Portfolio Management

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Selecting the Optimum Portfolio

- In this chapter, we will discuss in details the case of how to select among the portfolios in the opportunity set.

- The subject of this chapter is picking the optimum portfolio. In what follows, we discuss various techniques that have been proposed for selecting the optimum portfolio.

An introduction to preference functions

- We start our formal discussion of the choice between risky assets with a simple example. Consider the two alternatives shown in Table 11.2. Investment A and investment B each have three possible outcomes, each equally likely. Investment A has less variability in its outcomes but has a lower average outcome.
- One approach to choosing between them is to specify how much more valuable the large outcomes are relative to the small outcomes and then to weight the outcomes by their value and find the expected value of these weighted outcomes.
- The idea of adding up or averaging weighted outcomes is very common. Consider, for example, how the winning team is selected in hockey. Table 11.3 shows the hypothetical records for two hockey teams.

Table 11.2 Two Alternative Investments

Investment A		Investment B	
Outcome	Probability of Outcome	Outcome	Probability of Outcome
15	1/3	20	1/3
10	1/3	12	1/3
5	1/3	4	1/3

- Current practice weights wins by two, ties by one, and losses by zero. With this weighting scheme, the Islanders would be leading the Flyers 100 to 95. But there is nothing special about this weighting scheme. A league interested in deemphasizing the incentive for ties might weight wins by four, ties by one, and losses by zero. In this case, the Flyers would be considered the dominant team, 185 to 180. If we denote W as the result (win, tie, lose), $U(W)$ as the value of this result, and $N(W)$ as the number of times (games) that W occurs, then to determine the better team, we calculate:

$$\sum_{W} U(W)N(W)$$

Table 11.3 Data for Ranking Hockey Teams

	Islanders	Flyers
Wins	40	45
Ties	20	5
Losses	10	20

- The team with the higher U is considered the better team. For example, utilizing current practice, $U\ (win) = 2$, $U\ (tie) = 1$, and $U\ (loss) = 0$. Applying the formula to the Islanders yields:

$$U = 2(40) + 1(20) + 0(10) = 100$$

- This is the 100 we referred to earlier. While the particular function $U(W)$ differs between situations, the principle is the same. Traditionally, instead of using the number of outcomes of a particular type, the proportion is used.
- There were 70 hockey games in our example. If $P(W)$ is the proportion of the total games that resulted in outcome W, then $P(W) = N(W)/70$. Dividing through by 70 will not affect the ordering of *teams*. Weighting a function by the proportion of each outcome is equivalent to calculating an average or expected value. Letting $E(U)$ designate the expected value of U yields:

$$E(U) = \sum_{W} U(W)P(W)$$

- When we apply this principle to the decision problem shown in Table 11.2, we have special names for the principle. The weighting function is called a *utility function* and the principle is called the *expected utility theorem*. Consider the example shown in Table 11.2 and a set of weights as shown in Table 11.4.

Table 11.4 A Weighing Function

Outcome	Weight	Value of Outcome
20	0.9	18
15	1.0	15
12	1.1	13.2
10	1.2	12
5	1.4	7
4	1.5	6

[1] Should be read as the sum over all results.

- We have called the last column in the table the value of the outcome. Alternatively, it could be called the utility of an outcome. If this was the weighting function the investor felt was appropriate, then she would compare the expected utility of investments A and B using this function. For example, the expected utility of A is:

$$U(15)(1/3) + U(10)(1/3) + U(5)(1/3)$$

- Referring to the weighting function, we have:

$$(15)\left(\frac{1}{3}\right) + (12)\left(\frac{1}{3}\right) + (7)\left(\frac{1}{3}\right) = \frac{34}{3}$$

- and the expected utility of investment B is:

$$U(20)\left(\frac{1}{3}\right) + U(12)\left(\frac{1}{3}\right) + U(4)\left(\frac{1}{3}\right)$$

$$= (18)\left(\frac{1}{3}\right) + (13.2)\left(\frac{1}{3}\right) + (6)\left(\frac{1}{3}\right) = \frac{37.2}{3}$$

- In this situation, the investor would select investment B because it offers the higher average or expected utility. In general, we can say that the investor will choose among alternatives by maximizing expected utility or maximizing:

$$E(U) = \sum_{W} U(W)P(W)$$

- Consider a second example. Table 11.5 lists three separate investments. Assume the investor has the following utility function:

$$U(W) = 4W - (1/10)W^2$$

- Then the utility of 20 is: $80 - \left(\frac{1}{10}\right)(400) = 40$;
- the utility of 18 is: $72 - \left(\frac{1}{10}\right)(324) = 39.6$;
- and the utility of 14 is: $56 - \left(\frac{1}{10}\right)(196) = 36.4$.
- The rest of the values are shown in Table 11.6.

Table 11.5 Outcomes and Associated Probabilities for Three Investments

Investment A		Investment B		Investment C	
Outcome	Probability	Outcome	Probability	Outcome	Probability
20	3/15	19	1/5	18	1/4
18	5/15	10	2/5	16	1/4
14	4/15	5	2/5	12	1/4
10	2/15			8	1/4
6	1/15				

- The expected utility of the three investments is found by multiplying the probability of each outcome times the value of the outcome:
- $Expected\ utility\ A = (40)\left(\frac{3}{15}\right) + (39.6)\left(\frac{5}{15}\right) + (36.4)\left(\frac{4}{15}\right) + (30)\left(\frac{2}{15}\right)$
  $+ (20.4)\left(\frac{1}{15}\right) = \frac{544}{15} = 36.3$
- $Expected\ utility\ B = (39.9)\left(\frac{1}{5}\right) + (30)\left(\frac{2}{5}\right) + (17.5)\left(\frac{2}{5}\right) = \frac{134.9}{5} = 26.98$
- $Expected\ utility\ C = (39.6)\left(\frac{1}{4}\right) + (38.4)\left(\frac{1}{4}\right) + (33.6)\left(\frac{1}{4}\right) + (25.6)\left(\frac{1}{4}\right)$
  $= \frac{137.2}{4} = 34.4$
- Thus an investor with the utility function discussed earlier would select investment A.

Table 11.6 Including Utility

Investment A			Investment B			Investment C		
Outcome	Utility of Outcome	Probability	Outcome	Utility of Outcome	Probability	Outcome	Utility of Outcome	Probability
20	40	3/15	19	39.9	1/5	18	39.6	1/4
18	39.6	5/15	10	30	2/5	16	38.4	1/4
14	36.4	4/15	5	17.5	2/5	12	33.6	1/4
10	30	2/15				8	25.6	1/4
6	20.4	1/15						

- Note that the weighting function in Table 11.4 values small outcomes more heavily than large outcomes. Most investors prefer more wealth to less wealth and would prefer money with certainty rather than engage in a gamble with the same expected value.