

# SHEAR FORCE AND BENDING MOMENT

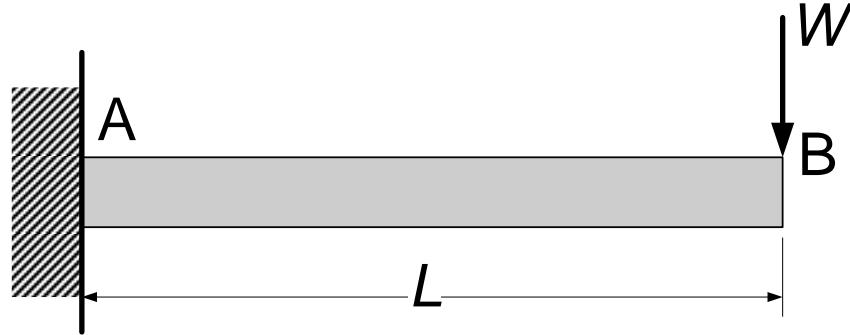
A beam must develop in its sections, a distribution of internal bending moments and shear forces, in order to resist any transversal (shear) Loads.

We will see that shear force and bending moment distributions are closely related, so it is convenient to consider them simultaneously.

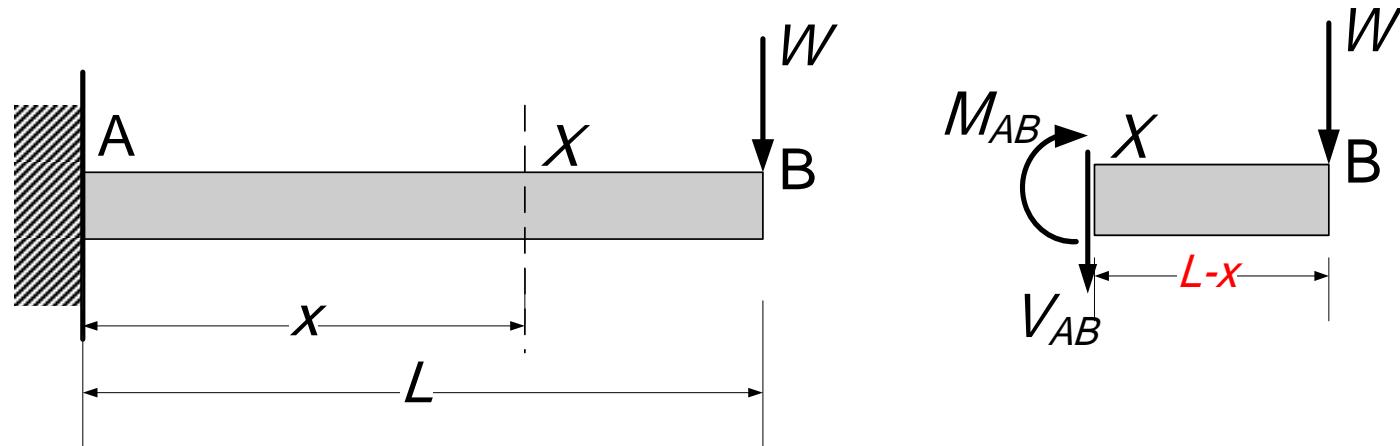
Since loading discontinuities, such as concentrated loads and/or a sudden change in the intensity of a distributed load, cause discontinuities in the distribution of shear force and bending moment, it is necessary to consider a series of sections, one between each loading discontinuity.

The method of construction shear force and bending moment diagrams will be illustrated by examples.

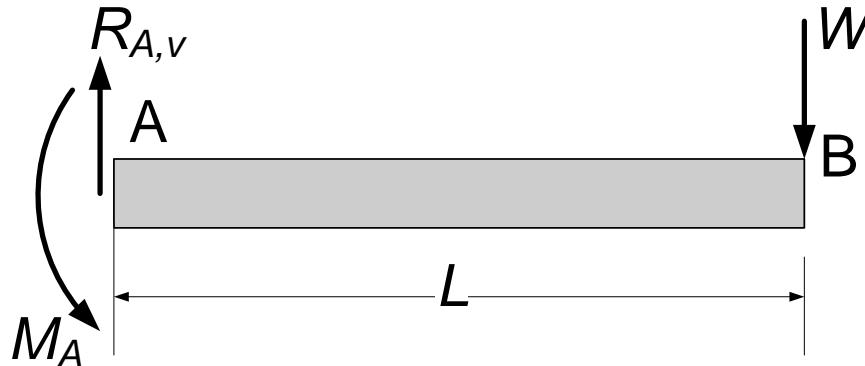
Ex. 1. Cantilever beam with a concentrated load at the free end.



In this example there are no loading discontinuities between the built-in end A and the free end B so that we may consider a section  $X$  at any point between A and B.



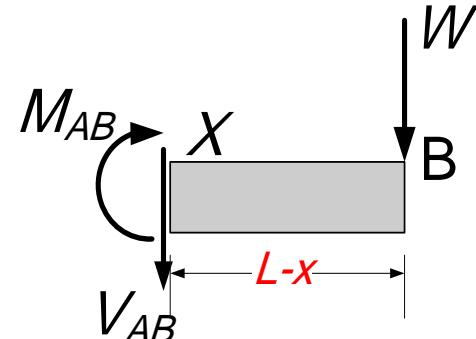
## Step 1, Reactions Computing (Not Necessary here)



$$\begin{aligned}\uparrow \sum F_y &= 0 \Rightarrow -W + R_{A,y} = 0 \Rightarrow R_{A,y} = W. \\ \downarrow \uparrow \sum M_{z,A} &= 0 \Rightarrow M_A - LW = 0 \Rightarrow M_A = LW\end{aligned}$$

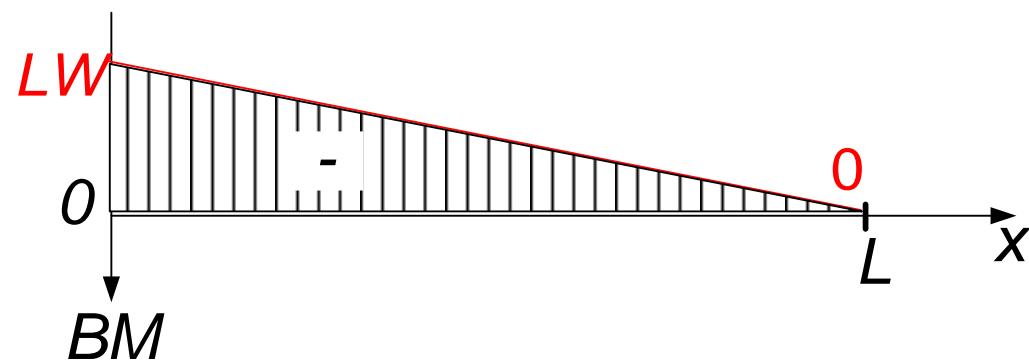
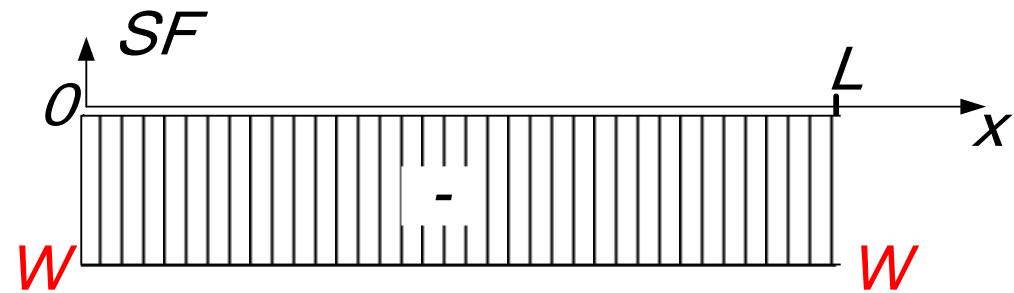
## Step 2, Internal Forces

- No point of load discontinuity between A & B, so
- Computing internal forces in segment AB:  $0 \leq x \leq L$



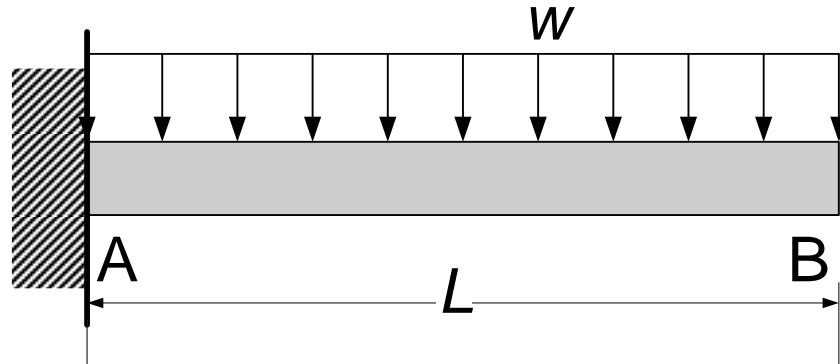
$$\begin{aligned}\uparrow \sum F_y &= 0 \Rightarrow -V_{AB} - W = 0 \Rightarrow V_{AB} = -W. \text{ (Const. Fun.)} \\ \downarrow \uparrow \sum M_{z,X} &= 0 \Rightarrow -M_{AB} - (L-x)W = 0 \Rightarrow M_{AB} = -(L-x)W. \text{ (Linear. Fun.)}\end{aligned}$$

## Step 3, Drawing the SF & BM diagrams



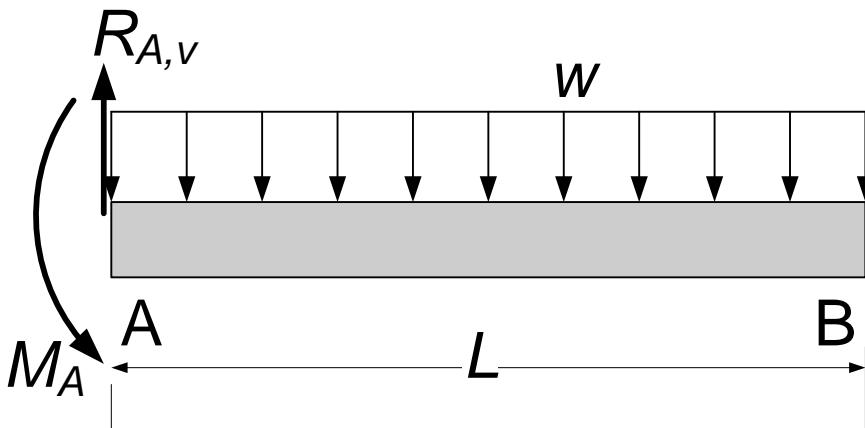
- 1- Draw axis  $ox$ ,
- 2- determine points A (0) & B ( $L$ )
- 3- fix the (+ve) direction of the diagram
- 4- Compute the values at A & B, then indicate on the diagram,
- 5- Draw the correct line between the two values (Const. , Linear,...)
- 6- Hatch the diagram and put the sign

**Ex 2.** Cantilever beam carrying a uniformly distributed load of intensity  $w$ .



## Solution

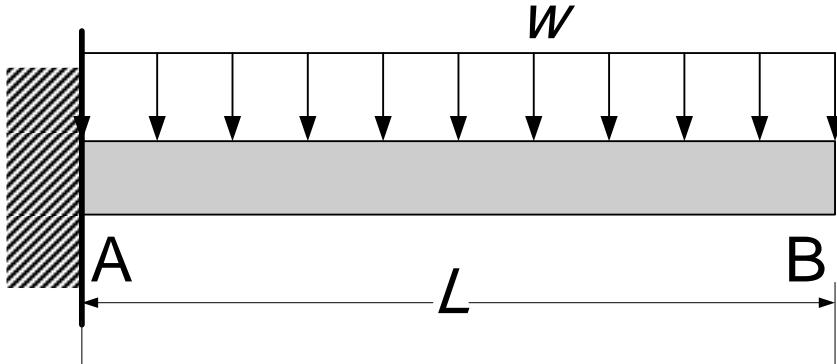
**Step 1,** Reactions Computing (Not Necessary in this example)



$$\begin{aligned} \uparrow \sum F_y &= 0 \Rightarrow -wL + R_{A,v} = 0 \Rightarrow R_{A,v} = wL. \\ \downarrow \uparrow \sum M_{z,A} &= 0 \Rightarrow M_A - (L/2)wL = 0 \Rightarrow M_A = wL^2/2 \end{aligned}$$

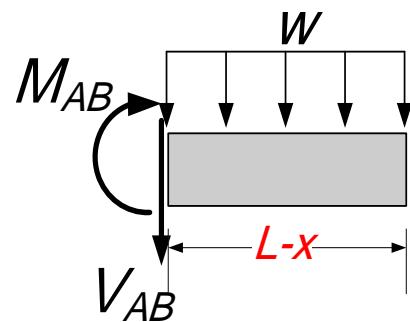
## Step 2, Internal Forces

- No point of load discontinuity between A & B, so
- Computing internal forces in segment AB:  $0 \leq x \leq L$



$$\uparrow \sum F_y = 0 \Rightarrow -V_{AB} - w(L-x) = 0 \Rightarrow V_{AB} = -w(L-x). \text{(Linear.Fun.)}$$

$$V_{AB}(A)|_{x=0} = -wL, V_{AB}(B)|_{x=L} = 0$$



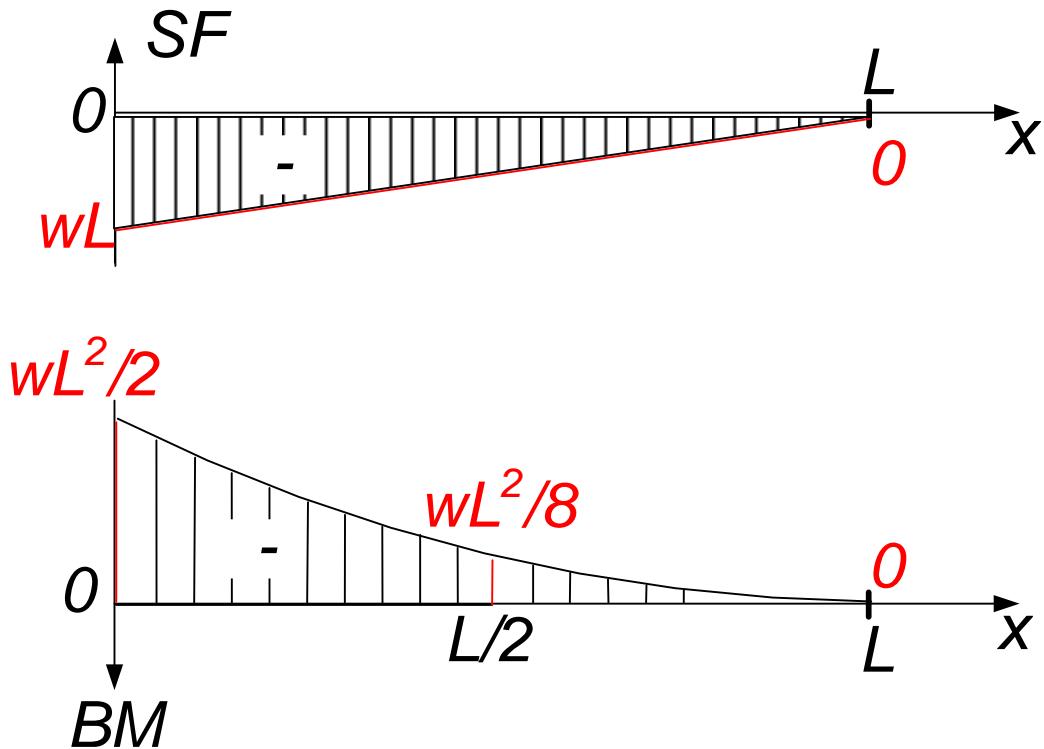
$$\downarrow \uparrow \sum M_{z,x} = 0 \Rightarrow -M_{AB} - [(L-x)/2]w(L-x) = 0$$

$$\Rightarrow M_{AB} = -w(L-x)^2 / 2. \text{(Quadratic.Fun.)}$$

$$M_{AB}(A)|_{x=0} = -wL^2 / 2, M_{AB}(B)|_{x=L} = 0$$

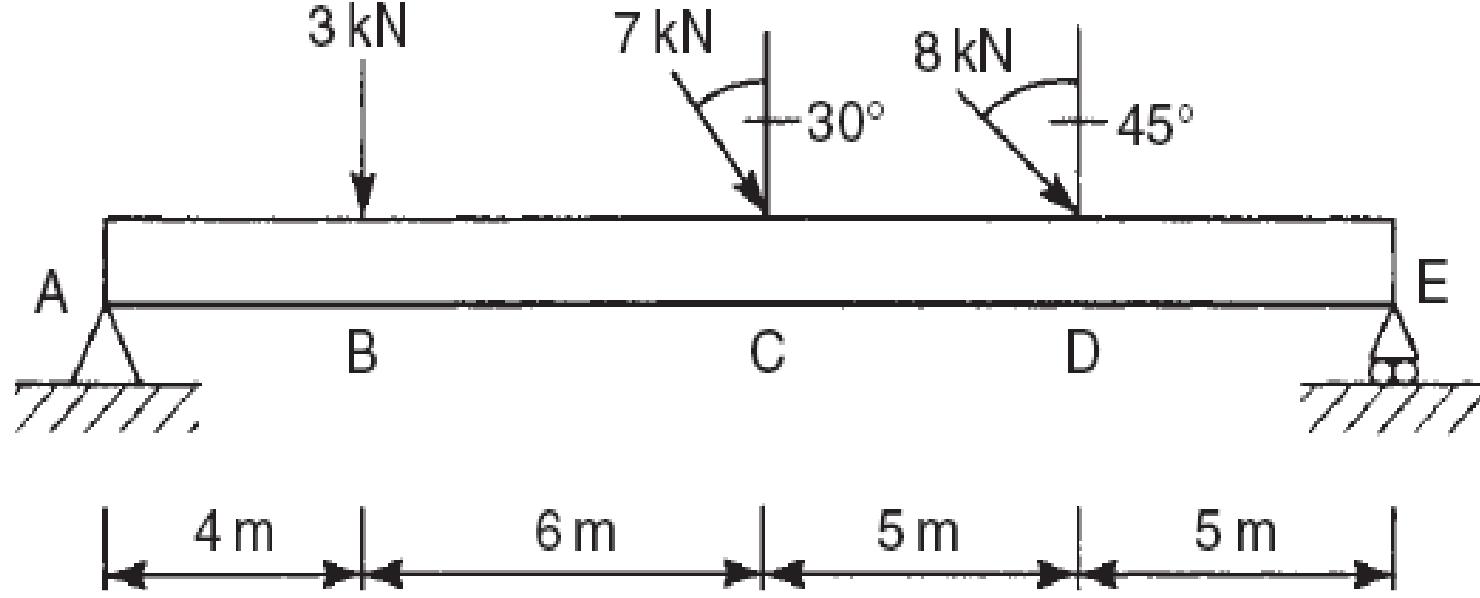
$$M_{AB}|_{x=L/2} = -wL^2 / 8.$$

## Step 3, Drawing the SF & BM diagrams



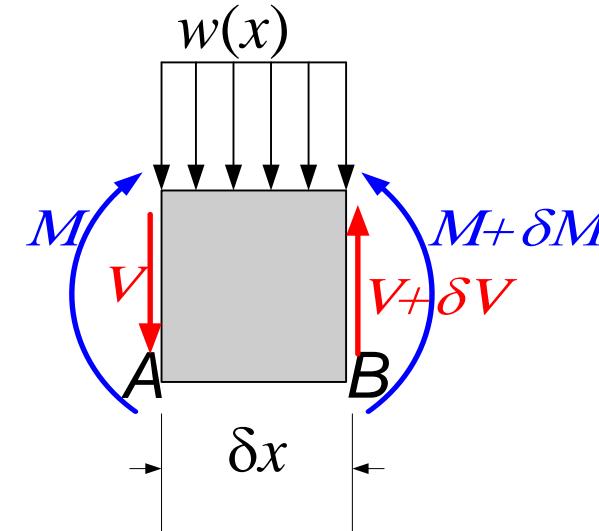
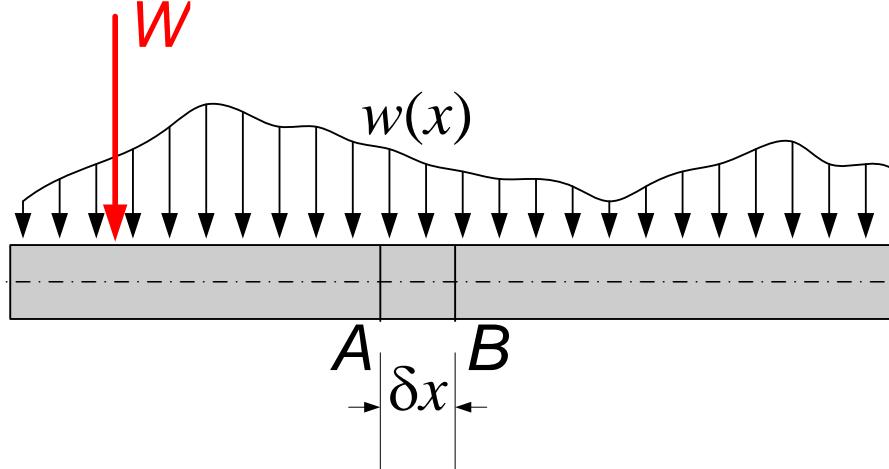
- 1- Draw axis ox,
- 2- determine points A (0) & B (L)
- 3- fix the (+ve) direction of the diagram
- 3- Compute the values at A & B, then indicate on the diagram,
- 4- Draw the correct line between the two values (Const. , Linear,...)
- 5- Hatch the diagram and put the sign

# Problem.1 Construct the normal force, shear force and bending moment diagrams for the beam shown in Fig.



# LOAD, SHEAR FORCE AND BENDING MOMENT RELATIONSHIPS

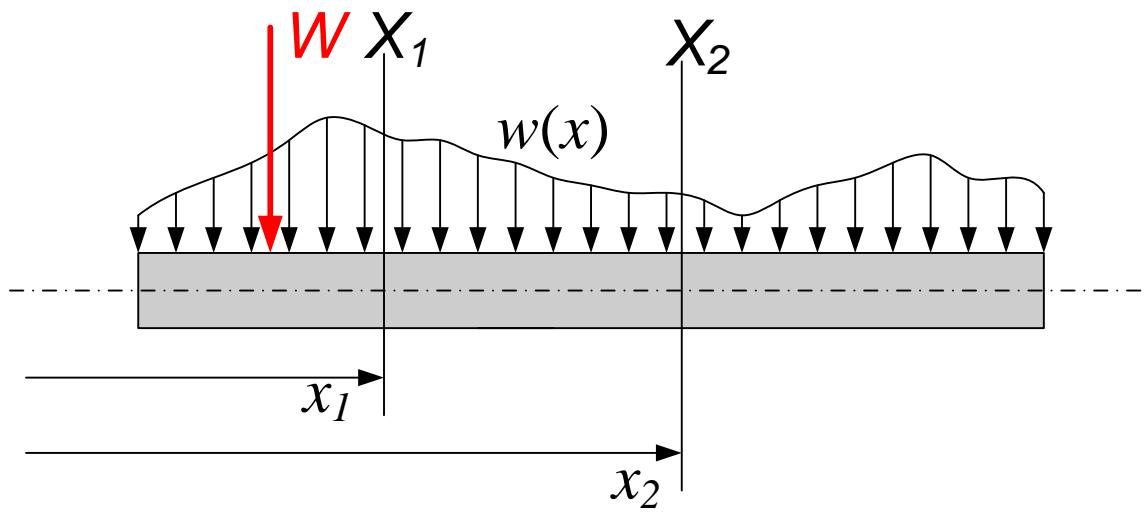
Examples have shown certain relationship between, load shear force and bending moment functions.



$$\uparrow \sum F_y = 0 \Rightarrow V + \delta V - V - w \delta x = 0 \Rightarrow \delta V - w \delta x = 0 \Rightarrow \boxed{\frac{dV}{dx} = w(x)}$$

$$\downarrow \uparrow \sum M_{Z,B} = 0 \Rightarrow M + \delta M - M + V \delta x + \frac{1}{2} \delta x (w \delta x) = 0 \Rightarrow$$

$$\delta M + V \delta x + \frac{1}{2} w (\delta x)^2 = 0 \Rightarrow \boxed{\frac{dM}{dx} = -V}$$

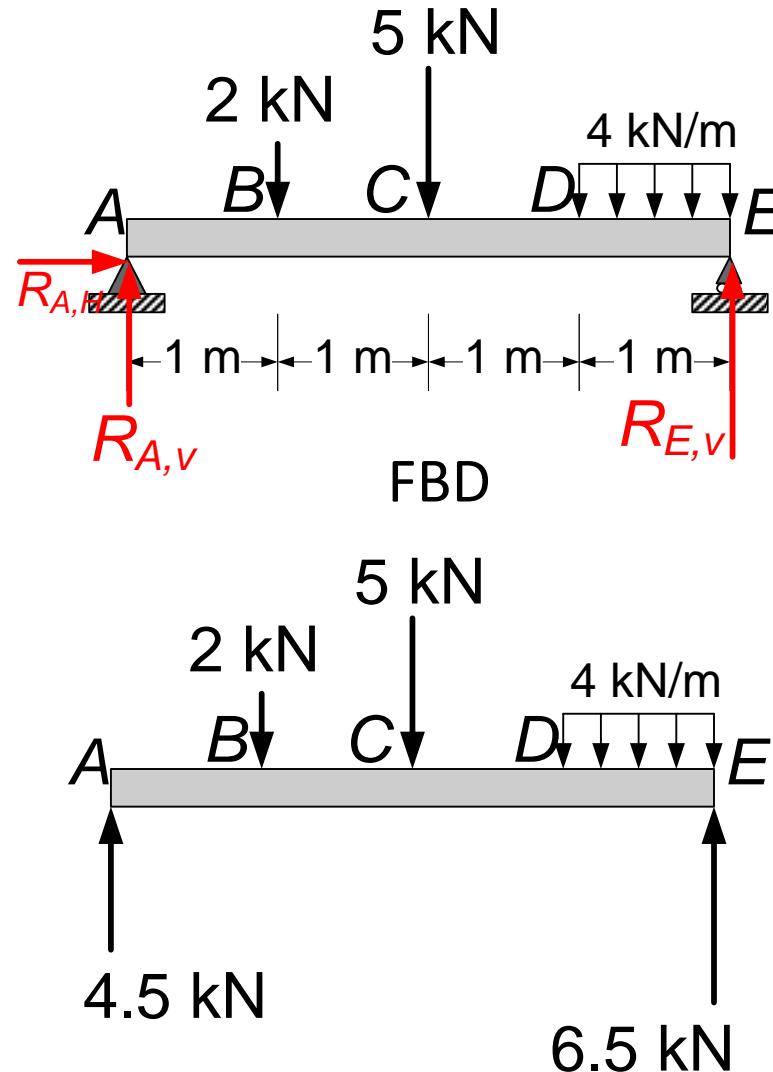


$$\boxed{\frac{dV}{dx} = w(x)} \Rightarrow V_2 - V_1 = \int_{x_1}^{x_2} w(x) dx$$

$$\boxed{\frac{dM}{dx} = -V} \Rightarrow M_2 - M_1 = - \int_{x_1}^{x_2} V(x) dx$$

$$\boxed{\frac{d^2M}{dx^2} = -w}$$

# EXAMPLE 1. Construct shear force and bending moment diagrams for the beam shown in Fig.



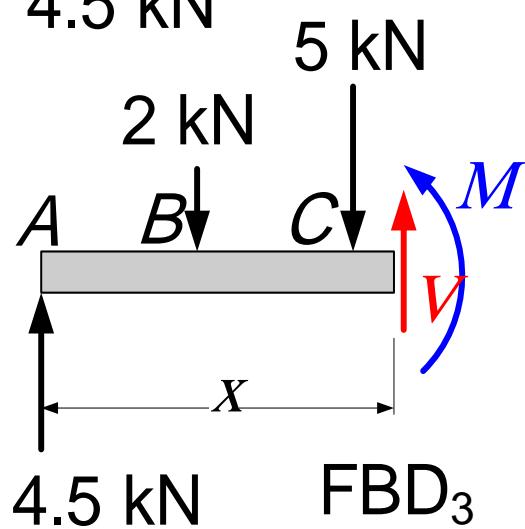
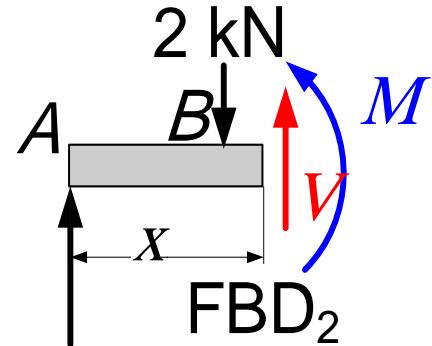
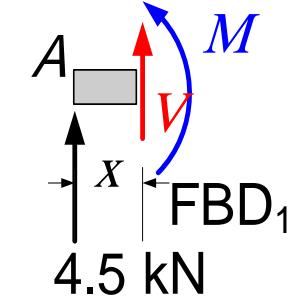
**Step 1, Reactions. From the FBD**

$$\sum_{\rightarrow}^+ F_x = 0 \Rightarrow R_{A,H} = 0.$$

$$\downarrow \uparrow \sum M_{z,A} = 0: -(1)(2) - (2)(5) - (3.5)(4)(1) + (4)R_{E,V} = 0 \\ \Rightarrow R_{E,V} = 6.5 \text{ kN}$$

$$\downarrow \uparrow \sum M_{z,B} = 0: (3)(2) + (2)(5) + (0.5)(4)(1) - (4)R_{A,V} = 0 \\ \Rightarrow R_{A,V} = 4.5 \text{ kN}$$

Check:  $\uparrow \sum F_y = 0 \Rightarrow 4.5 + 6.5 - 2 - 5 - (4)(1) = 0.$



## Step2, Internal Forces:

- Segment AB,  $0 < x < 1$ . From FBD<sub>1</sub>

$$\downarrow \uparrow \sum M_{z,X} = 0: -(x)(4.5) + M_{AB} = 0 \Rightarrow$$

$$M_{AB} = 4.5x \text{ kN.m}; M(A^+) = 0, M(B^-) = 4.5 \text{ kN.m}$$

$$\uparrow \sum F_y = 0: 4.5 + V_{AB} = 0 \Rightarrow V_{AB} = -4.5 \text{ kN} = \text{Const.}$$

- Segment BC,  $1 < x < 2$ . From FBD<sub>2</sub>

$$\downarrow \uparrow \sum M_{z,X} = 0: -(x)(4.5) + (x-1)(2) + M_{BC} = 0 \Rightarrow$$

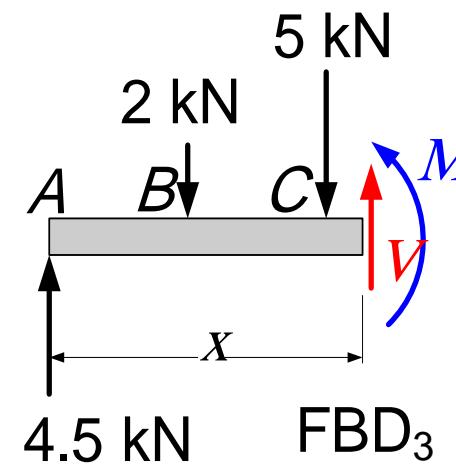
$$M_{BC} = 2.5x + 2, \text{ kN.m}; M(B^+) = 4.5, M(C^-) = 7 \text{ kN.m}$$

$$\uparrow \sum F_y = 0: 4.5 - 2 + V_{BC} = 0 \Rightarrow V_{BC} = -2.5 \text{ kN} = \text{Const.}$$

- Segment CD,  $2 < x < 3$ . From FBD<sub>3</sub>

$$\downarrow \uparrow \sum M_{z,X} = 0: -(x)(4.5) + (x-1)(2) + (x-2)(5) + M_{CD} = 0$$

$$M_{CD} = -2.5x + 12 \text{ kN.m}; M(C^+) = 7, M(D^-) = 4.5 \text{ kN.m}$$



- Segment CD,  $2 < x < 3$ . From FBD<sub>3</sub>

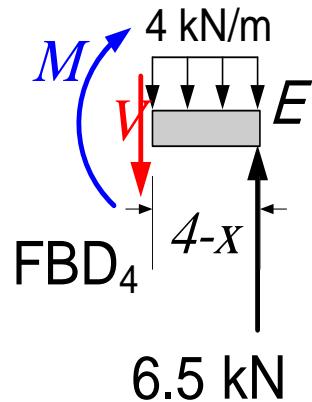
$$M_{CD} = -2.5x + 12 \text{ kN.m}; M(C^+) = 7, M(D^-) = 4.5 \text{ kN.m}$$

$$\uparrow \sum F_y = 0: 4.5 - 2 - 5 + V_{CD} = 0 \Rightarrow V_{CD} = 2.5 \text{ kN} = \text{Const.}$$

- Segment DE,  $3 < x < 4$ . From FBD<sub>4</sub>, (-ve face)

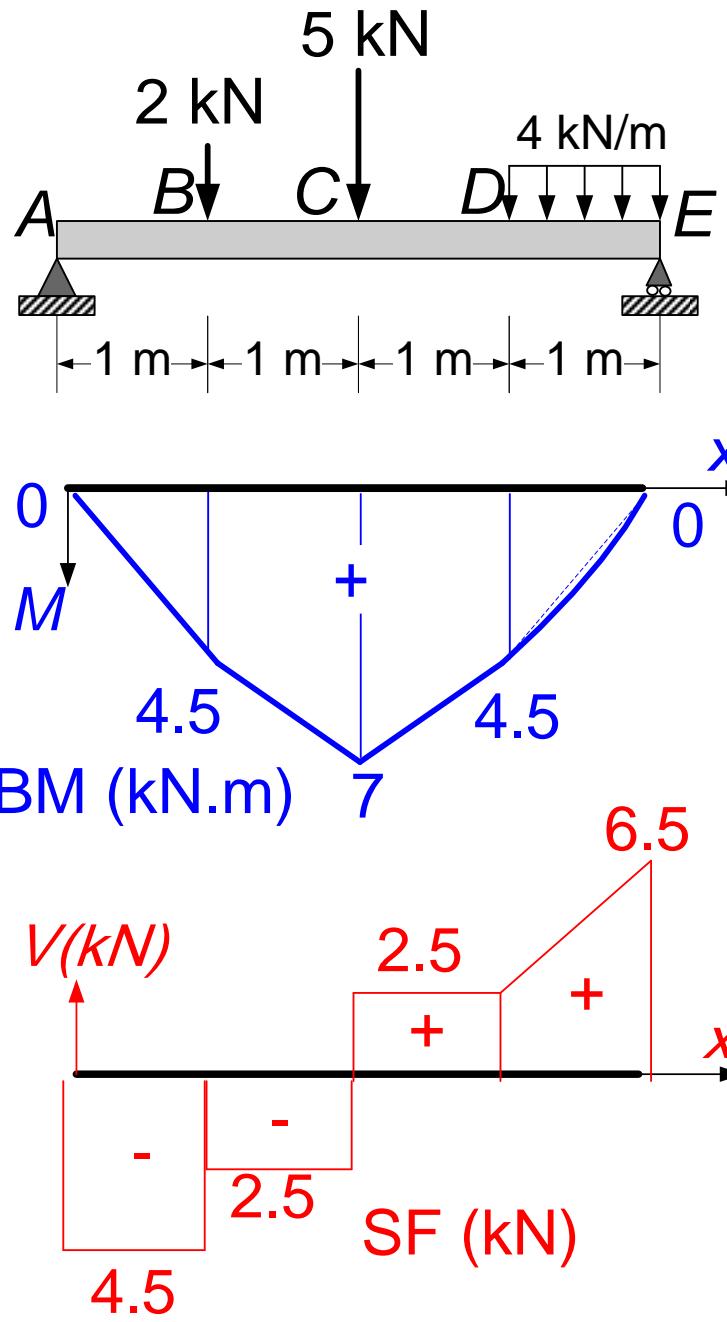
$$\downarrow \uparrow \sum M_{z,X} = 0: (4-x)(6.5) - \frac{1}{2}(4-x)(4)(4-x) - M_{DE} = 0$$

$$\Rightarrow M_{DE} = -2(4-x)^2 + 6.5(4-x) \text{ kN.m}; M(D^+) = 4.5 \text{ kN.m}, M(E^-) = 0.$$



$$\uparrow \sum F_y = 0: -V_{DE} - (4)(4-x) + 6.5 = 0$$

$$\Rightarrow V_{DE} = -4(4-x) + 6.5 \text{ kN}, V(D^+) = 2.5 \text{ kN}, V(E^-) = 6.5 \text{ kN}$$



### Step3, SF & BM Diagrams

$$M_{AB} = 4.5x \text{ kN.m}; M(A^+) = 0, M(B^-) = 4.5 \text{ kN.m}$$

$$V_{AB} = -4.5 \text{ kN} = \text{Const.}$$

$$M_{BC} = 2.5x + 2, \text{ kN.m}; M(B^+) = 4.5, M(C^-) = 7 \text{ kN.m}$$

$$V_{BC} = -2.5 \text{ kN} = \text{Const.}$$

$$M_{CD} = -2.5x + 12 \text{ kN.m}; M(C^+) = 7, M(D^-) = 4.5 \text{ kN.m}$$

$$V_{CD} = 2.5 \text{ kN} = \text{Const.}$$

$$M_{DE} = -2(4-x)^2 + 6.5(4-x) \text{ kN.m}; M(D^+) = 4.5 \text{ kN.m}, M(E^-) = 0.$$

$$V_{DE} = -4(4-x) + 6.5 \text{ kN}, V(D^+) = 2.5 \text{ kN}, V(E^-) = 6.5 \text{ kN}$$

**Problem 1.** Construct shear force and bending moment diagrams for the shown beam.

$L=1.5 \text{ m}$ ,  $W_2=8 \text{ kN}$ ,  $W_1=6 \text{kN}$ ,  $w=4\text{kN/m}$ .

