Portfolio Management

Manara University
Department of Banking and Finance

Second Term

2021-2022

Dr Hayan Omran

- We begin the problem of simplifying the inputs to the portfolio problem. We start with a discussion of the amount and type of information needed to solve a portfolio problem. We then discuss the oldest and most widely used simplification of the portfolio structure: the single-index model. The nature of the model as well as some estimating techniques are examined.

The inputs to portfolio analysis

- Let us return to a consideration of the portfolio problem. First, we should be able to determine the expected return and standard deviation of return on a portfolio. We can write the expected return on any portfolio as:

$$\bar{R}_P = \sum_{i=1}^{N} X_i \bar{R}_i \quad (1)$$

- While the standard deviation of return on any portfolio can be written as:

$$\sigma_P = \left[\sum_{i=1}^{N} X_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} X_iX_j\sigma_i\sigma_j\rho_{ij}\right]^{1/2}$$

(2)

- These equations define the input data necessary to perform portfolio analysis.
- From Equation (1) we see that we need estimates of the expected return on each security that is a candidate for inclusion in our portfolio.
- From Equation (2) we see that we need estimates of the variance of each security, plus estimates of the correlation between each possible pair of securities for the stocks under consideration. The need for estimates of correlation coefficients differs from the two previous requirements.

- The principal job of the security analyst traditionally has been to estimate the future performance of stocks he follows. At a minimum, this means producing estimates of expected returns on each stock he follows.
- With the increased attention that “risk” has received in recent years, more and more analysts are providing estimates of risk as well as return. The analyst who estimates the expected return of a stock should also be in a position to estimate the uncertainty of that return.

- Correlations are an entirely different matter. Portfolio analysis calls for estimates of the pairwise correlation between all stocks that are candidates for inclusion in a portfolio.
- One analyst might follow steel stocks or, perhaps in a smaller firm, all metal stocks. A second analyst might follow chemical stocks. But portfolio analysis calls for these analysts not only to estimate how a particular steel stock will move in relationship to another steel stock but also how a particular steel stock will move in relationship to a particular chemical stock or drug stock.

- The problem is made more complex by the number of estimates required. Most financial institutions follow between 150 and 250 stocks. To employ portfolio analysis, the institution needs estimates of between 150 and 250 expected returns and 150 and 250 variances.
- Let us see how many correlation coefficients it needs. If we let *N* stand for the number of stocks a firm follows, then it has to estimate ρ_{ij} for all pairs of securities i and j. The first index i can take on *N* values (one for each stock); the second can take on (*N*-1) values (remember $j \neq i$). This gives us *N*(*N*-1) correlation coefficients. However, because the correlation coefficient between stocks i and j is the same as that between stocks j and i, we have to estimate only *N*(*N*-1)/2 correlations. The institution that follows between 150 and 250 stocks needs between 11,175 and 31,125 correlation coefficients.

- It seems unlikely that analysts will be able to directly estimate correlation structures. Their ability to do so is severely limited by the nature of feasible organizational structures and the huge number of correlation coefficients that must be estimated. Recognition of this has motivated the search for the development of models to describe and predict the correlation structure between securities.
- The models developed for forecasting correlation structures fall into two categories: index models and averaging techniques. The most widely used technique assumes that the co-movement between stocks is due to a single common influence or index. This model is appropriately called the *single-index model*.

Single-index models: An overview

- Casual observation of stock prices reveals that when the market goes up (as measured by any of the widely available stock market indexes), most stocks tend to increase in price, and when the market goes down, most stocks tend to decrease in price. This suggests that one reason security returns might be correlated is because of a common response to market changes, and a useful measure of this correlation might be obtained by relating the return on a stock to the return on a stock market index. The return on a stock can be written as:

$$R_i = a_i + \beta_i R_m$$

- Where:
- a_i is the component of security i's return that is independent of the market's performance—a random variable.
- R_m is the rate of return on the market index—a random variable.
- β_i is a constant that measures the expected change in R_i given a change in R_m.

- This equation simply breaks the return on a stock into two components, that part due to the market and that part independent of the market. Variable β_i in the expression measures how sensitive a stock's return is to the return on the market. A β_i of 2 means that a stock's return is expected to increase (decrease) by 2% when the market increases (decreases) by 1%.

- The term a_i represents that component of return insensitive to (independent of) the return on the market. It is useful to break the term a_i into two components. Let α_i denote the expected value of a_i, and let e_i represent the random (uncertain) element of a_i. Then:

$$a_i = \alpha_i + e_i$$

- Where e_i has an expected value of zero. The equation for the return on a stock can now be written as:

$$R_i = \alpha_i + \beta_i R_m + e_i \quad (3)$$

- Up to this point we have made no simplifying assumptions. We have written return as the sum of several components, but these components, when added together, must by definition be equal to total return. It is convenient to have e_i uncorrelated with R_m. Formally, this means that:

$$cov(e_i R_m) = E[(e_i - 0)(R_m - \bar{R}_m)] = 0$$

- If e_i is uncorrelated with R_m, it implies that how well Equation (3) describes the return on any security is independent of what the return on the market happens to be.