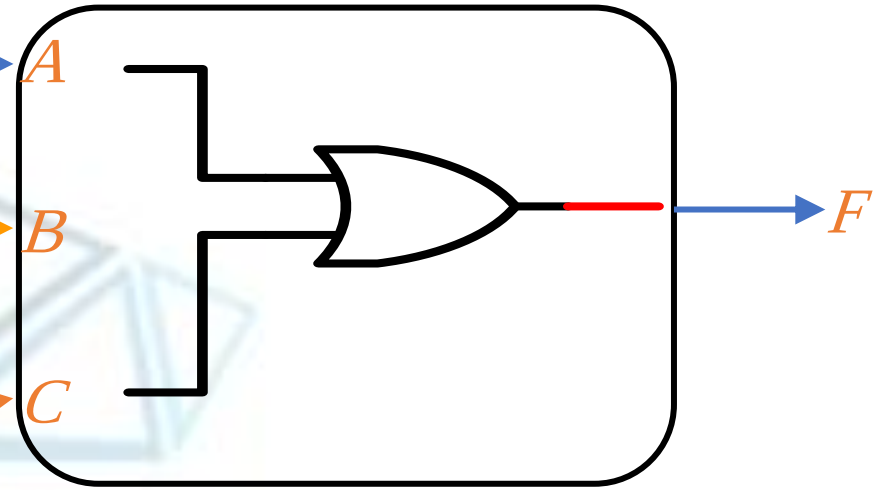
$$F(A,B,C) = \sum(1, 4)$$

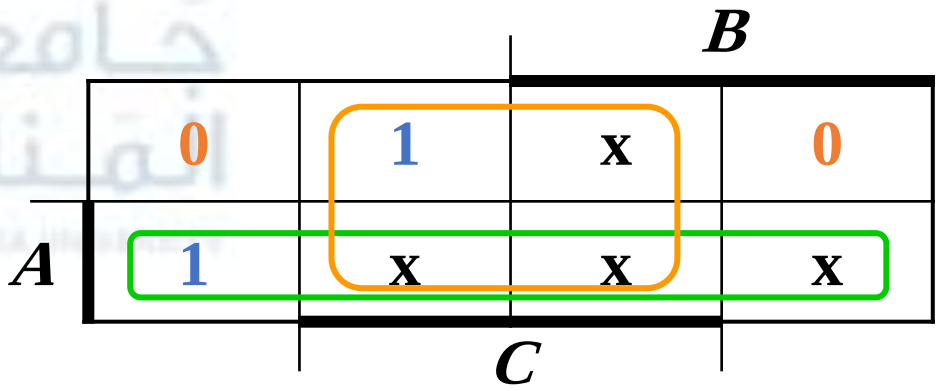
$$d(A,B,C) = \sum(3, 5, 6, 7)$$

Don’t care what value F may take

Don’t-Care Condition

- Example





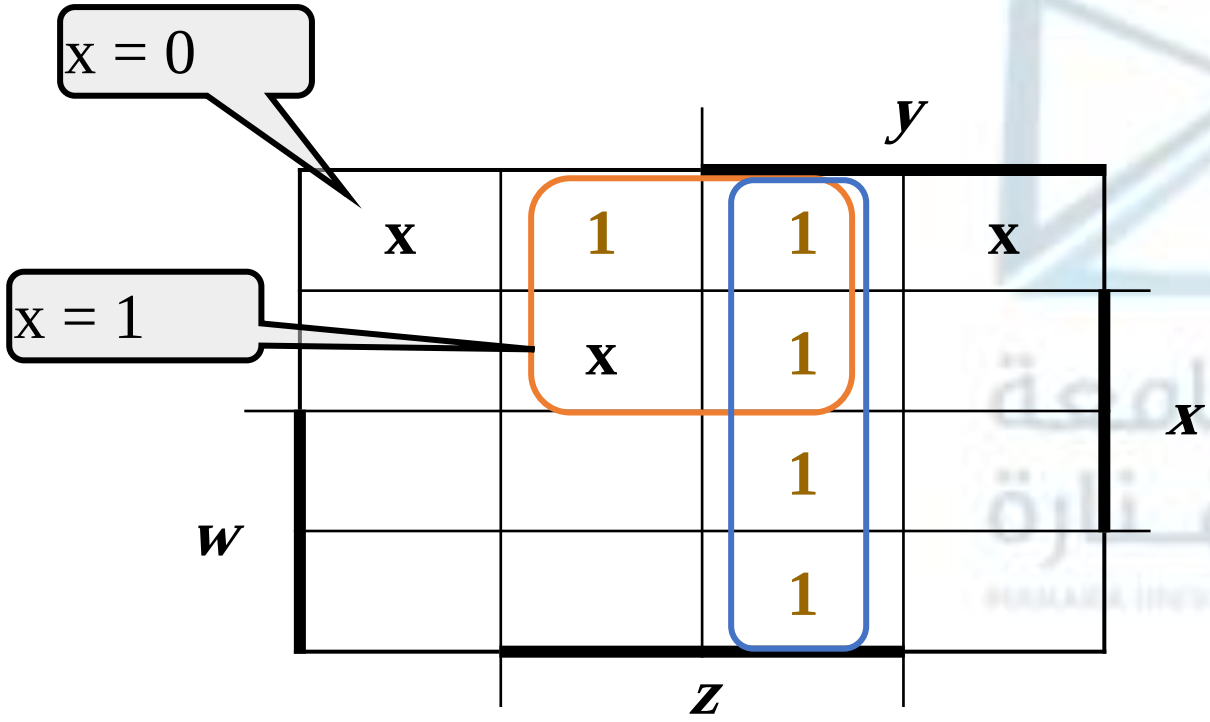
$$F = \overline{A}\overline{B}C + A\overline{B}\overline{C}$$

$$F = A + C$$

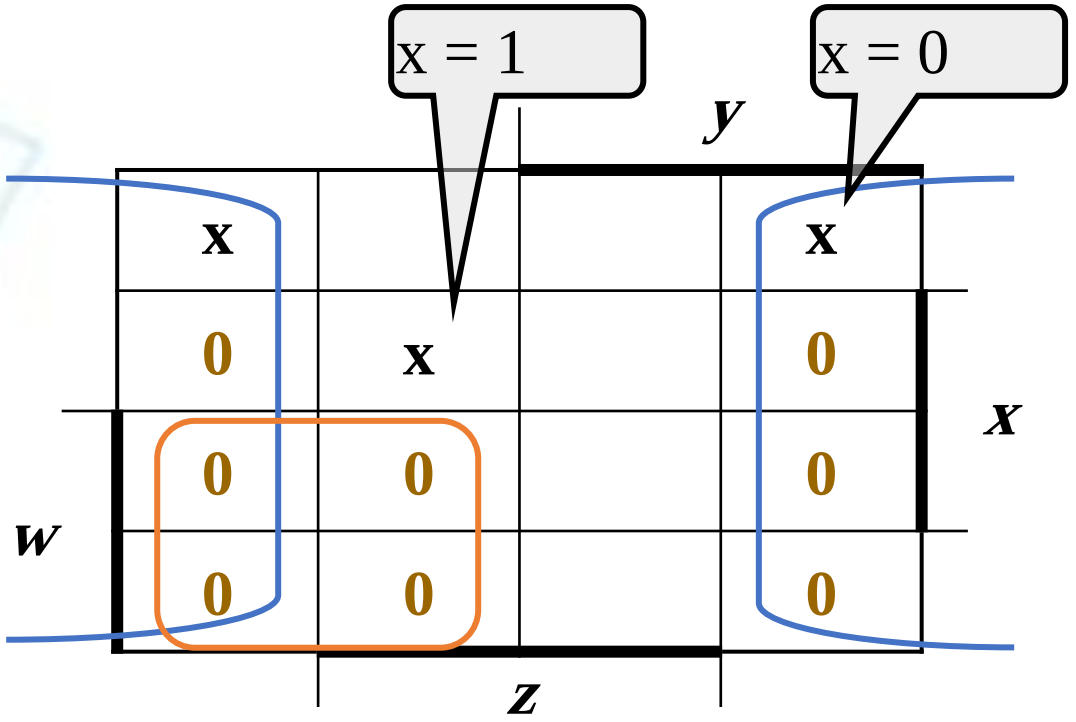
Don’t-Care Condition

- Example

$$F(w, x, y, z) = \sum(1, 3, 7, 11, 15)$$
$$d(w, x, y, z) = \sum(0, 2, 5)$$



$$F = y\,z + \overline{w}\,z$$



$$\overline{F} = \overline{z} + w\,\overline{y}$$

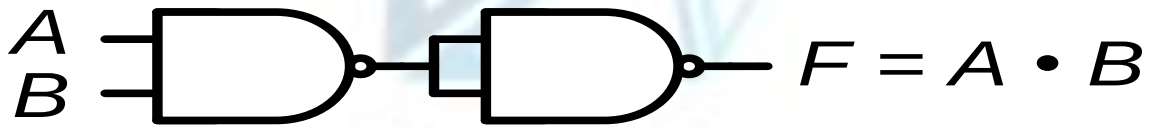
Universal Gates

- One Type
  - Use as many as you need (quantity), but one type only.
- Perform Basic Operations
  - AND, OR, and NOT
- NAND Gate
  - NOT-AND functions
  - OR function can be obtained from AND by Demorgan’s
- NOR Gate
  - NOT-OR functions (AND by Demorgan’s)

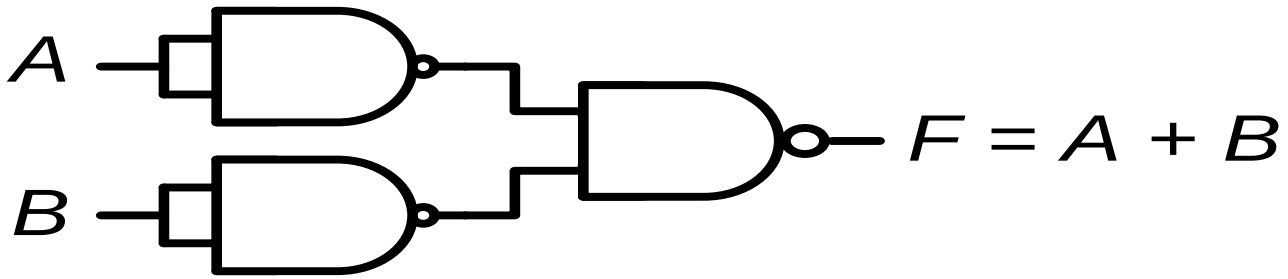
Universal Gates

- **NAND Gate**
  - NOT:

  - AND:



  - OR:

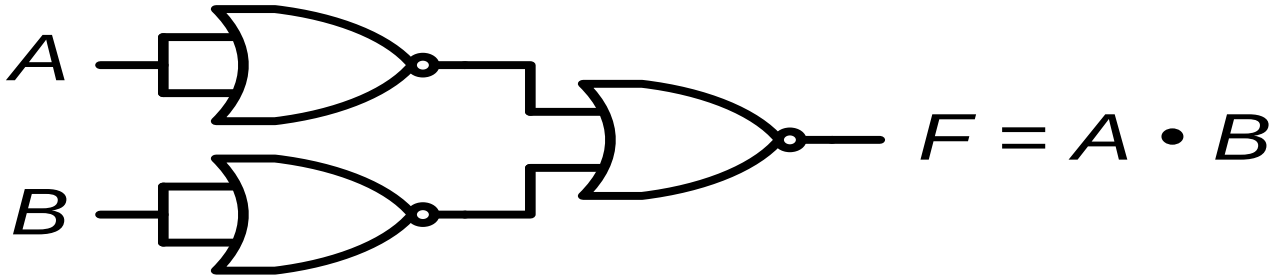


Universal Gates

- NOR Gate
  - NOT:

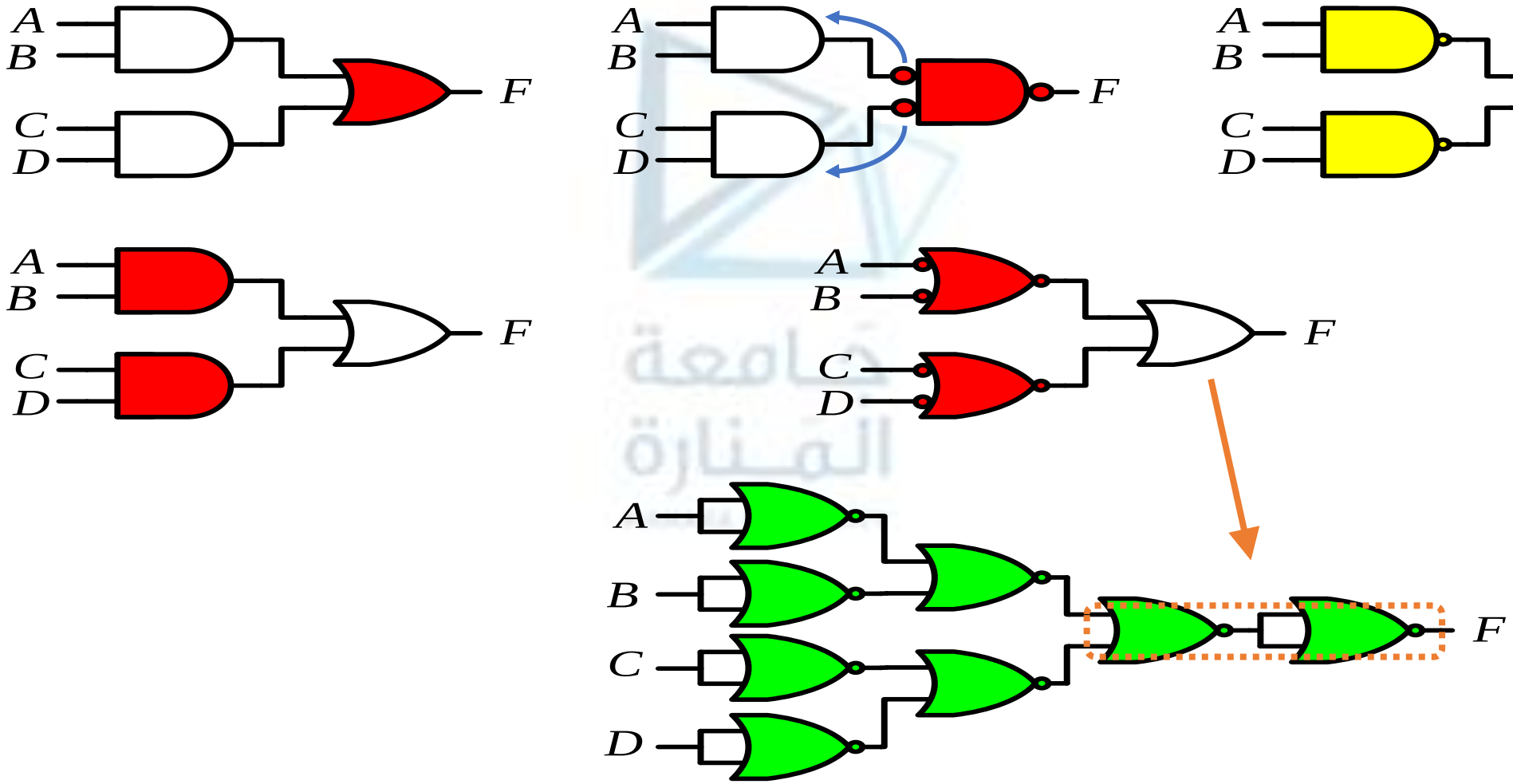
  - OR:

  - AND:



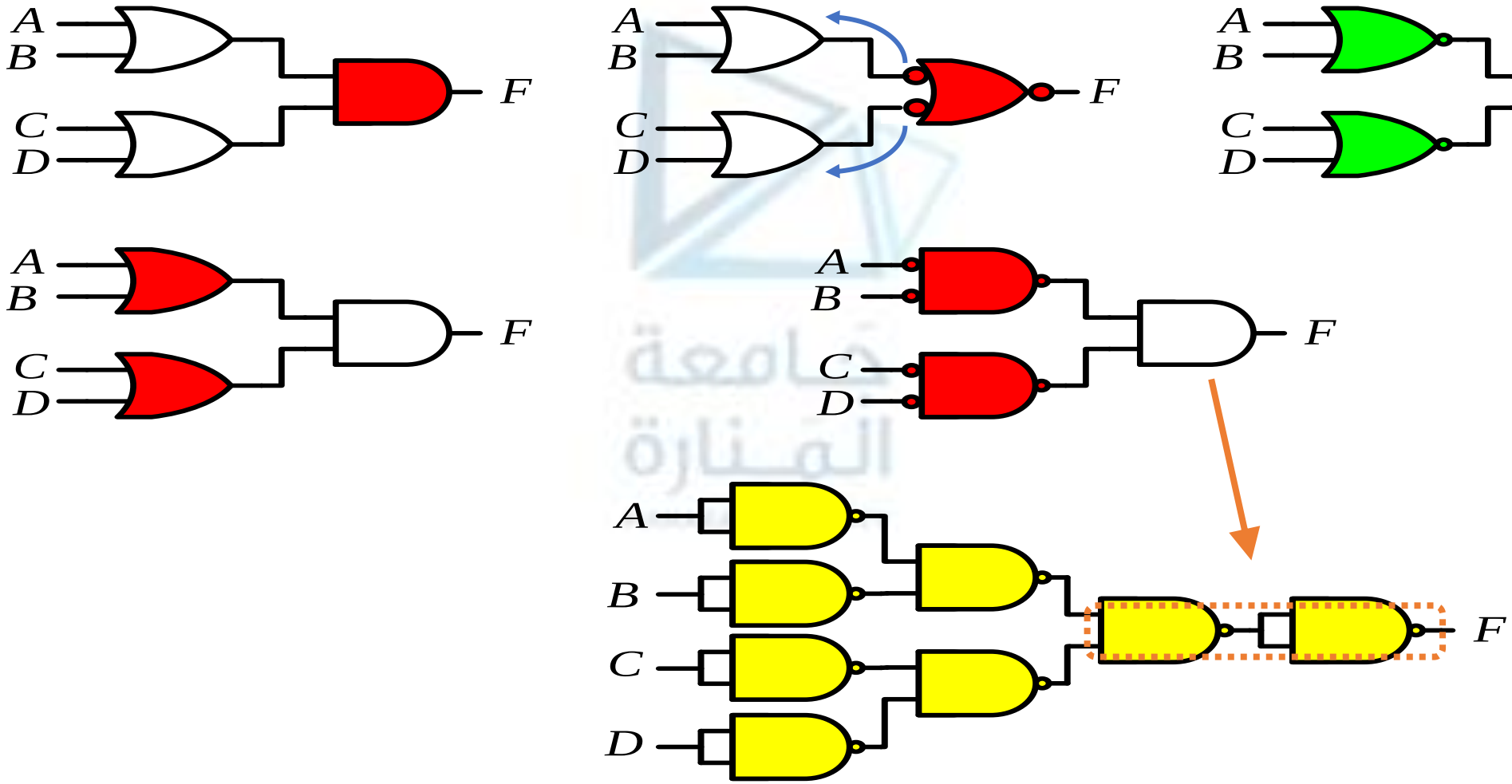
NAND & NOR Implementation

- Two-Level Implementation



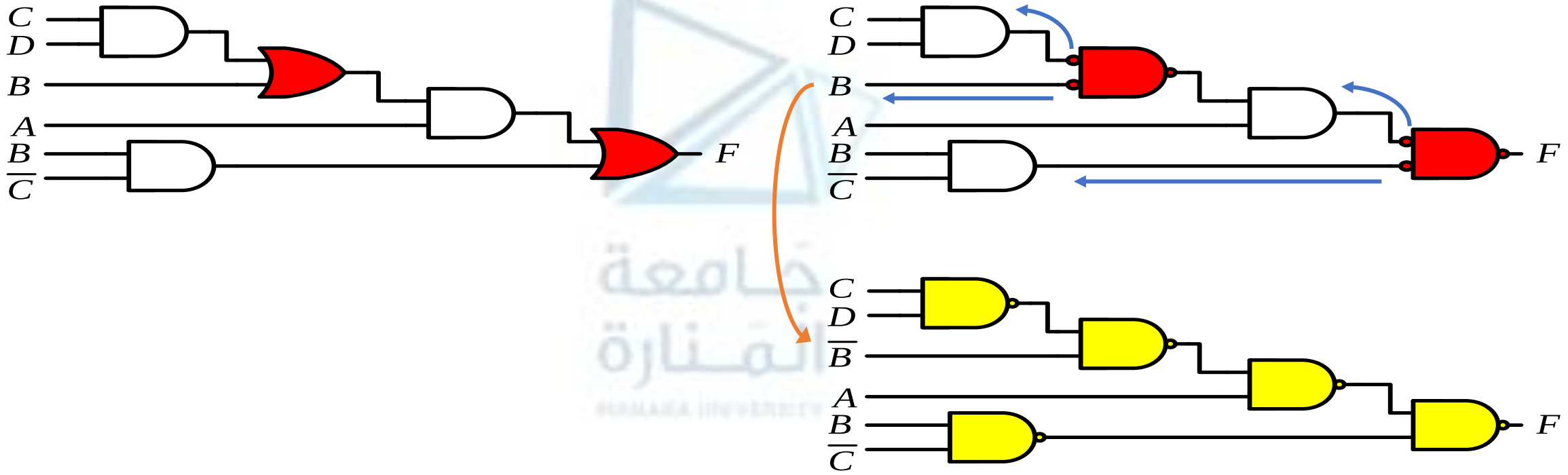
NAND & NOR Implementation

- Two-Level Implementation



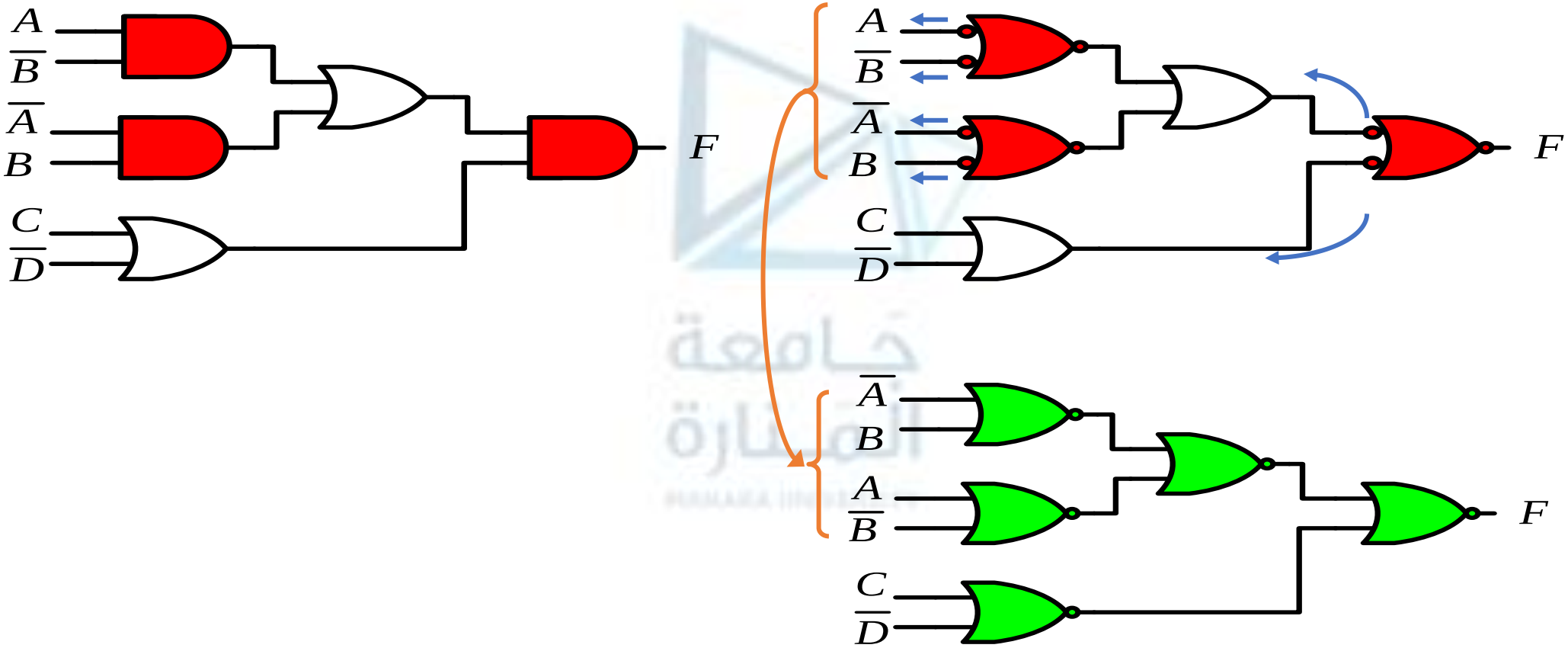
NAND & NOR Implementation

- Multilevel NAND Implementation



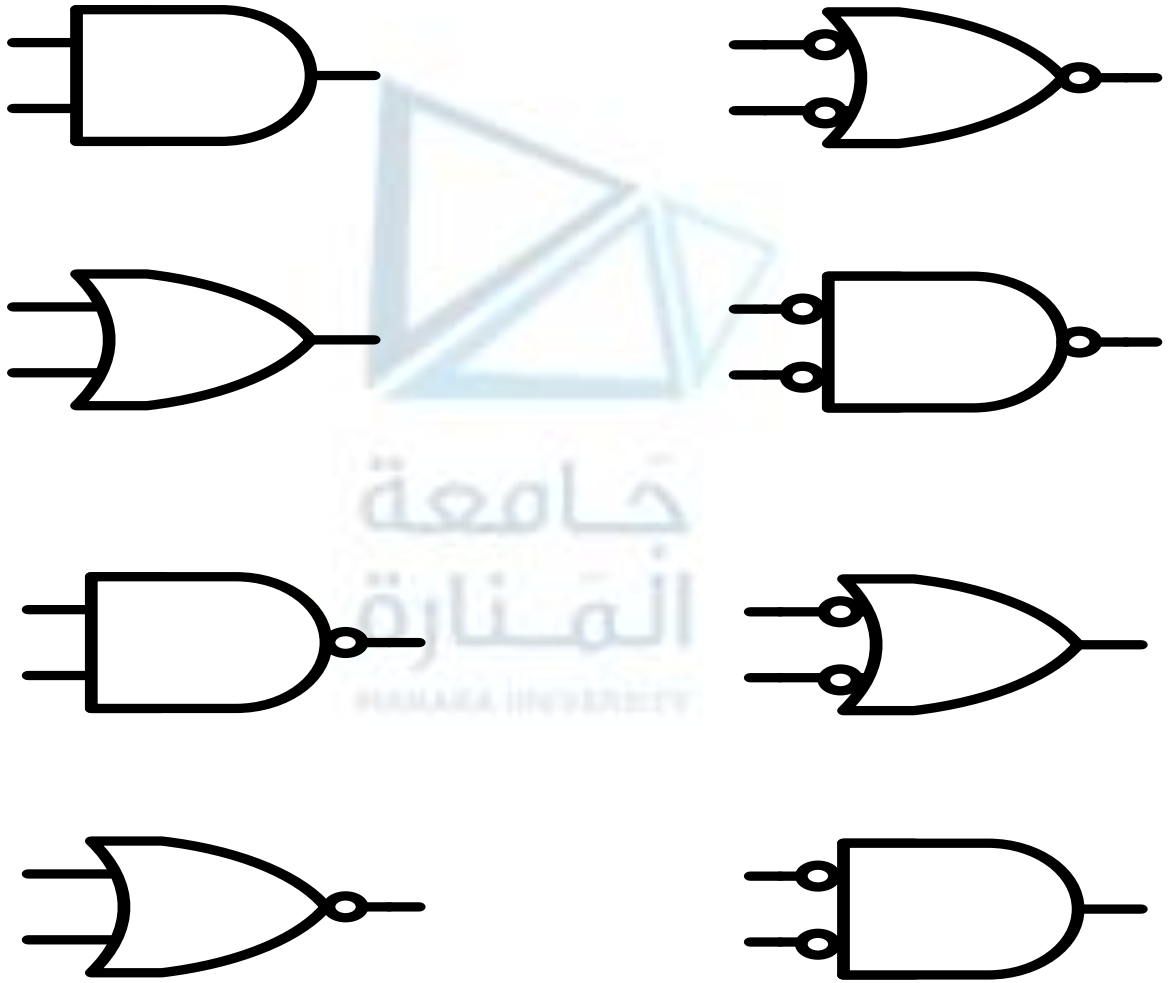
NAND & NOR Implementation

- Multilevel NOR Implementation



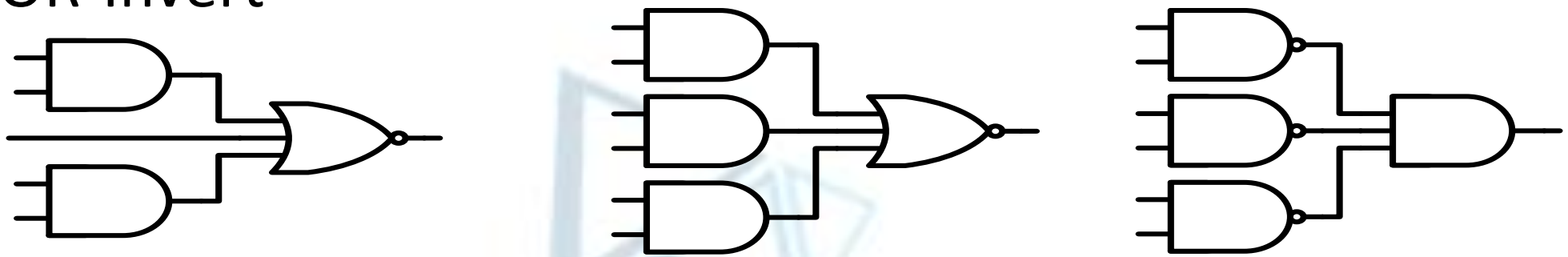
Gate Shapes

- AND
- OR
- NAND
- NOR

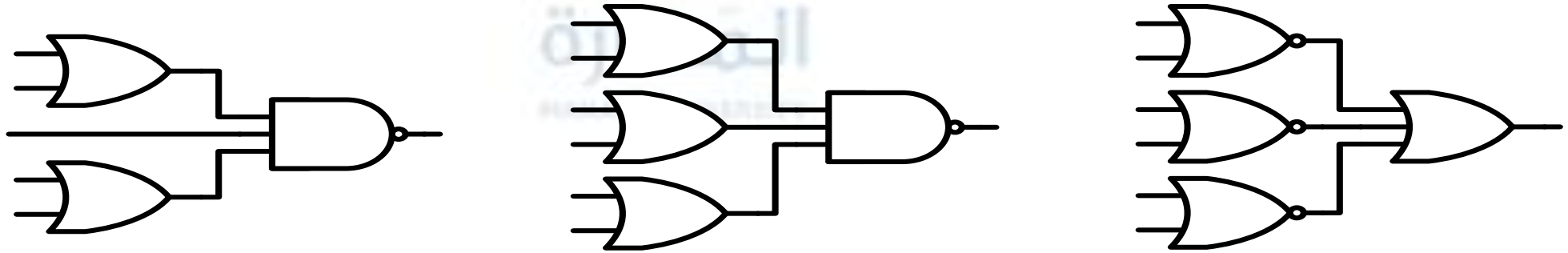


Other Implementations

- AND-OR-Invert



- OR-AND-Invert



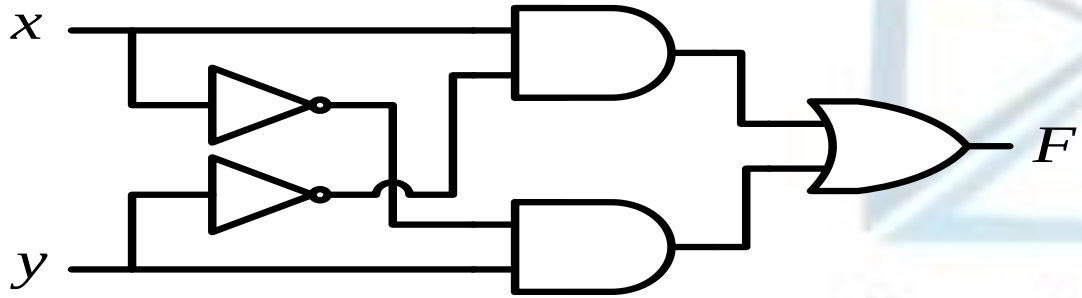
Implementations Summary

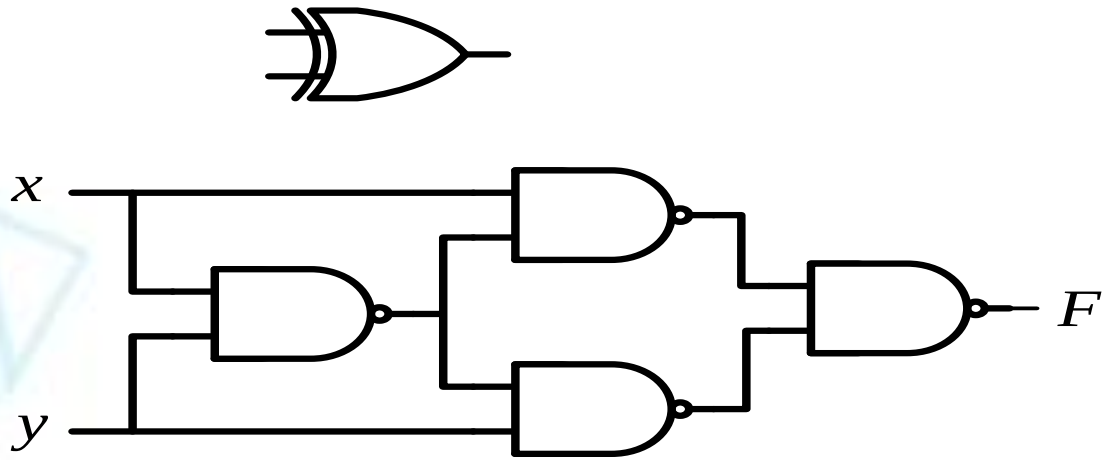
- Sum Of Products:
  - AND-OR
  - AND-OR-Invert = AND-NOR = NAND-AND
- Products Of Sums
  - OR-AND
  - OR-AND-Invert = OR-NAND = NOR--OR

Exclusive-OR

- XOR

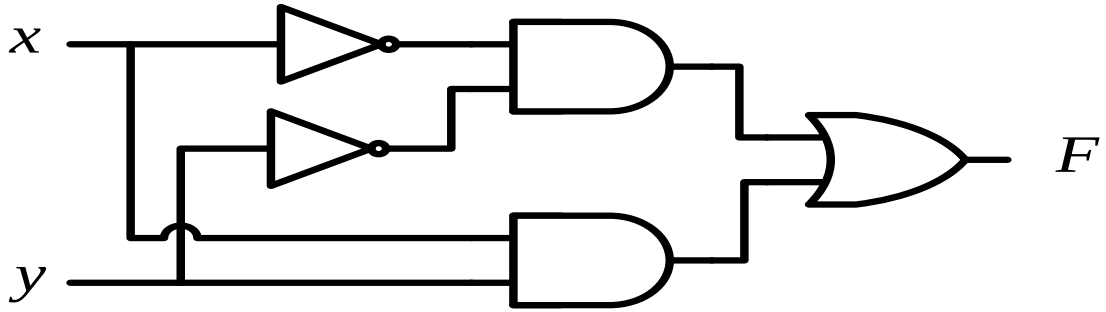
$F = x \oplus y = x\overline{y} + \overline{x}y$

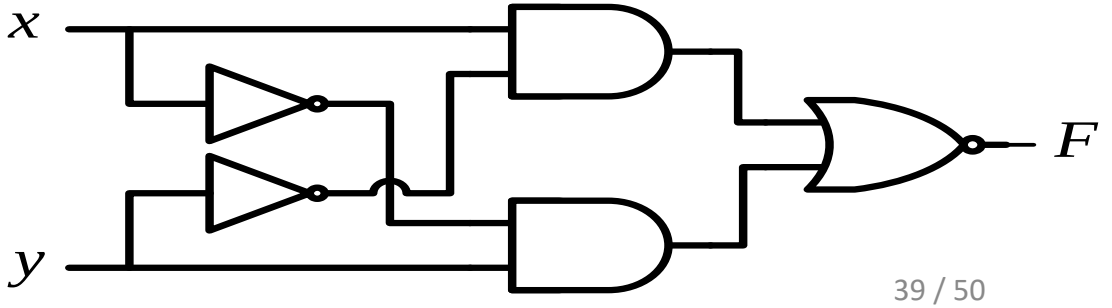




- XNOR

$F = \overline{x \oplus y} = x \odot y = \overline{x}\,\overline{y} + x\,y$





Exclusive-OR

- Identities
  - $x \oplus 0 = x$
  - $x \oplus 1 = \overline{x}$
  - $x \oplus x = 0$
  - $x \oplus \overline{x} = 1$
  - $x \oplus \overline{y} = \overline{x} \oplus y = \overline{x \oplus y}$
- Commutative & Associative
  - $x \oplus y = y \oplus x$
  - $(x \oplus y) \oplus z = x \oplus (y \oplus z) = x \oplus y \oplus z$

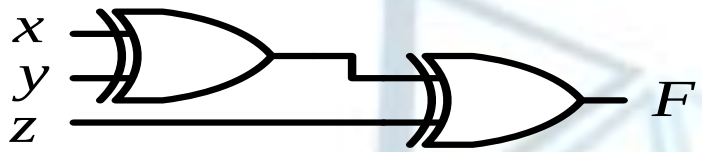
x	*y*	*XOR*
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive-OR Functions

- Odd Function

  $F = x \oplus y \oplus z$

  $F = \sum(1, 2, 4, 7)$

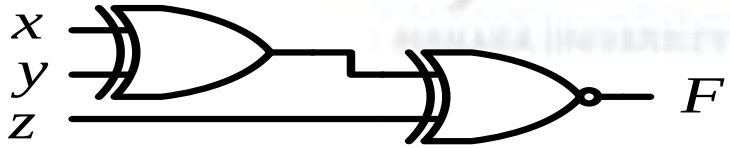


x	y	z	XOR	XNOR
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

- Even Function

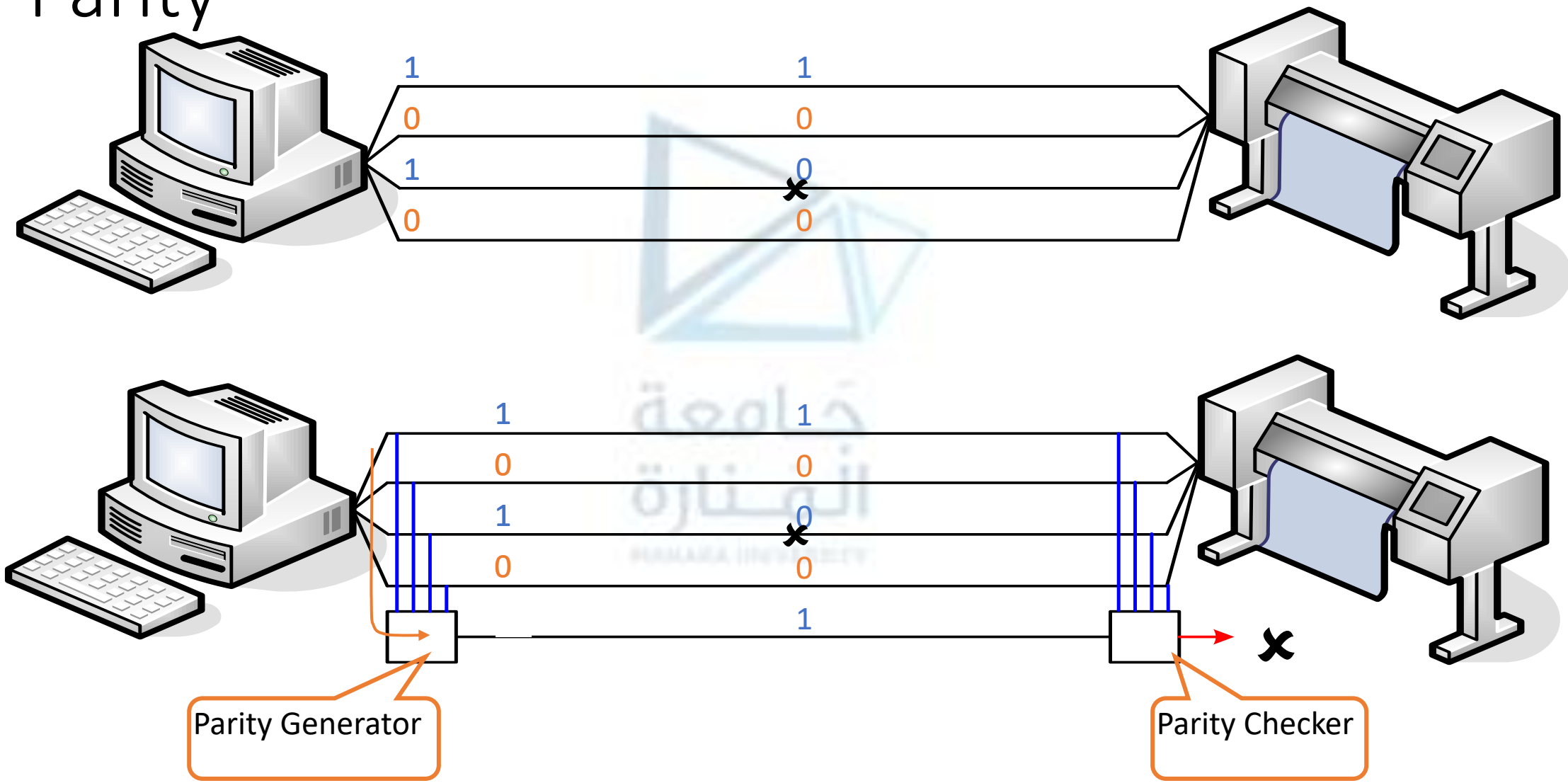
  $F = \overline{x \oplus y \oplus z}$

  $F = \sum(0, 3, 5, 6)$



x \ yz	00	01	11	10
0	0	1	0	1
1	1	0	1	0

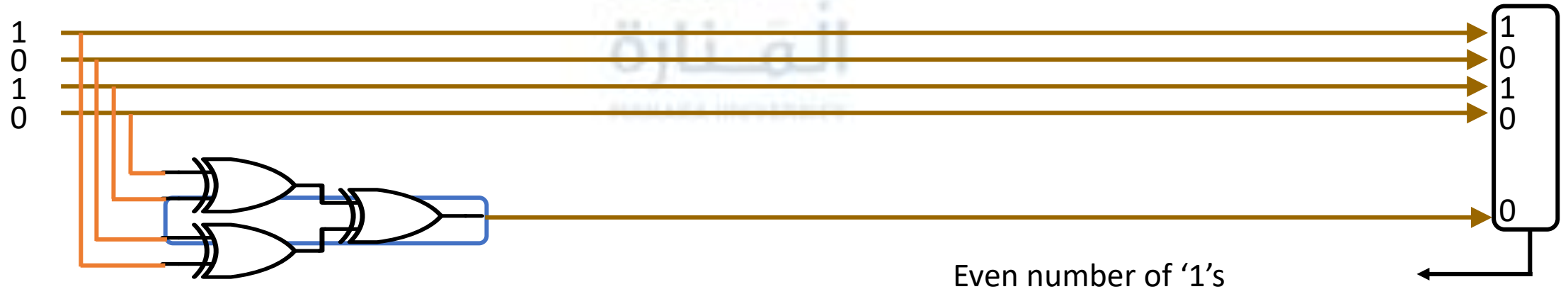
Parity



Parity Generator

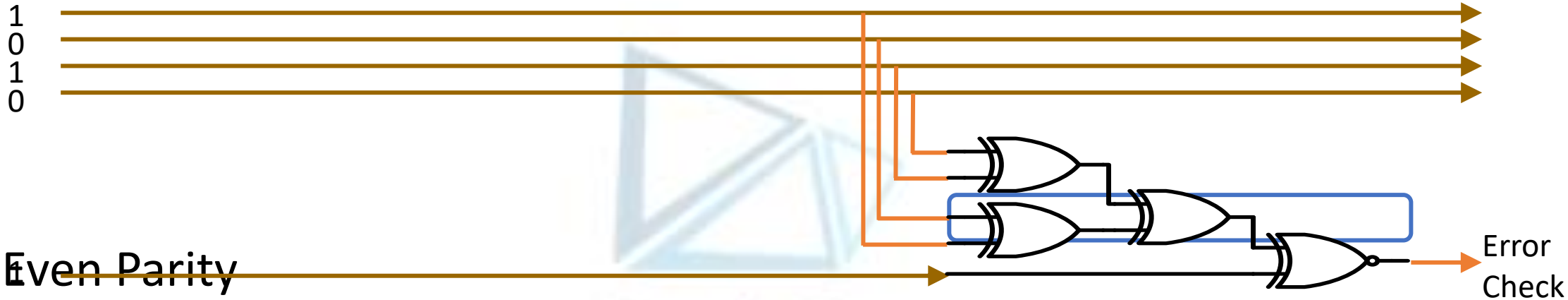
- Odd Parity

- Even Parity

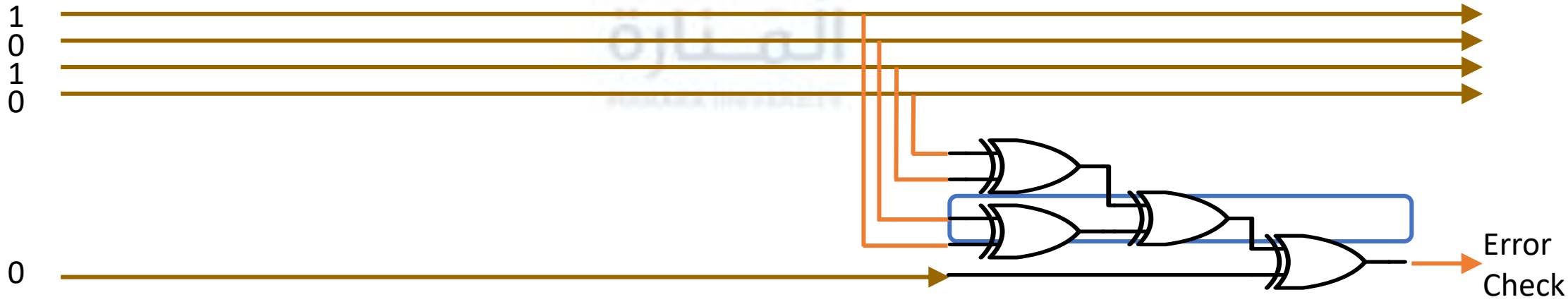


Parity Checker

- Odd Parity



- Even Parity



Homework

- Mano
  - Chapter 3
    - 3-1
    - 3-3
    - 3-5
    - 3-7
    - 3-9
    - 3-15
    - 3-16
    - 3-18
    - 3-22

Homework

- Mano

3-1 **Simplify the following Boolean functions, using three-variable maps:**

(a) $F(x, y, z) = \sum(0, 2, 6, 7)$

(b) $F(A, B, C) = \sum(0, 2, 3, 4, 6)$

(c) $F(a, b, c) = \sum(0, 1, 2, 3, 7)$

(d) $F(x, y, z) = \sum(3, 5, 6, 7)$

Homework

3-3 **Simplify the following Boolean functions, using three-variable maps:**

(a) $xy + x'y'z' + x'yz'$

(b) $x'y' + xz + x'yz'$

(c) $A'B + BC' + B'C'$

3-5 **Simplify the following Boolean functions, using four-variable maps :**

(a) $F(w, x, y, z) = \sum(1, 4, 5, 6, 12, 14, 15)$

(b) $F(A, B, C, D) = \sum(0, 1, 2, 4, 5, 7, 11, 15)$

(c) $F(w, x, y, z) = \sum(2, 3, 10, 11, 12, 13, 14, 15)$

(d) $F(A, B, C, D) = \sum(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

Homework

3-7 **Simplify the following Boolean functions, using four-variable maps:**

(a) $w'z + xz + x'y + wx'z$ **(b)** $B'D + A'BC' + AB'C + ABC'$

(c) $AB'C + B'C'D' + BCD + ACD' + A'B'C + A'BC'D$

(d) $wxy + yz + xy'z + x'y$

3-9 **Find the prime implicants for the following Boolean functions, and determine which are essential:**

(a) $F(w, x, y, z) = \sum(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

(b) $F(A, B, C, D) = \sum(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

(c) $F(A, B, C, D) = \sum(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

Homework

3-15 **Simplify the following Boolean function *F*, together with the don't-care conditions *d*, and then express the simplified function in sum of products:**

(a) $F(x, y, z) = \sum(0, 1, 2, 4, 5)$

$d(x, y, z) = \sum(3, 6, 7)$

(b) $F(A, B, C, D) = \sum(0, 6, 8, 13, 14)$

$d(A, B, C, D) = \sum(2, 4, 10)$

(c) $F(A, B, C, D) = \sum(1, 3, 5, 7, 9, 15)$

$d(A, B, C, D) = \sum(4, 6, 12, 13)$

Homework

3-16 **Simplify the following expressions, and implement them with two-level NAND gate circuits:**

(a) $AB' + ABD + ABD' + A'C'D' + A'BC'$

(b) $BD + BCD' + AB'C'D'$

3-18 **Draw a logic diagram using only two-input NAND gates to implement the following expression:**

$$(AB + A'B')(CD' + C'D)$$

Homework

3-22 **Convert the logic diagram of the circuit shown in Fig. 4-4 into a multiple-level NAND circuit.**