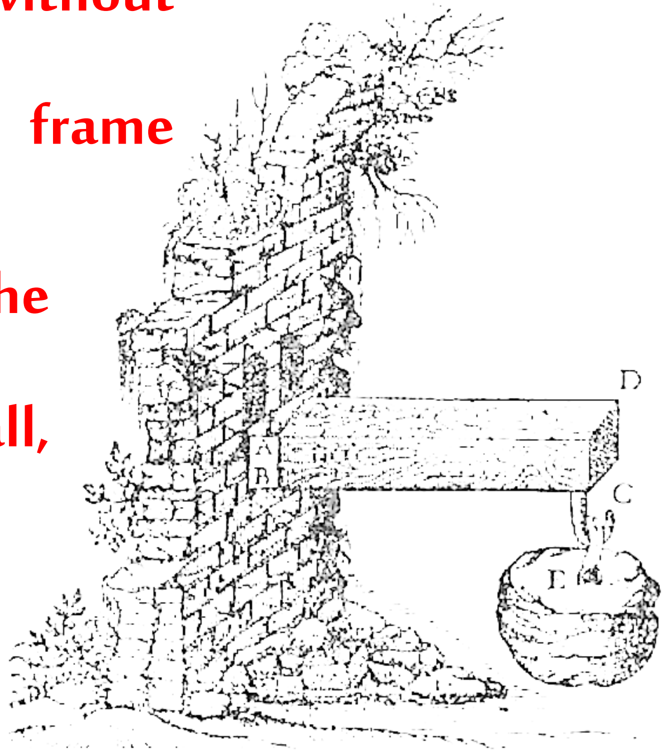
# Opening Remarks -- الافتتاحية

Stepping on a car's accelerator increases the power produced by the engine. The generated torque must be transmitted from the engine to the wheels without any part of the system failing.

When the car is loaded with passengers, the chassis distorts. The frame deflections must be small so that the doors can continue to open and close.




A bridge must be sufficiently strong to support loads due to traffic and the environment without any component breaking.

The bridge should also be stiff so that its deflections are relatively small, otherwise commuters would feel apprehensive about using the bridge.

Micro-electromechanical systems (MEMS) must be designed with the proper stiffness (force–deflection response) to perform their functions as sensors and actuators.

Biomedical implants such as hip and heart valve replacements must be biocompatible and strong enough to function for long periods of time under cyclic loads.

**Dominic J. Bello, Frederick A. Leckie; Strength and Stiffness of Engineering Systems.**

Strength of Materials ⟶ Mechanics of Materials ⟶ Mechanics of Deformable Solids ⊂ Continuum Mechanics

# Review of Statics - - مراجعة علم السكون (الستاتيك)

**Statics Analysis of an Engineering System**, aims at:

1- Determining how it supports load externally and internally.

2- Determining how it deforms under load.

يهدف التحليل الستاتيكي إلى إيجاد:(1) قوى الاستناد الخارجية وآليات انتقال الحمل داخلياً إليها. (2) التشوه الناتج عن ذلك.

**Statics Equilibrium (or simply Equilibrium) Conditions**:

1. Vector sum of all the forces $\boldsymbol{F}_i$, that act on a system must equal zero.
2. Vector sum of all the moments $\boldsymbol{M}_i$, about an arbitrary point and acting on the system must equal zero.

$$\sum \boldsymbol{F}_i = 0, \quad \& \quad \sum \boldsymbol{M}_i = 0 \quad \text{manually} \quad \sum \vec{F}_i = 0, \quad \& \quad \sum \vec{M}_i = 0$$

شرطا التوازن: (1) المجموع الشعاعي للقوى المؤثرة معدوم. (2) المجموع الشعاعي للعزوم حول نقطة ما معدوم

**Free body diagrams (FBDs)**:

1. The body is isolated from its surroundings (loads and supports).

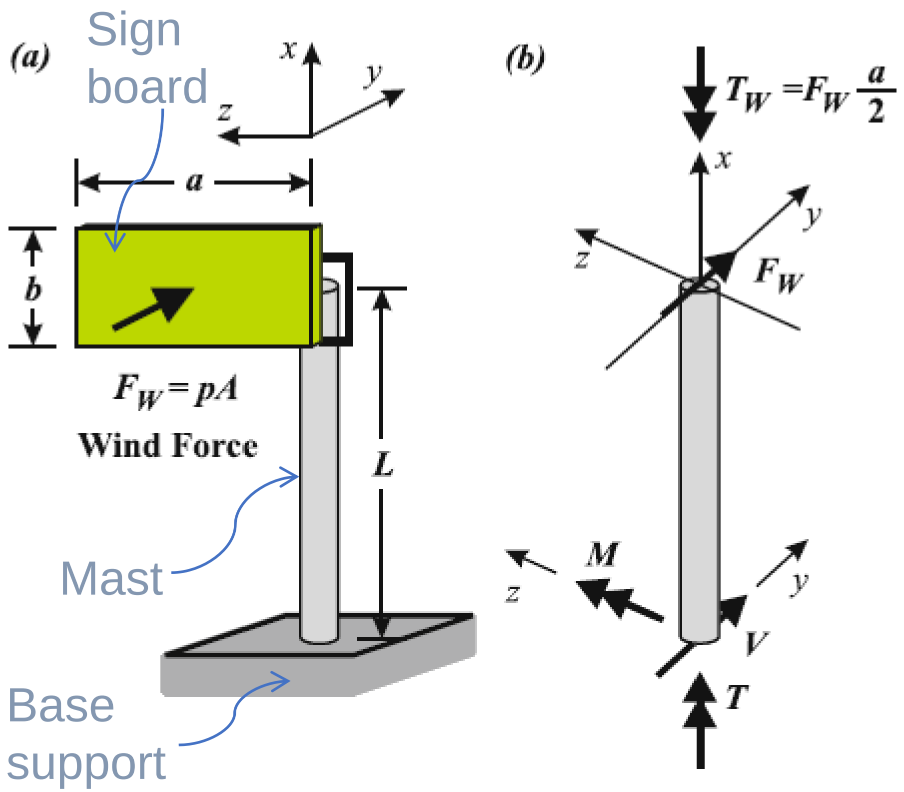
## Example of a FBD: A mast holding highway sign is acted on by wind load $F_W$:

**Free body diagrams (FBDs):**

1. The body is isolated from its surroundings (loads and supports).

   * Make a choice for a 3D right hand coordinate system $x$ - $y$ - $z$.

2. All of the forces & moments acting on the body from its surroundings are then represented by vectors.

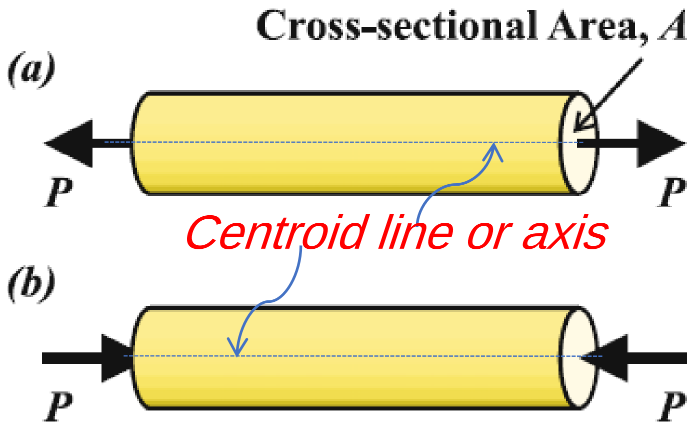



** In this simplified model the weight of the sign and the mast are ignored.

*** The FBDs are the basis for the equations of equilibrium. Every problem & solution should include a FBD.

# Axial Members

An *axial member is a straight component* supporting a force *P* parallel to its axis. The force must pass through the *centroid* of the cross-section, so that the response (internal force) at any cross-section is the same (uniform axial force). Loads that stretch the component are *tensile loads (a), and those* that shorten the component are *compressive loads (b).*




tensile internal force is positive

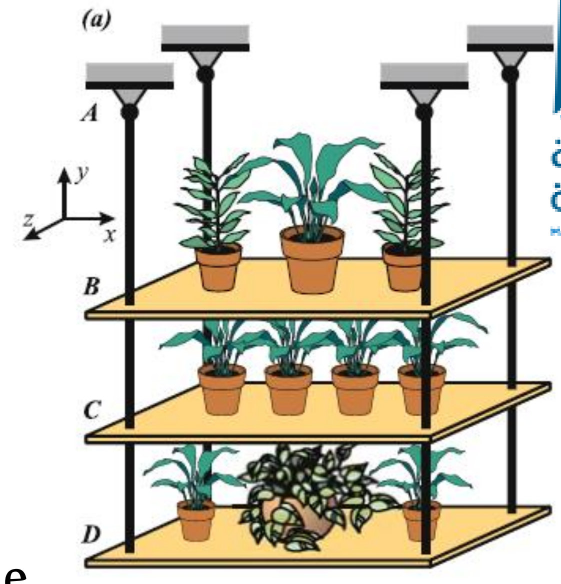
Compressive internal force is negative

An axial member is typically a ***two-force member.*** The forces that act at each end of a two-force member are **equal**, **opposite**, and **co-linear**. A straight two-force member supports a constant force *P* normal to any cross-section along its entire length.

*An* assembly of straight two-force members appropriately pinned together is a *truss.*

A few statics examples with axial members follow.

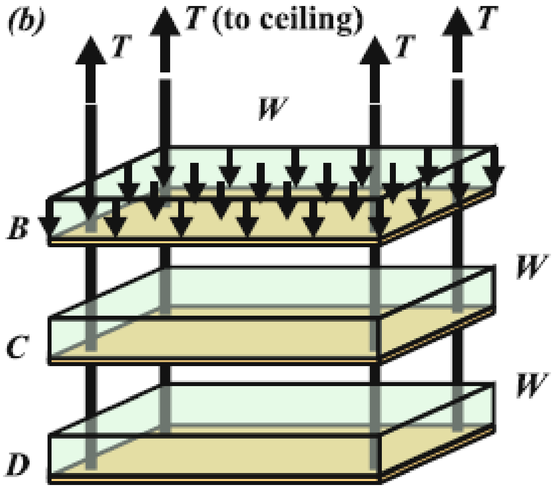
## Ex.1 Flower Pots on Hanging Shelves




**Given:** Three shelves are hung from the ceiling by four rope cords. Plants are evenly distributed on each shelf as shown in *Fig. (a)*. The total weight of the plants, pots, and earth on each shelf is $W$ = 500 N, which is assumed to be uniformly distributed over the shelf area.

**Required:** Determine the force in each segment of each cord.

**Solution:** Assuming that weight W is evenly distributed on each shelf, that the cords are symmetrically placed, and that the shelves are level, then the four cords share equally in supporting each shelf. Figure (b).




***Step 1.*** A FBD of the entire system (*Figure b*) is made by taking a cut at A, replacing the physical ceiling supports with reaction forces $T$; the plants are replaced with total weight W on each shelf. Equilibrium in the vertical ($y$) direction gives the reaction at each ceiling support:

$$\sum F_y = 0:\ 4T - 3W = 0 \ \Rightarrow\ T = (3/4)W = 0.75W = 375 \text{ N}$$ *External Equilibrium*

***Step 2.*** By taking a cut between levels A and B, the same equation is used to show that each cord segment AB carries tensile load $T_{AB}$ = 375 N. The FBD in 2D is shown in Figure c.

$$\sum F_y = 0:\ 4T_{AB} - 3W = 0 \ \Rightarrow\ T_{AB} = (3/4)W = 0.75W = 375 \text{ N}$$ *Internal Equilibrium*

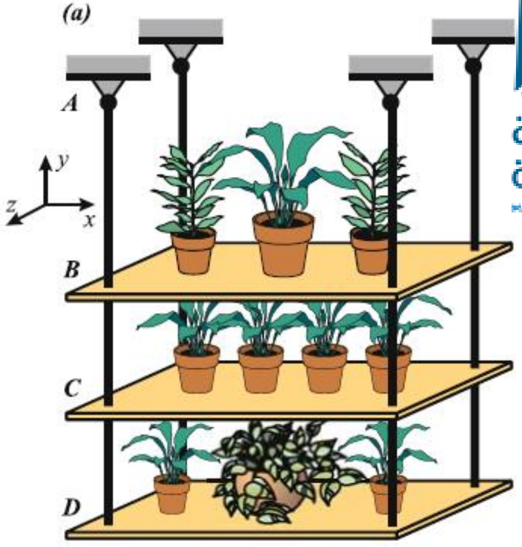
***Step 3.*** By taking a cut through the cords below the top shelf (*Figure d*), equilibrium gives the tension in each cord segment between *B* and *C*:

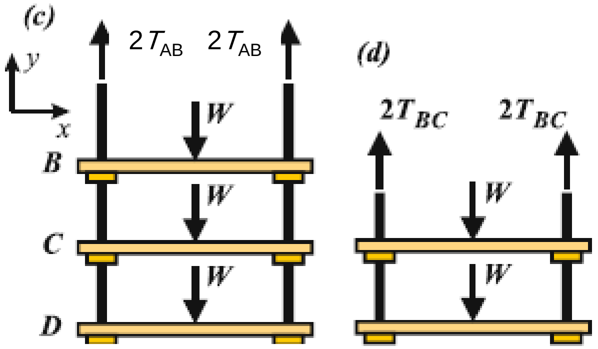
$$\sum F_y = 0:\ 4T_{BC} - 2W = 0 \ \Rightarrow\ T_{BC} = (2/4)W = 0.5W = 250 \text{ N}$$ *Internal Equilibrium*

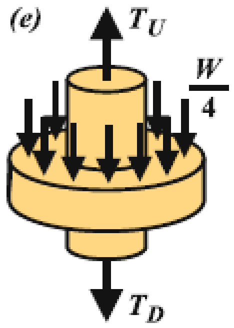
***Step 4.*** You are kindly asked to verify by cutting the cords between *C* and *D* that the tension in each of the lowest cords segment is $T_{CD}$ = ??? N. *Internal Equilibrium*

* A connector is necessary to transfer the load from each shelf to the cord (*Figure e*). Each connector supports a load *W/4* applied by the shelf, and the connector must be strong enough to transfer that load to the cord. The difference between the tension in the cord above the connector $T_U$ and the tension in the cord below the connector $T_D$, equals its share of the weight of the shelf: *W/4*.

*Internal Equilibrium*



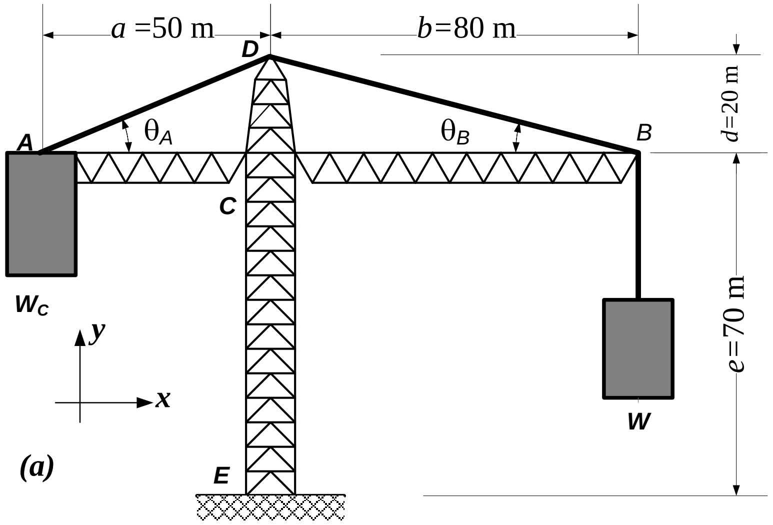



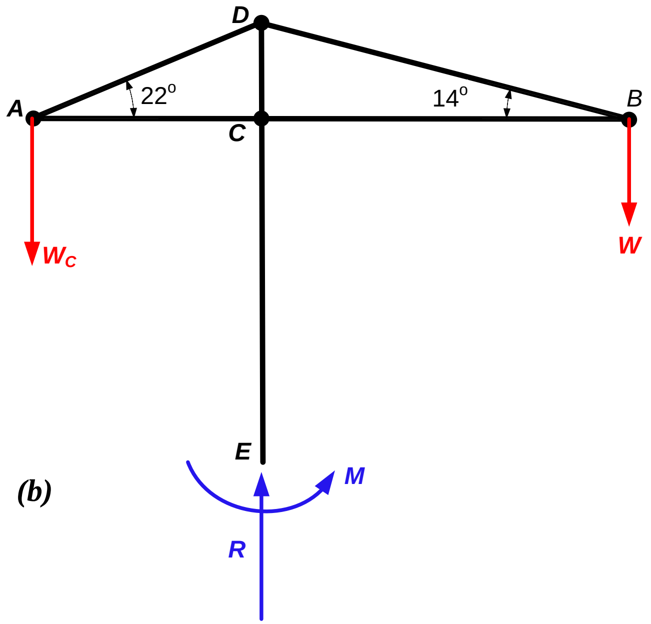



# Ex. 2 Tower Crane – Method of Joints

**Given:** The tower crane shown in (*Figure a*) consists of tower ***DCE*** fixed at the ground, and two jibs ***AC*** and ***CB***. The jibs are supported by tie bars ***AD*** and ***DB***, and are assumed to be attached to the tower by pinned connections. The counterweight $W_C$ weighs 1750 kN and the crane has a lifting capacity of $W_{max}$ = 1200 kN. Neglect the weight of the crane itself.

**Required:** Determine (a) the reactions at the base of the tower when the crane is lifting its capacity, (b) the axial forces in tie bars ***AD*** and ***DB***, and jibs ***AC*** and ***CB***, and (c) the internal forces and moment in the tower at point ***F***, 12 m below joint ***D***.

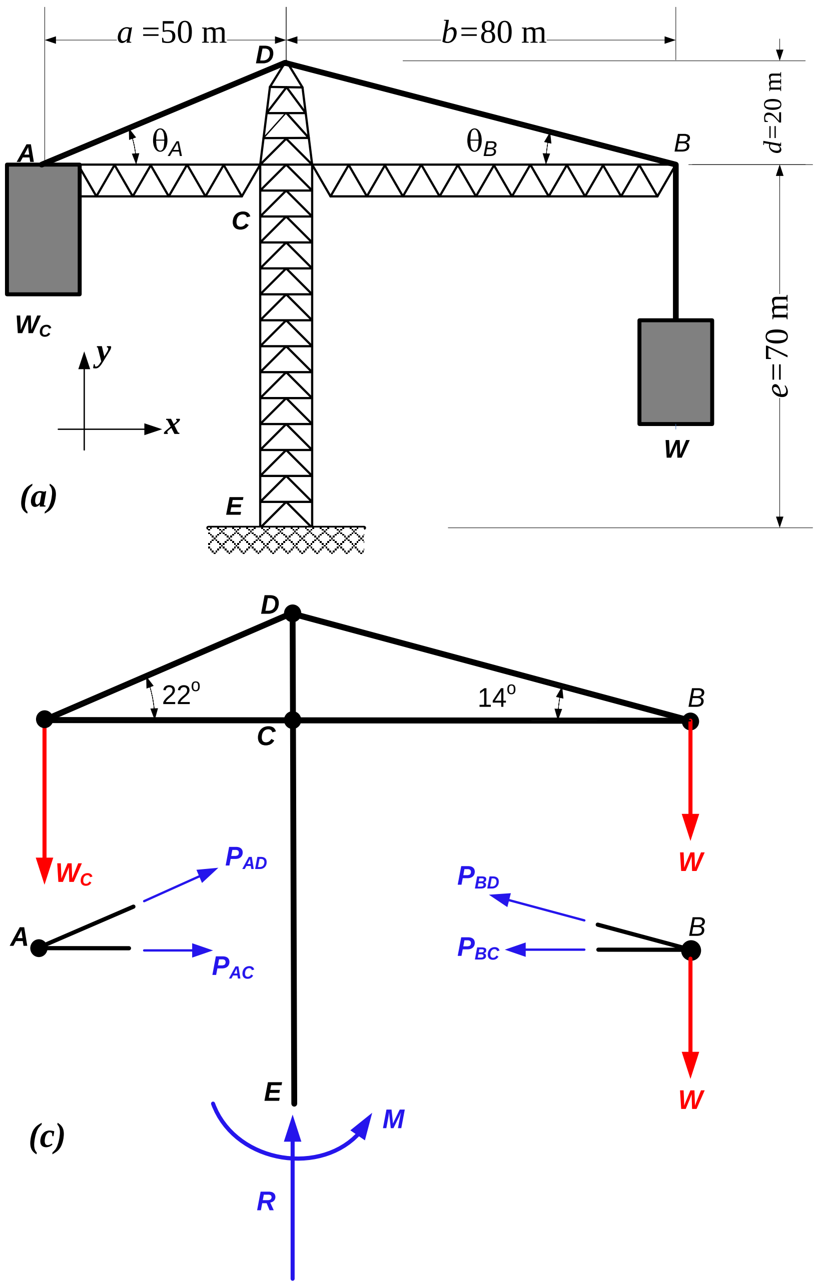


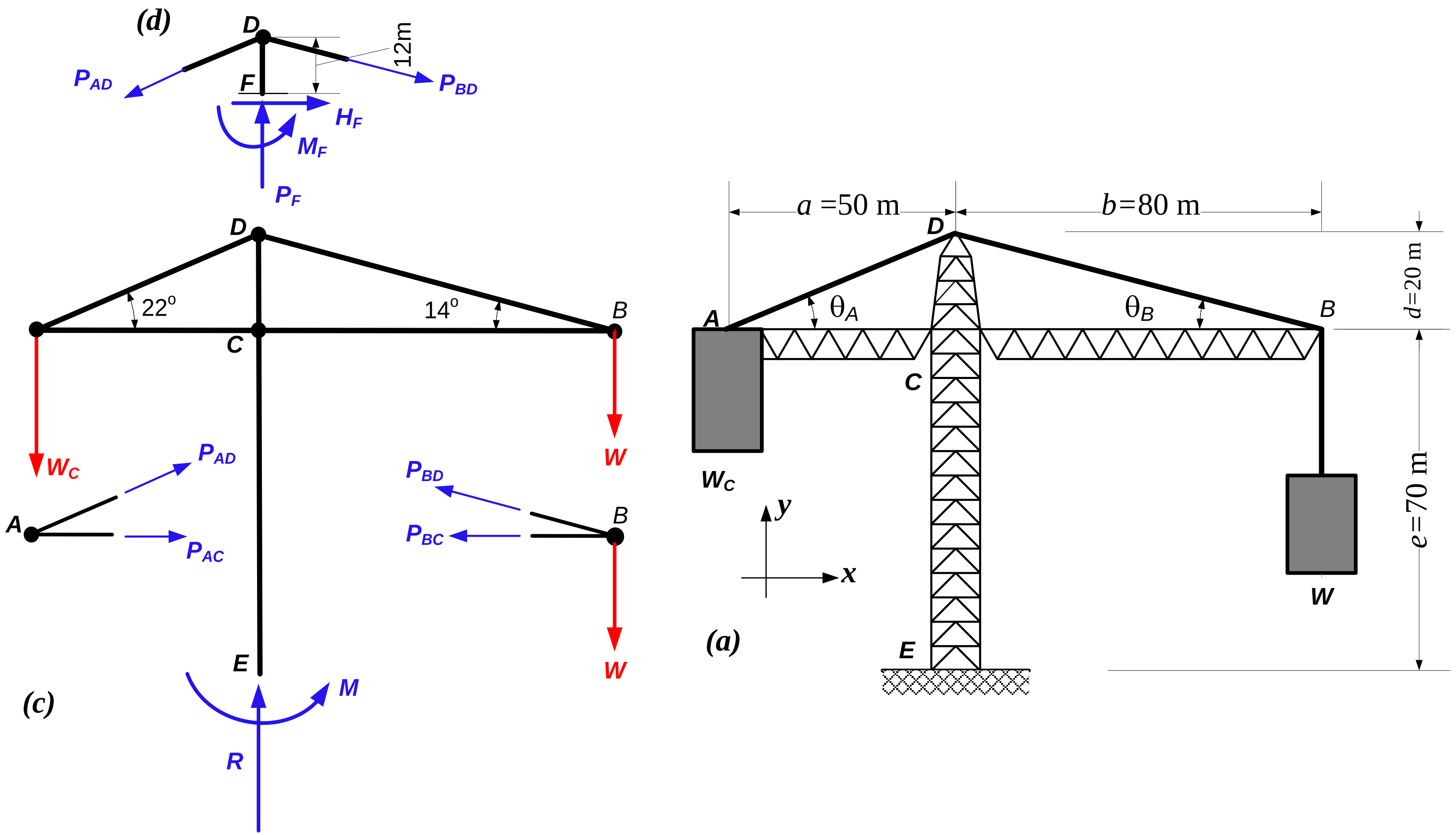

## Solution:

*Step 1. Reactions.*

*Step 2. Forces in members* ***AC, AD, BD*** & ***BC*** are solved using the Method of Joints.

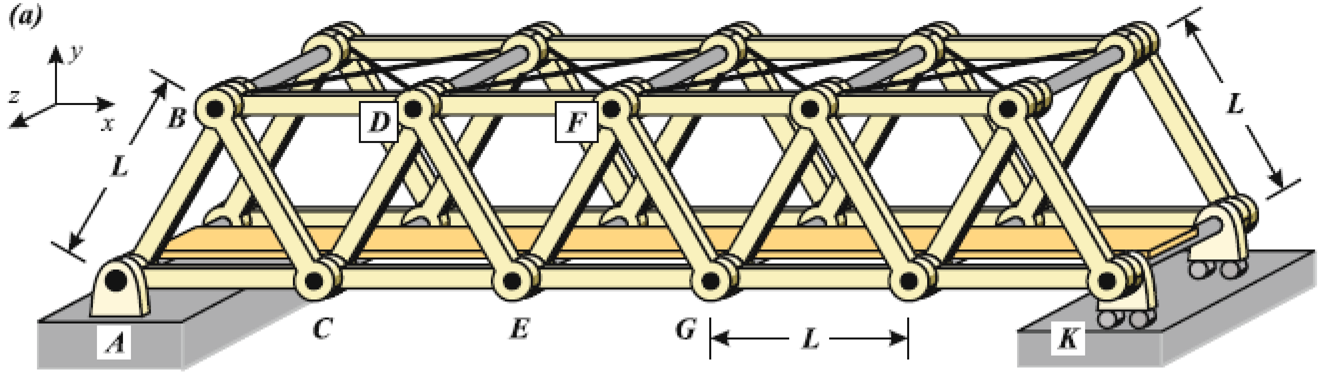


# *Step 3. Applying equilibrium at point **F**.*



## Ex. 3 Truss System – Method of Joints and Method of Sections

**Background:** Trusses are used in such applications as cranes, railway bridges, supermarket roofs, ships, aircraft, and space structures.




Axial members are pinned together to form a beam-like structure. Trusses are a very effective means of spanning large distances.

**Given:** A simple truss bridge is shown in (*Figure a*). Two plane trusses are constructed of 4.5 m long axial members assembled with pins into equilateral triangles. Crossbeams connected to the trusses at the pin (node) locations maintain the spacing between the trusses, while diagonal bracing keeps them from moving laterally with respect to each other. The lower crossbeams support a roadway (deck) 6 m wide. For the design, the roadway is to carry a uniformly distributed load of 4 kN/m$^2$. Neglect the weight of the bridge itself.

**Required:** Considering only the roadway load, determine the forces (a) in members *AB and AC using the Method of Joints and (b) in DF, EF and EG using the Method of Sections.*

**Solution:** *Step 1. Load path*:

Each lower crossbeam supports a deck *tributary area* of 4.5 (m) × 6 (m) = 27 (m$^2$).

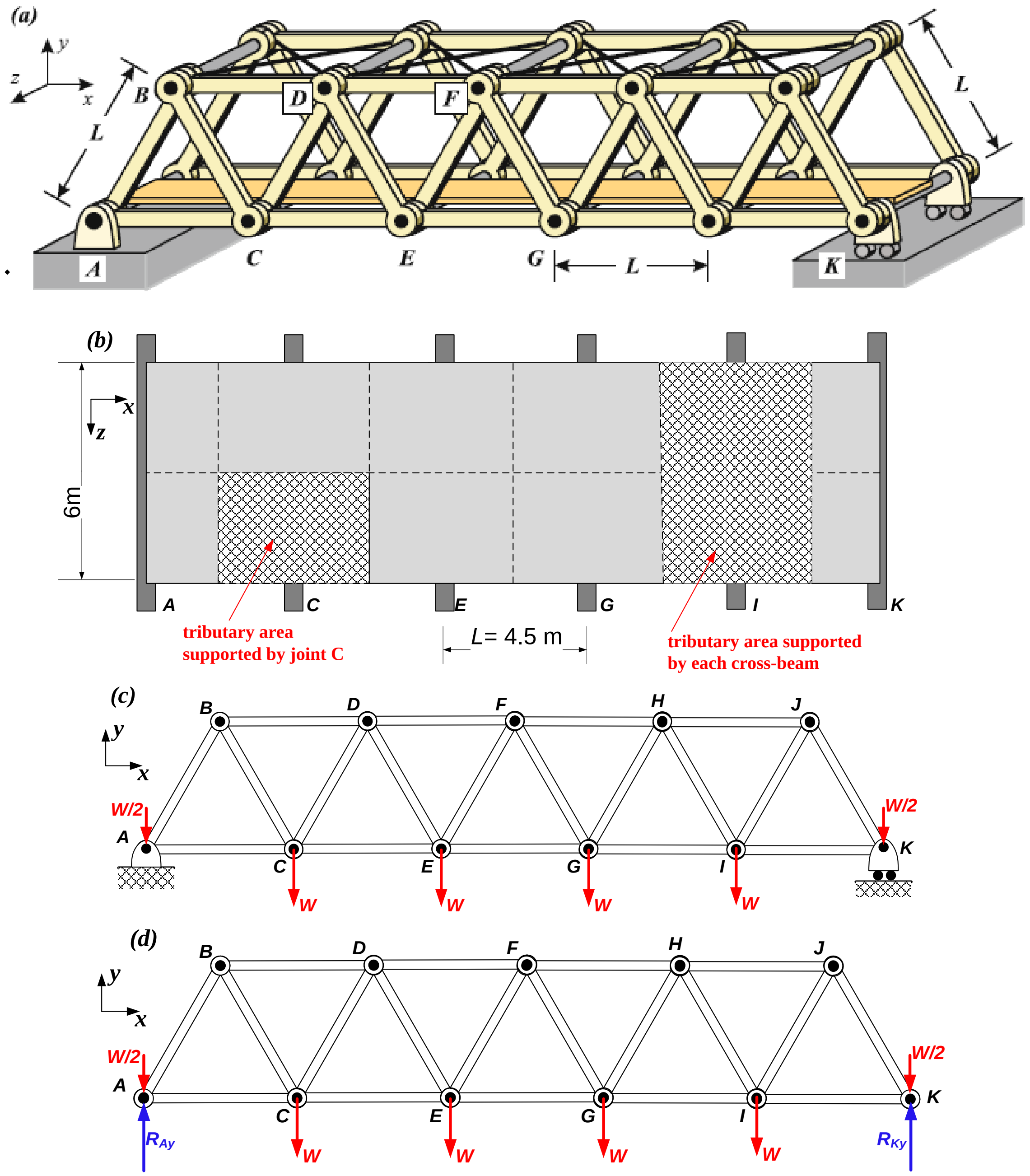
The force on each crossbeam is 27 (m$^2$) × 4 (kN/m$^2$)= 108 (kN).

So by symmetry, The force supported at any lower pin is half the value of the load on each crossbeam, or: $W$=54 kN.

The supports, are loaded by: $W/2$.

Because the tributary area is reduced by 50% at the two supports.

*Step 2. Reactions.* From equilibrium & symmetry, the vertical reaction loads are: $R_{Ay}=R_{ky}$= 2.5 $W$=135 N.



*Step 3. Forces in members AB and AC (Figure e)*. Applying the method of joints at joint *A*, the forces in members AB & AC are determined using Eq. Eqs. of joint A:

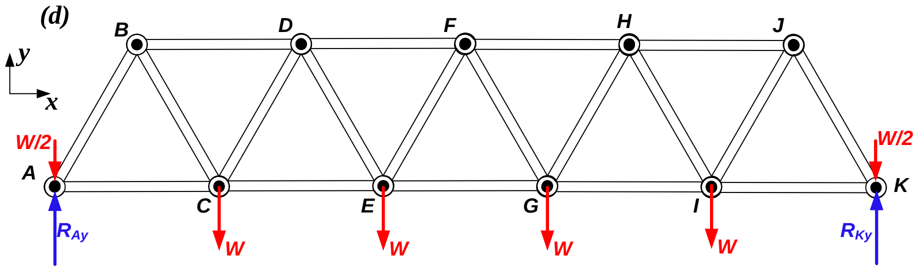
$\sum F_x=0$: $P_{AC}+P_{AB}\cos60^\circ=0$. (1).

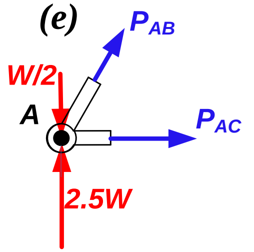
$\sum F_y=0$: $2.5W-0.5W+P_{AB}\sin60^\circ=0$, (2).

$\Longrightarrow$ Solving the 2 Eqs. gives

$P_{AB}=-2W/\sin60^\circ=-2.31W. = -125$ kN &

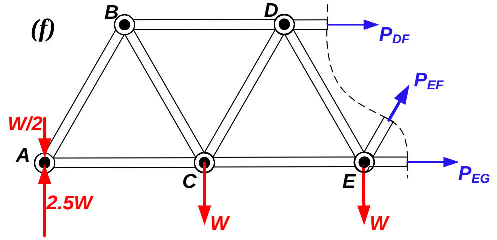
$P_{AC}=-(-2W/\sin60^\circ)\cos60^\circ=1.15W=62$ kN




*Step 4. Forces in DF, EF, & EG, (Figure f)*. The method of joints can be used, one joint at a time, to solve for all the member forces.

However, using the method of sections, the force in any inner member is directly determined. For example, take a cut through members *DF, EF* and *EG*, as shown in (*Figure f*). Using Eq. Eqs. of the complete left part:




$\sum M_{Ez}=0$: $2L(0.5W)-2L(2.5W)+L(W)-L\sin60^\circ(P_{DF})=0$, (1).

$\sum M_{Fz}=0$: $2.5L(0.5W)-2.5L(2.5W)+1.5L(W)+0.5L(W)+L\sin60^\circ(P_{EG})=0$, (2).

$\sum F_y=0$: $-0.5W+2.5W-W-W+P_{EF}\sin60^\circ=0$, (3).

The good choice of the Eq. Eqs. has given us three uncoupled Eqs., solving gives:

$P_{DF}=-3W/\sin60^\circ=-3.46W=-187\ kN$

$P_{EG}=3W/\sin60^\circ=3.46W=187\ kN$

$P_{EF}=0$.

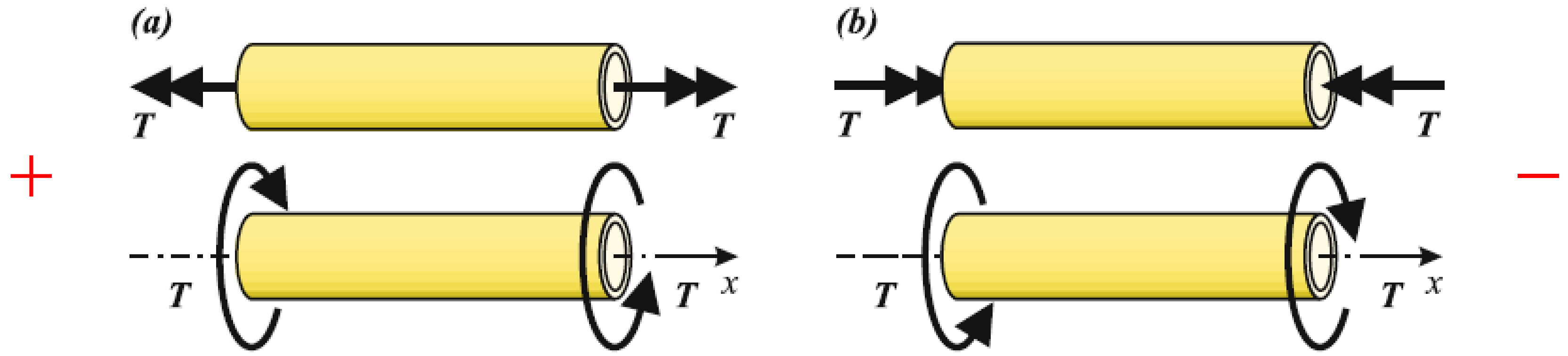
Check: $\sum F_x=0$:

$P_{DF}+P_{EF}\cos60^\circ+P_{EG}=-187+0+187=0$ ok.

# Torsion Members

**A torsion member is a longitudinal component that transmits torque T** The torque twists the member about its axis, which passes through the *centroid* (center of area) of its cross-section.

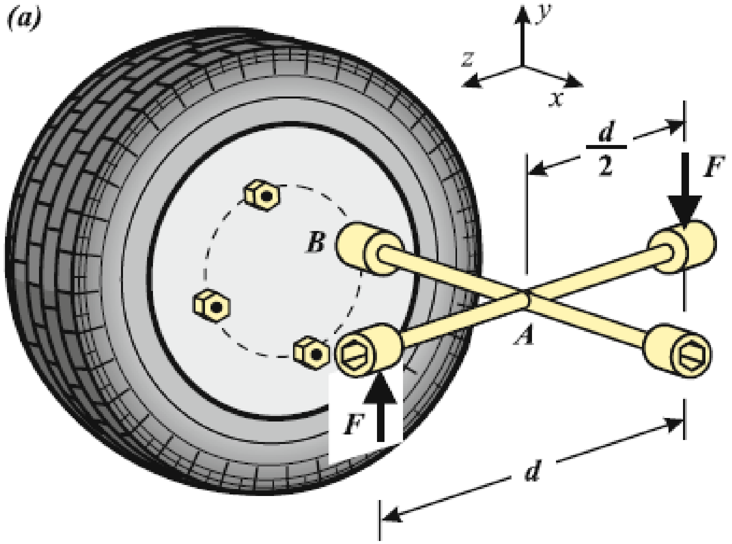
يكون عنصر الفتل متطاولاً ويقوم بنقل عزم محوري يسعى إلى فتل العنصر حول محوره الذي يمر من مراكز ثقل مقاطع العنصر.لذلك غالبا ما يسمى العزم المحوري بعزم الفتل.




A torque is represented alternatively by a double-headed vector or a curved arrow. A torque within a torsion member is termed positive if it points in the same direction as the outward-pointing normal vector of the cross-section (figure a). If It points in the opposite direction its negative (figure b).

# Ex. 4 Classic Lug Wrench مفتاح العزق الرباعي :

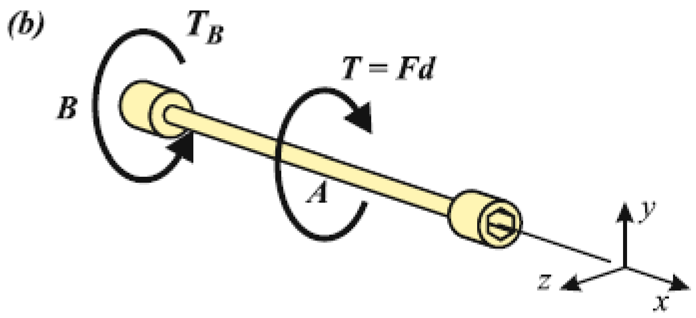
**Given**: A lug nut is tightened by applying a downward force *F* on the lug wrench's right arm and an upward force *F* on the wrench's left arm (*Figure a*). Linear motion is converted into angular motion; force is converted into torque. The forces are assumed to be of equal magnitude.




**Required:** Determine the magnitude of the torque applied to the lug nut.

**Solution:** The torque**,** or couple, applied to the wrench stem at point *A* is: $T_A = F\,d$ , and is clockwise with respect to the +*x-axis.*

The reaction torque $T_B$ applied by the lug nut against the wrench's stem is shown in (*Figure b*). The torque applied to the lug nut by the stem is equal and opposite to the reaction $T_B = T_A = F\,d$ .

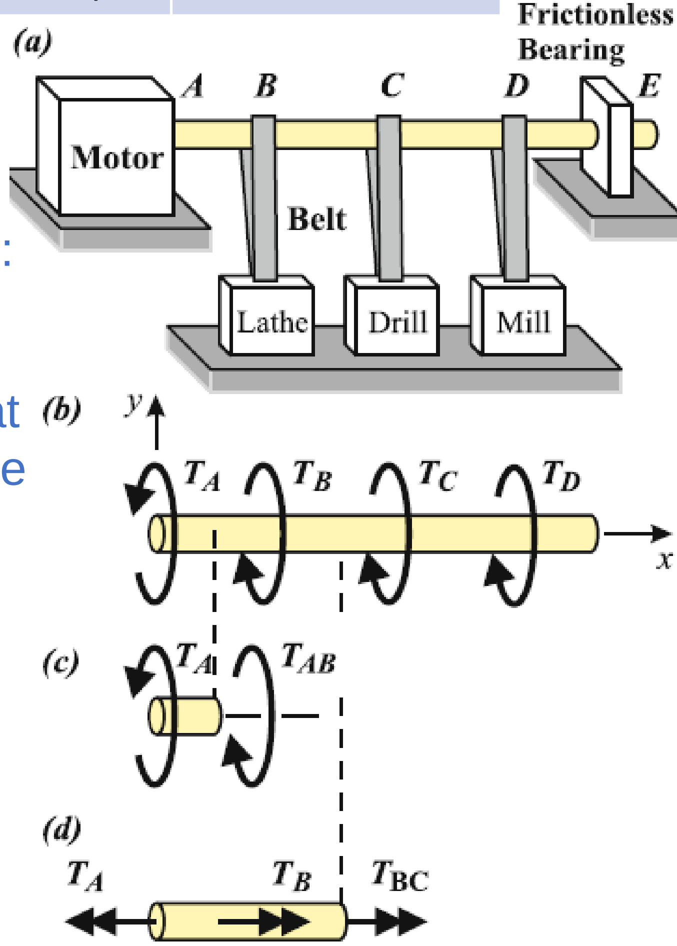


# Ex. 5 Drive Shaft in a Machine Shop المحور القائد في ورشة آلات

**Given:** The individual machines of classical machine shops were powered by belts driven by drive shafts. An example is shown in *Figure* (*a*) , in which three machines, *B*, *C*, and *D*, draw torque from the main shaft according to the next table. Bearing *E* is assumed to be frictionless, and therefore draws no torque.

| Machine | Torque (N.m) |
|---|---|
| B (Lathe مخرطة) | 20 |
| C (Drill مثقب) | 40 |
| D (Mill مقشطة) | 30 |

**Required:** (a) Determine the torque anywhere along the drive shaft and (b) draw the torque diagram, *T(x) vs. x.*




**Solution:** ***Step 1.*** The FBD of the entire drive shaft is shown in (*Figure b*). Torque $T_A$ is the input torque. From equilibrium, the sum of the torques about the x-axis must be zero:

$$\sum T_x = 0: -T_A + T_B + T_C + T_D = 0 \Rightarrow T_A = 20+40+30 = 90 \text{ N.m}$$

***Step 2.*** The internal torque supported at any cross-section is found by taking a cut at that section, and a FBD of the remaining structure is considered. The torque carried inside the shaft between *A* and *B*, $T_{AB}$ , is found by taking a cut between *A and B* (*Figure c)* and applying equilibrium to the external and internal torques.

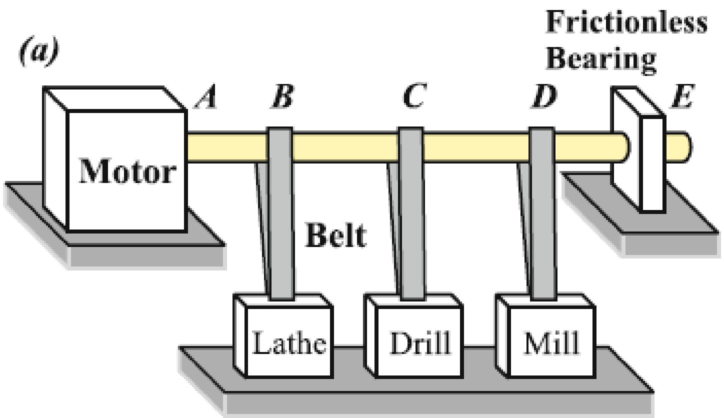
$$\sum T_x = 0: -T_A + T_{AB} = 0 \Rightarrow T_{AB} = T_A = 90 \text{ N.m}$$

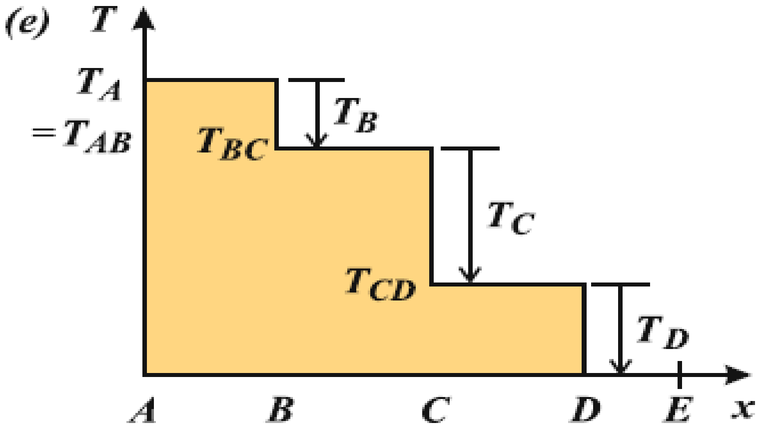
***Step 3.*** Likewise, the internal torque between B and C (*Figure d*) is:

$$\sum T_x = 0: -T_A + T_B + T_{BC} = 0 \Rightarrow T_{BC} = T_A - T_B = 90-20 = 70 \text{ N.m}$$

***Step 4.*** Verify for yourself that the torque in segment *CD* is $T_{CD}$ = 30 N.m.

| Machine | Torque (N.m) |
|---|---|
| B (Lathe مخرطة) | 20 |
| C (Drill مثقب) | 40 |
| D (Mill مقشطة) | 30 |

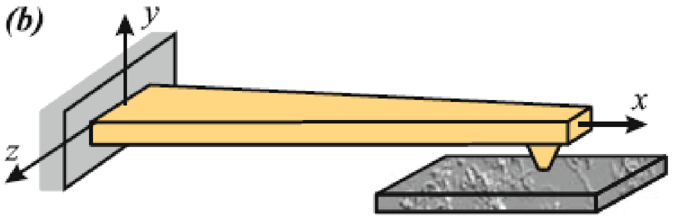


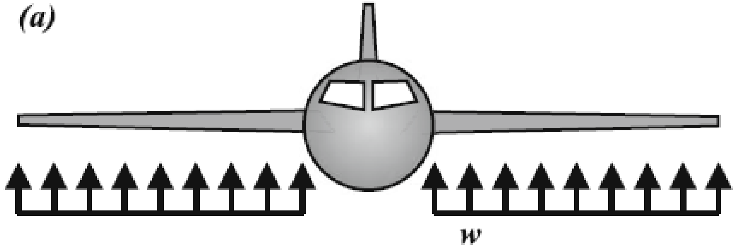




***Step 5.*** The torque diagram (*Figure e*) is used to display the internal torque carried by the shaft. The torque diagram is analogous to the shear force and moment diagrams for beams as will be seen later.

# Beams

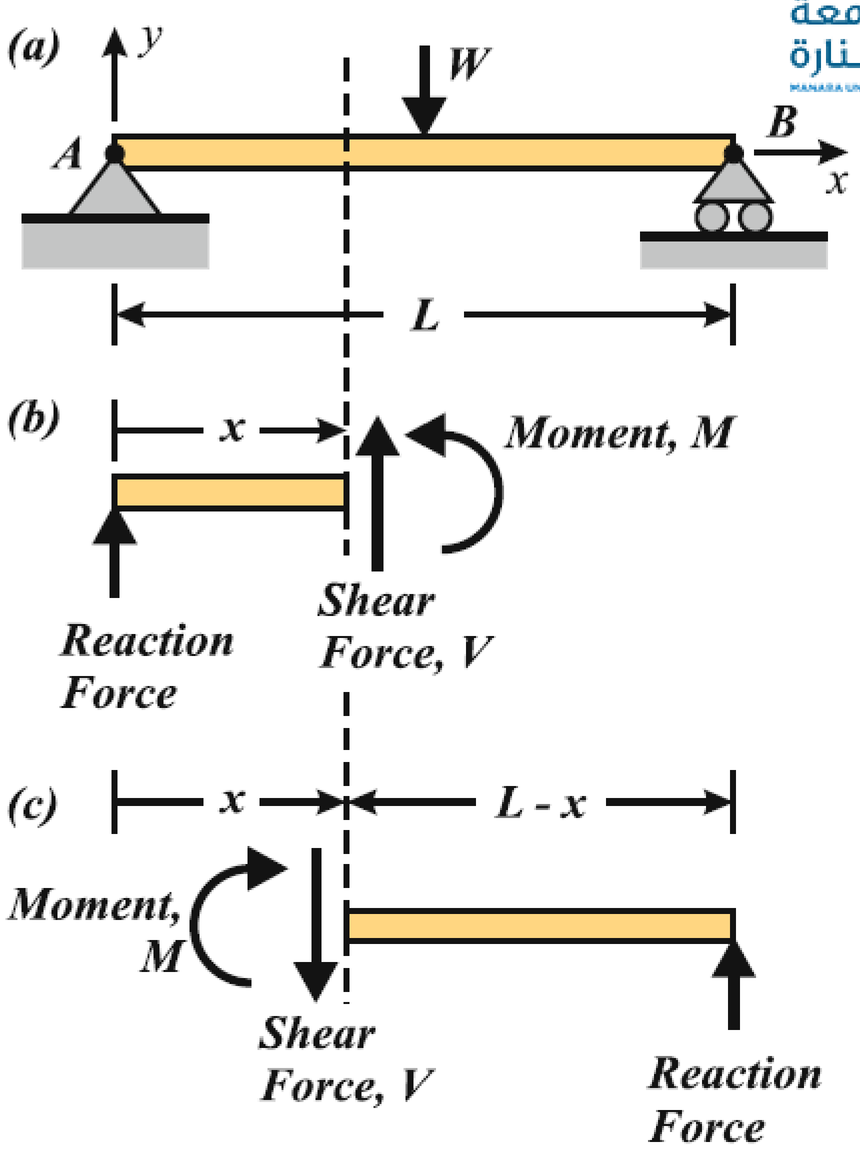
***Beams*** are structural components that support loads transverse to their main axis. Examples include aircraft wings, floor, and ceiling joists in buildings, bridges, atomic force microscopes, robotic arms in space structures, tree branches, etc. (Figures).



The internal loads in beams are bending moments and shear forces (Figure).

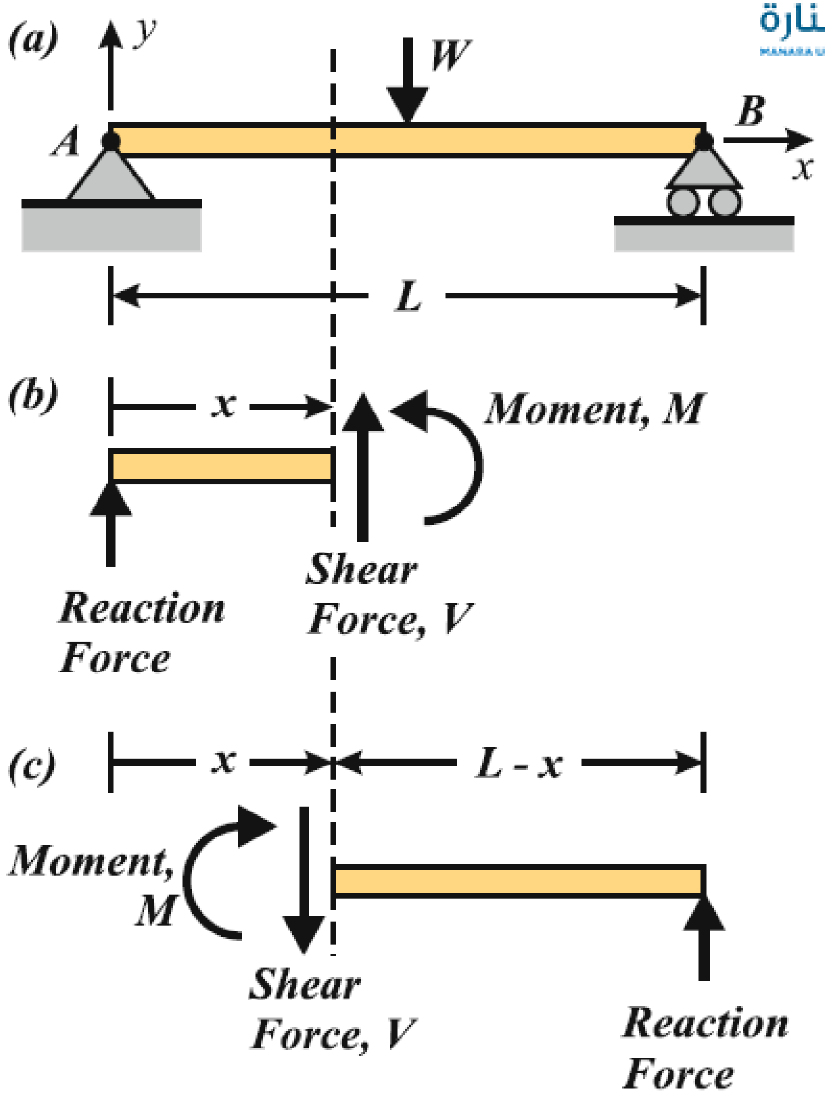
In general, the internal bending moment *M* and shear force *V* vary with distance x along the beam. In (*Figure b*), they are drawn in their positive senses as defined by the convention of this text, and described in the following paragraphs.




The internal bending moment *M* is *positive* if it causes *compression* on the top of the beam; the moment is *negative* if it causes *compression* on the bottom of the beam.

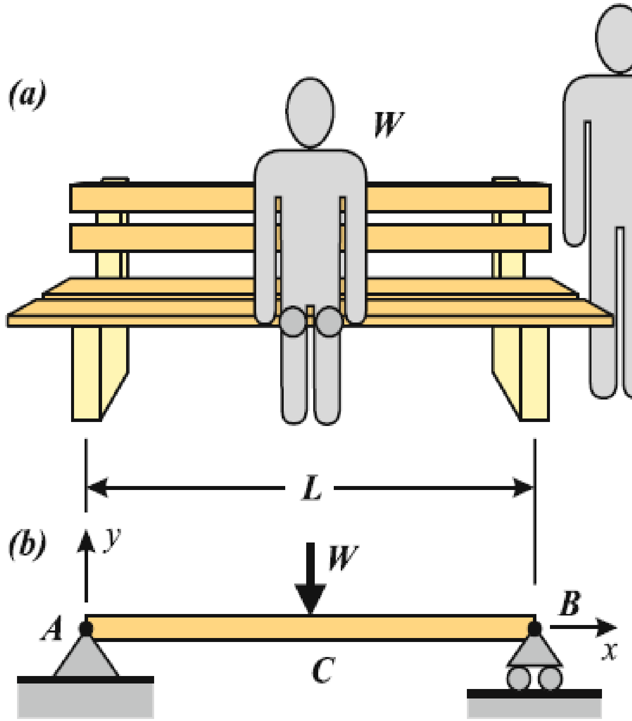
The shear force *V* is *positive* if it acts on a positive face in the *y* direction, or on a negative face in the *y.* Otherwise, the shear force is negative.

(*Figure b*) is a FBD of a *x* length of the beam exposing a cross-section that faces in the +*x-direction* (*positive face*). At the cut, a positive moment *M* is drawn acting about the +*z-axis,* out of the paper (check this with the right-hand rule), and a positive shear force *V* is drawn acting in the +*y-direction.*




(*Figure c*) is the complementary FBD of (*Figure b*). The FBD of *L–x* exposes a cross-section that faces in the *negative x-direction.* Drawn in their positive senses, moment and shear force both act on a *negative* face in a negative direction: the moment about the − *z-axis* and the shear force in the − *y-direction.*

# *Ex. 6 Park Bench*: *Modeled as a Simply-Supported Beam under a Point Load*




**Given:** A person sits in the middle of a park bench. The slats of the bench are supported at each end by a set of legs. (*Figure a*).

The bench is modeled as a beam supported by a *pin* at the left end and a *roller* at the right end, with a *point load* applied at the center (*Figure b*).

Pinned supports allow rotation and cannot support or resist a moment. A beam with pinned supports (e.g., a pin and a roller) is called a *simply-supported* beam.

**Required:** (a) Determine the *shear force* & the *bending moment* as functions of x along the length of the beam. (b) Draw the *shear force* & *bending moment diagrams.*

**Solution:** ***Step 1.*** Reactions. The FBD of the entire beam is shown in (*Figure c*). Since the loading and geometry are both *symmetric,* then the vertical reactions are equal:

$$R_{Ay} = R_{By} = R = W/2$$

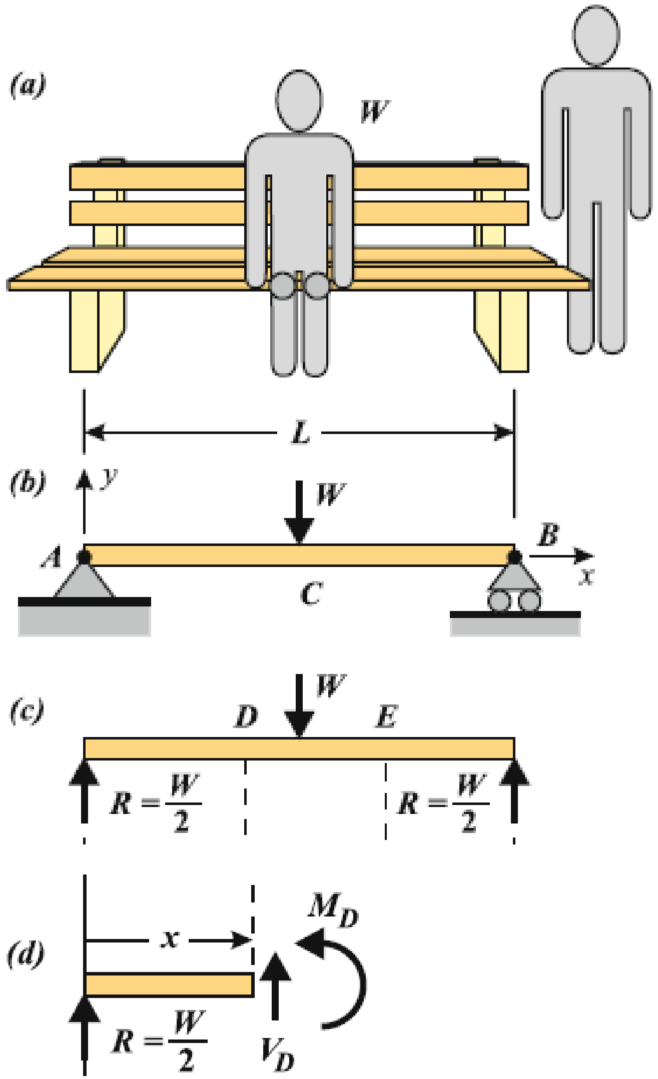
Since there is no load applied horizontally, the horizontal reaction at point *A* is zero*.*

***Step 2.*** Shear force and bending moment for 0 < *x* < *L/2.*

The shear force and bending moment at any section *D* to the left of the load are found from the FBD in (*Figure d*)*.* From equilibrium of segment *AD* (*taking moments* about point *D*):

$$\sum F_y = 0 : (W/2) + V_D = 0 \Rightarrow V_D = -(W/2)$$

$$\sum M_{z,D} = 0 : -(W/2)x + M_D = 0 \Rightarrow M_D = (W/2)x$$




If equilibrium of the right-hand FBD (a FBD from point *D* to point *B* ) is considered, then the same results are obtained. Check this statement.

***Step 3.*** Shear force and bending moment *for L /2 < x < L.*

*The shear force and bending moment on any cross-section E* to the right of the load are found from the FBD in (*Figure e*). From equilibrium of segment *AE:*

$$\sum F_y = 0 : (W/2) - W + V_E = 0 \Rightarrow V_E = (W/2)$$

$$\sum M_{z,E} = 0 : -(W/2)x + W[x - (L/2)] + M_E = 0$$

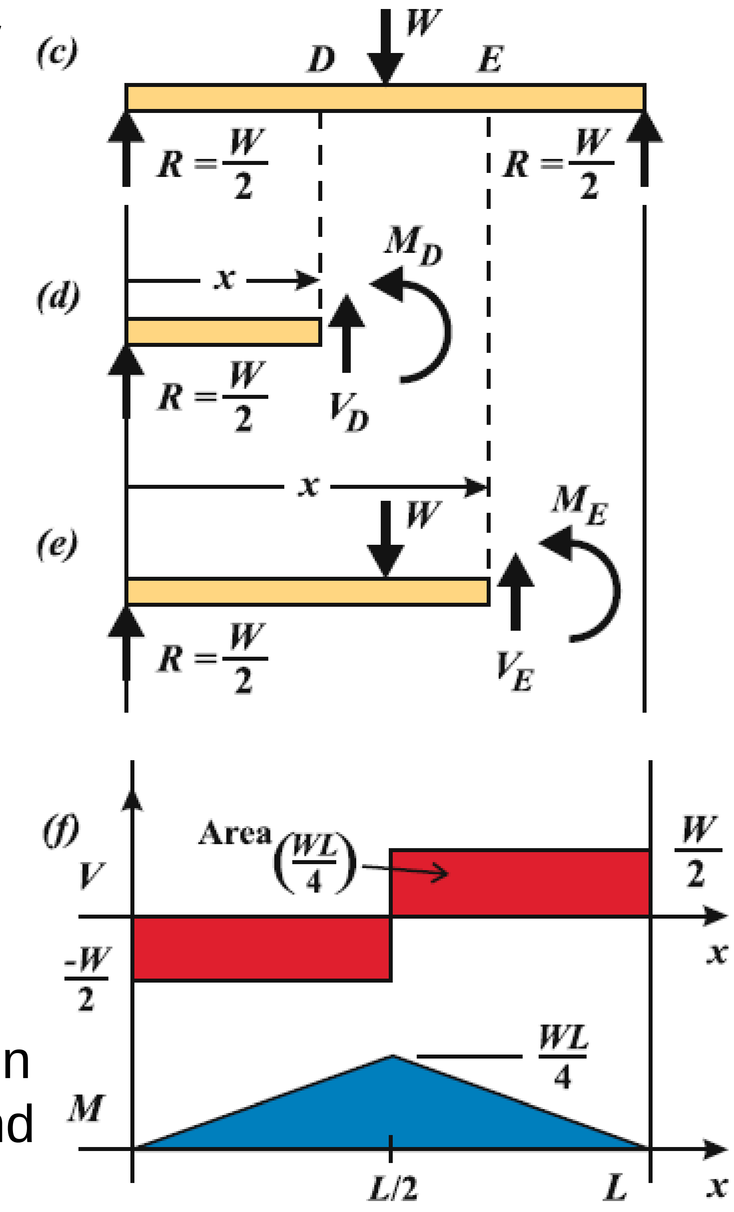
$$\Rightarrow M_E = (W/2)(L - x)$$

In summary:

For $0 < x < L/2 : V_D = -W/2$ and $M_D = (w/2)x$

For $L/2 < x < L : V_E = W/2$ and $M_E = (w/2)(L - x)$

*Step 4.* The variations of shear force and bending moment along the beam are shown in (*Figure f*). These plots are the shear force and bending moment diagrams.

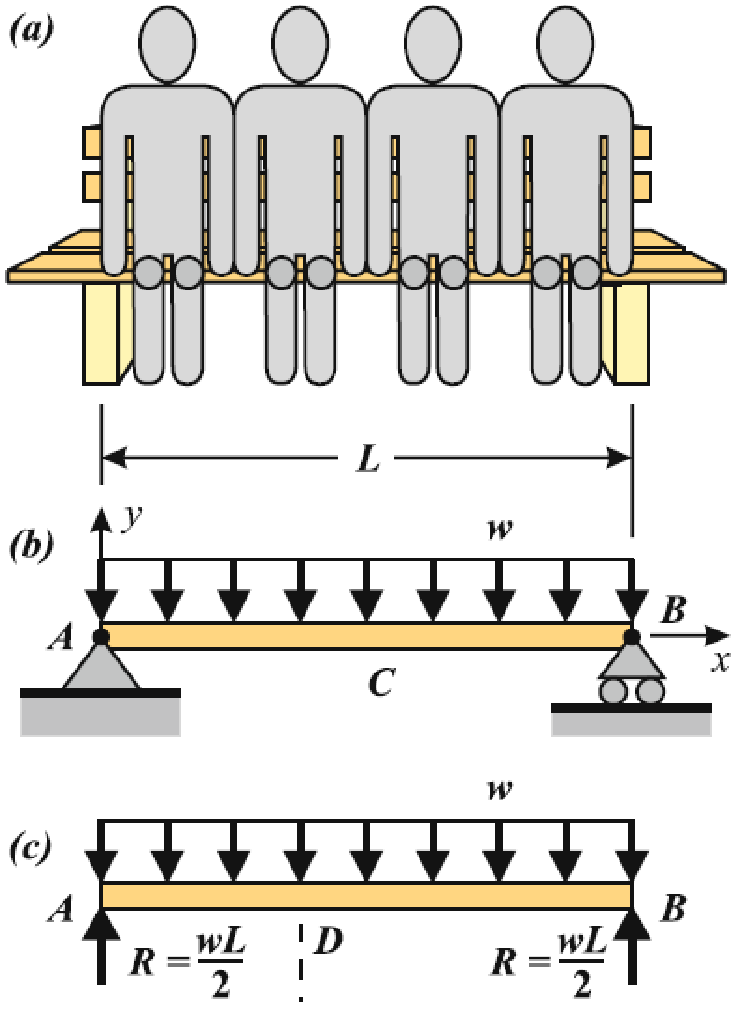


# *Ex. 7 Park Bench: Modeled as a Simply-Supported Beam; Uniformly Distributed Load*

**Given:** The park bench in the previous example is now completely full (*Figure a*). The beam is assumed to have the same geometric boundary conditions (simple supports). Since the beam is full, the load is modeled as a *uniformly distributed load* (force per unit length). The distributed load is:

$$w = nW/L$$

where $n$ is the number of people on the bench, *$W$ is the weight of each person* (assumed to be the same), and *$L$ is the* distance between supports. The beam model is shown in (*Figure b*).




**Required:** (a) Determine the shear force and the bending moment along the length of the beam. (b) Draw the shear force and bending moment diagrams.

**Solution:** ***Step 1.*** The FBD of the entire beam is shown in Figure 2.12c. Since the loading and geometry are both symmetric. $R_{Ay} = R_{By} = R$. From vertical equilibrium:

$$\sum F_y = 0: -wL + R_{Ay} + R_{By} = 0$$

$$\Rightarrow R_{Ay} = R_{By} = R = wL/2$$

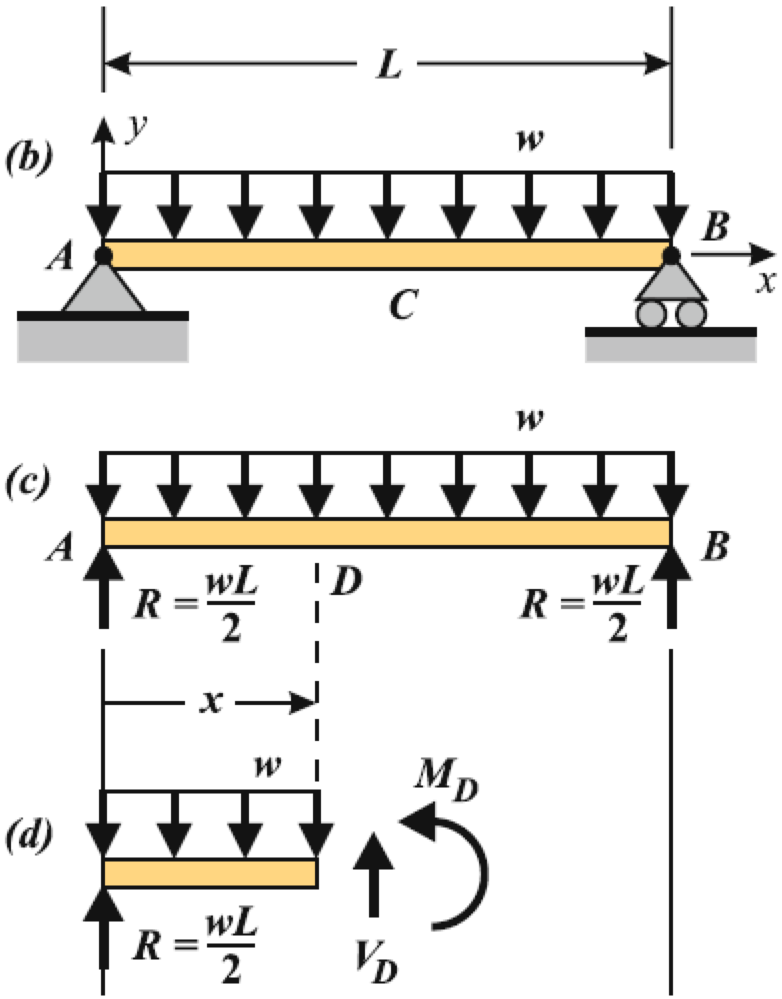
*Step 2. The internal shear force and bending* moment at any section *D distance x from the* origin may be found from the FBD shown in (*Figure d*).

From vertical equilibrium:

$$\sum F_y = 0: wL/2 - wx + V_D = 0$$

$$\Rightarrow V_D = V(x) = w(x - \tfrac{1}{2}L)$$

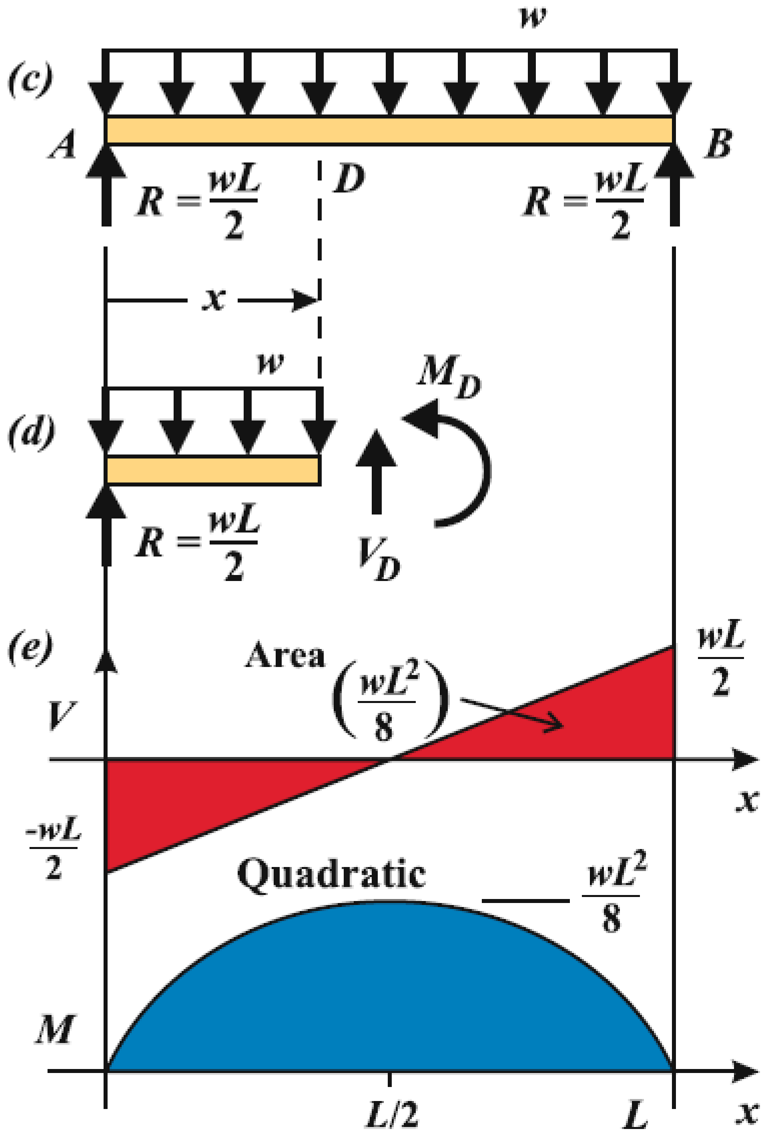
Moment equilibrium about point *D* gives:




$$\sum M_{z,D} = 0: -(wL/2)x + wx(x/2) + M_D = 0 \Rightarrow M_D = M(x) = (w/2)(Lx - x^2)$$

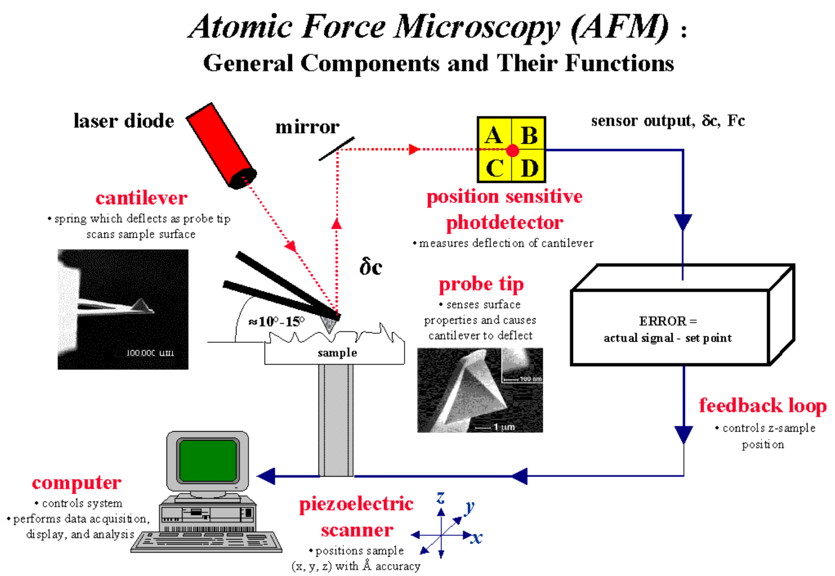
***Step 3.*** The shear force and bending moment diagrams are shown in (*Figure e*).

For this problem, the shear force is *linear,* with a maximum magnitude *of* $wL/2$ that occurs at each support. The bending moment is *parabolic,* with a maximum value of $wL^2/8$ at the center of the beam. The maximum bending moment occurs when the shear force is equal to zero. Because of the symmetry of the geometry and applied load, the response (shear force and moment) are symmetric.

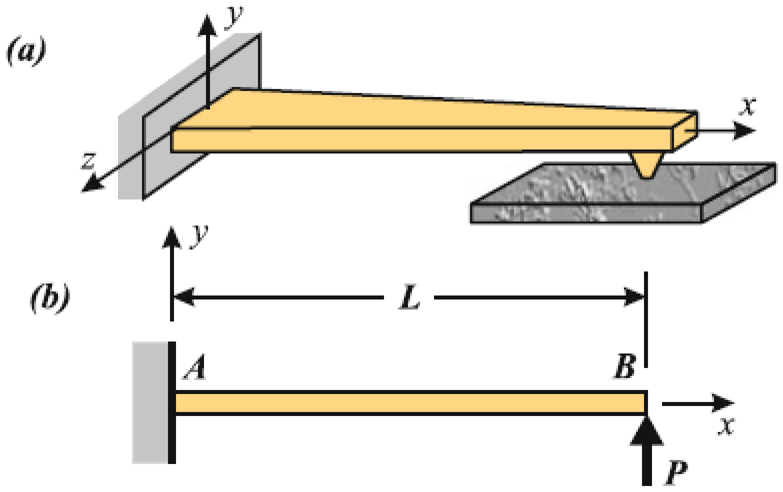


# Ex. 8 Atomic Force Microscope: A Cantilever with a Point Load

**Background:** The principal component of the atomic force microscope (AFM), used to measure the micro-geometry of surfaces and the forces in biological systems, consists of a *cantilever* beam*.* A cantilever beam is built-in (fixed against displacement and rotation) at one end and free at the other end (*Figures*)*.*




**Given:** Force P is applied at the free end of the AFM cantilever. A representative load at this scale is $P = 20$ nN ($20\times10^{-9}$ N) and the beam length is $L = 60$ μm ($60\times10^{-6}$ m).




**Required: (a**) Determine the shear force and the bending moment along the length of the beam. (b) Draw the shear force and moment diagrams.

**Solution:** ***Step 1.*** The FBD of the entire beam is shown in (*Figure c*).

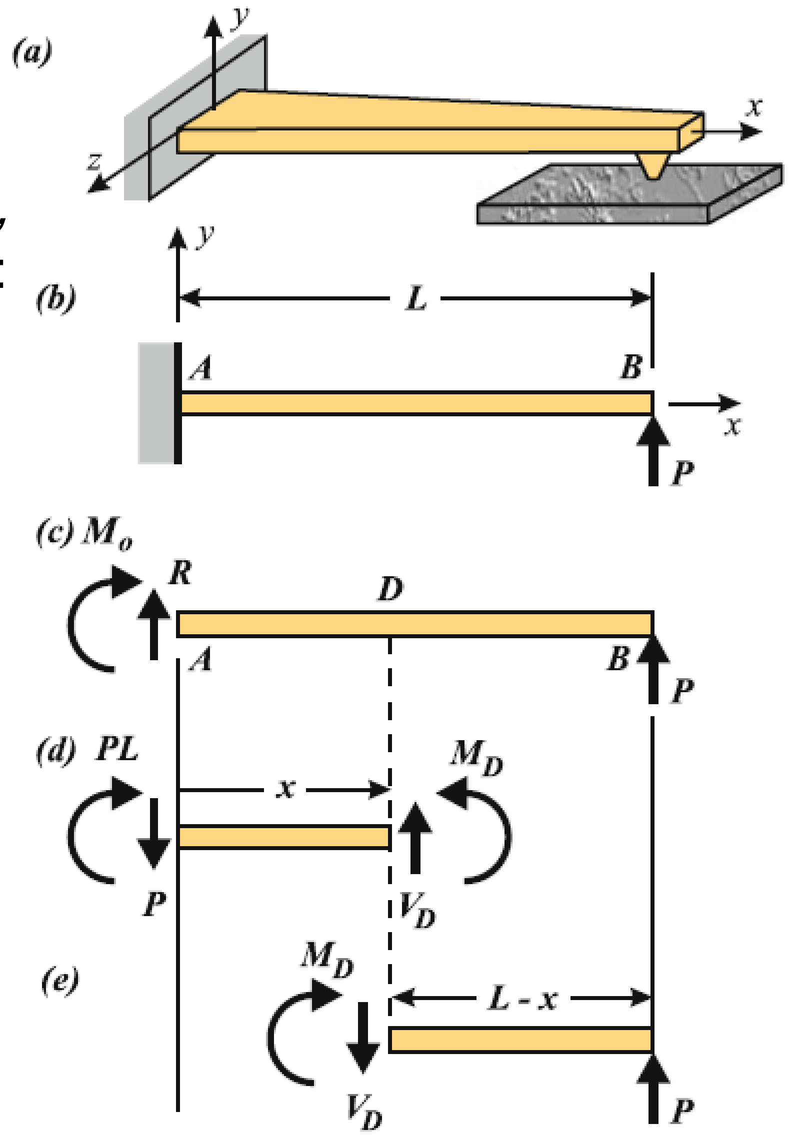
From force equilibrium in the *y direction* and moment equilibrium about the *z-axis,* the reaction force and reaction moment are:

$$R = -P \text{ and } M_0 = PL$$

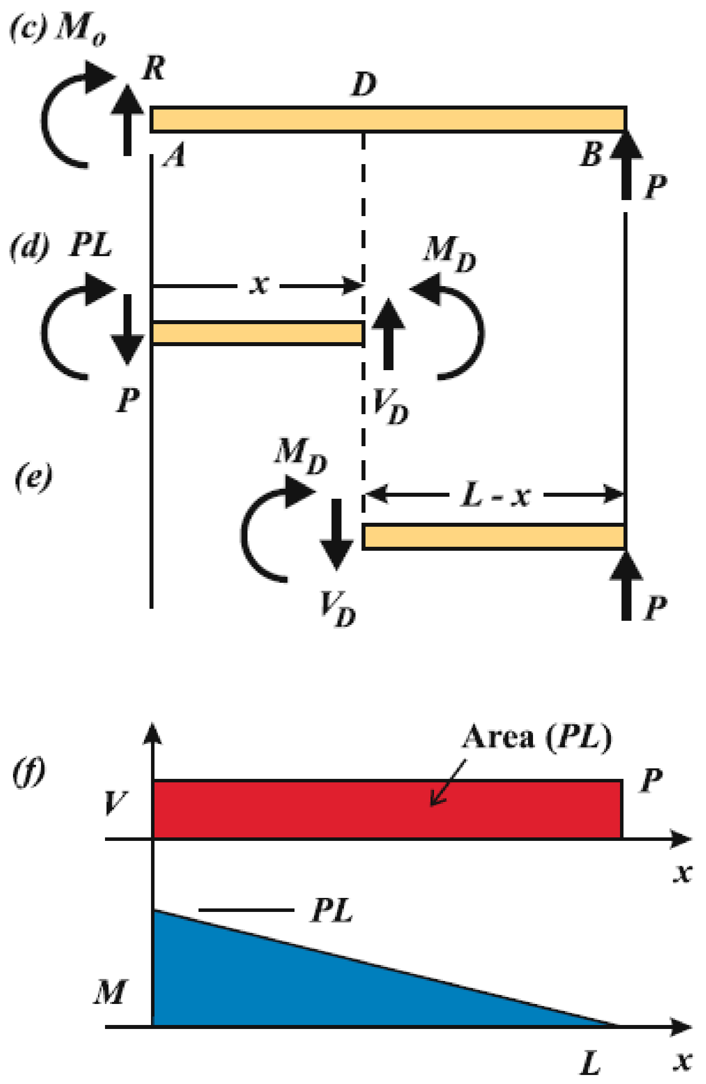
***Step 2.*** To investigate how the shear force and moment vary with distance *x* along the beam, a cut is taken at an arbitrary cross-section *D.* Since the load on the beam does not change over its length, only one cut needs to be taken.

Taking equilibrium of the right-hand FBD, segment DB (*Figure e*):

$$V_D = V(x) = P \text{ and } M_D = M(x) = P(L - x)$$



*Step 3.* The shear force and moment diagrams are shown in (*Figure f*). Using the given representative values, the maximum bending moment is $M_{max} = PL =$ (20 nN)(60 μm) =$1.2\times10^{-12}$ N·m. Shear force $V_D$ is plotted as positive since it acts upward on a +x-face (or downward on a −x-face). Moment $M_D$ is plotted positive as it causes compression on the top of the beam (it is a +z-moment on the +x-face or a −z-moment on a −x-face).

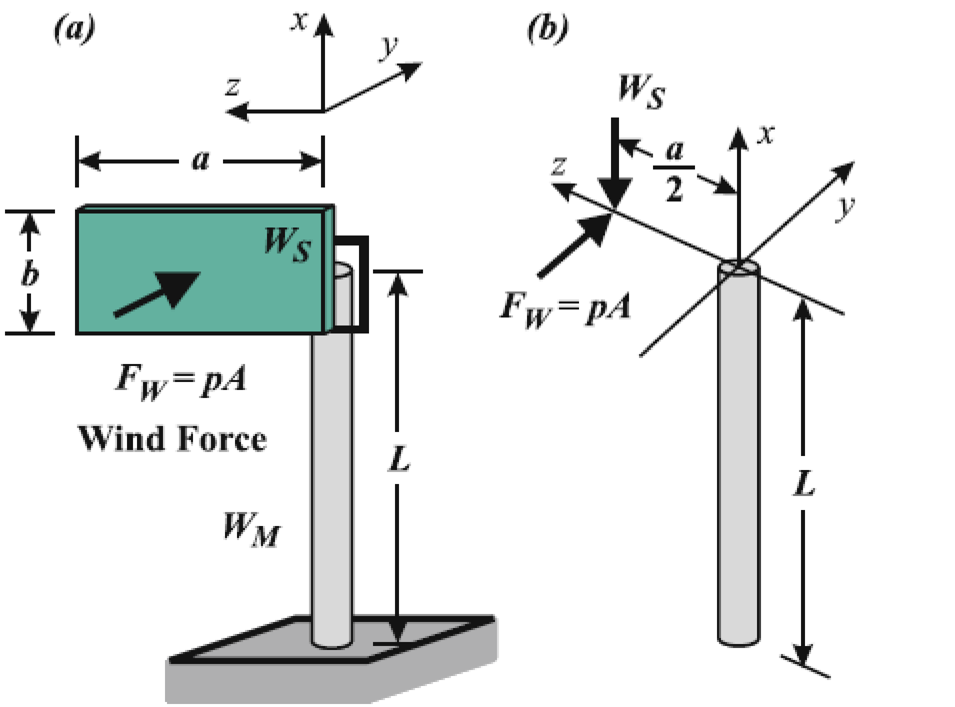


# Combined Loading

Components are frequently subjected to several types of loading at the same time. Two examples of combined loading follow.

## *Ex.9 Highway Sign with Wind Load*

**Given:** Signs overhanging highways are often supported by steel masts as shown in *Figure.* The sign is $b = 1.2$ m high and $a = 3.6$ m wide, and its center is $L = 4.8$ m above the road. Wind blows against the sign at $V = 160$ km/h. *The* sign weighs $W_S = 1.8$ kN and the mast weighs $W_M = 4.5$ kN.




**Required:** (a) Determine the reactions at the base of the mast.
(b) Determine the internal forces in the mast.

## Ex.10 Single-Arm Lug Wrench

**Given:** The loading on the compact lug wrenches that come in modern automobiles is similar to the wind loading on the sign of the previous example (*Figure*). In these lug wrenches, the lug nut is tightened by applying a downward force *F* at point C when the wrench arm is on the right side of the stem *AB.*