Portfolio Management

Manara University
Department of Banking and Finance
Second Term
2021-2022

Dr Hayan Omran

The Characteristics of the Opportunity Set under Risk

- The existence of risk means that the investor can no longer associate a single number or payoff with investment in any asset. The payoff must be described by a set of outcomes and each of their associated probabilities of occurrence, called a frequency function or return distribution. In this section, we start by examining the two most frequently employed attributes of such a distribution: a measure of central tendency, called the expected return, and a measure of risk or dispersion around the mean, called the standard deviation. Investors should not and, in fact, do not hold single assets; they hold groups or portfolios of assets.

جامعة المنارة
MANARA UNIVERSITY

- In this section, we are concerned with how one can compute the expected return and risk of a portfolio of assets given the attributes of the individual assets. One important aspect of this analysis is that the risk on a portfolio is more complex than a simple average of the risk on individual assets. It depends on whether the returns on individual assets tend to move together or whether some assets give good returns when others give bad returns. Actually there is a risk reduction from holding a portfolio of assets if assets do not move in perfect unison.

- In the certainty case, the investor's decision problem can be characterized by a certain outcome. For example, the 5% return on lending (or the 5% cost of borrowing) was known with certainty. Under risk, the outcome of any action is not known with certainty, and outcomes are usually represented by a frequency function. A frequency function is a listing of all possible outcomes along with the probability of the occurrence of each. Table 4.1 shows such a function.

- Table 4.1 Data on Three Hypothetical Events

Return	Probability	Event
12	$\frac{1}{3}$	1
9	$\frac{1}{3}$	2
6	$\frac{1}{3}$	3

- This investment has three possible returns. If event 1 occurs, the investor receives a return of 12%; if event 2 occurs, 9% is received; and if event 3 occurs, 6% is received. In our examples each of these events is assumed to be equally likely. Table 4.1 shows us everything there is to know about the return possibilities.

Determining the average outcome

- The concept of an average is intuitive. If someone earns $11,000 one year and $9,000 in a second, we say his average income in the two years is $10,000. If three children in a family are age 15, 10, and 5, then we say the average age is 10. In Table 4.1 the average return was 9%. Statisticians usually use the term *expected value* to refer to what is commonly called an average.

- We have for the expected value of the M equally likely returns for asset *i*:

$$\bar{R}_i = \sum_{j=1}^{M} \frac{R_{ij}}{M}$$

- If the outcomes are not equally likely and if P_{ij} is the probability of the jth return on the ith asset, then expected return is:

$$\bar{R}_i = \sum_{j=1}^{M} P_{ij} R_{ij}$$

- Certain properties of expected value are extremely useful:

1. The expected value of the sum of two returns is equal to the sum of the expected value of each return, that is:

$$E\left(R_{1j} + R_{2j}\right) = \overline{R_1} + \overline{R_2}$$

2. The expected value of a constant C times a return is the constant times the expected return, that is:

$$E\left(\left[C(R_{1j})\right]\right) = C\overline{R_1}$$

- These principles are illustrated in Table 4.2. For any event, the return on asset 3 is the sum of the return on assets 1 and 2. Thus the expected value of the return on asset 3 is the sum of the expected value of the return on assets 1 and 2. Likewise, for any event, the return on asset 3 is 3 times the return on asset 1. Consequently, its expected value is 3 times as large as the expected value of asset 1.
- Table 4.2 Return on Various Assets

Event	Probability	Asset 1	Asset 2	Asset 3
A	$\frac{1}{3}$	14	28	42
B	$\frac{1}{3}$	10	20	30
C	$\frac{1}{3}$	$\underline{6}$	$\underline{12}$	$\underline{18}$
	Expected return	10	20	30

A measure of dispersion

- Not only is it necessary to have a measure of the average return but it is also useful to have some measure of how much the outcomes differ from the average.
- It is noteworthy to mention that the average squared deviation is called the *variance*; the square root of the variance is called the *standard deviation*. In Table 4.3 we present the possible returns from several hypothetical assets as well as the variance of the return on each asset. The alternative returns on any asset are assumed equally likely. Examining asset 1, we find the deviations of its returns from its average return are (15 - 9), (9 - 9), and (3 - 9). The squared deviations are 36, 0, and 36, and the average squared deviation or variance is (36 + 0 + 36)/3 = 24.

- To be precise, the formula for the variance of the return on the ith asset (which we symbolize as σ_i^2) when each return is equally likely is:

$$\sigma_i^2 = \sum_{j=1}^{M} \frac{(R_{ij} - \bar{R}_i)^2}{M}$$

- Table 4.3 Returns on Various Investments

Market Condition	Return					Return
	Asset 1	Asset 2	Asset 3	Asset 5	Rainfall	Asset 4
Good	15	16	1	16	Plentiful	16
Average	9	10	10	10	Average	10
Poor	3	4	19	4	Poor	4
Mean return	9	10	10	10		10
Variance	24	24	54	24		24
Standard deviation	4.9	4.9	7.35	4.90		4.9

- The alternative returns on each asset are assumed equally likely, and thus each has a probability of $\frac{1}{3}$

- If the observations are not equally likely, then, as before, we multiply by the probability with which they occur. The formula for the variance of the return on the ith asset becomes:

$$\sigma_i^2 = \sum_{j=1}^{M} \left[P_{ij} \left(R_{ij} - \bar{R}_i \right)^2 \right]$$

- Occasionally, it is convenient to employ an alternative measure of dispersion called standard deviation. The standard deviation is just the square root of the variance and is designated by σ_i.
- The variance tells us that asset 3 varies considerably more from its average than asset 2. This is what we intuitively see by examining the returns shown in Table 4.3. The expected value and variance or standard deviation are the usual summary statistics utilized in describing a frequency distribution.