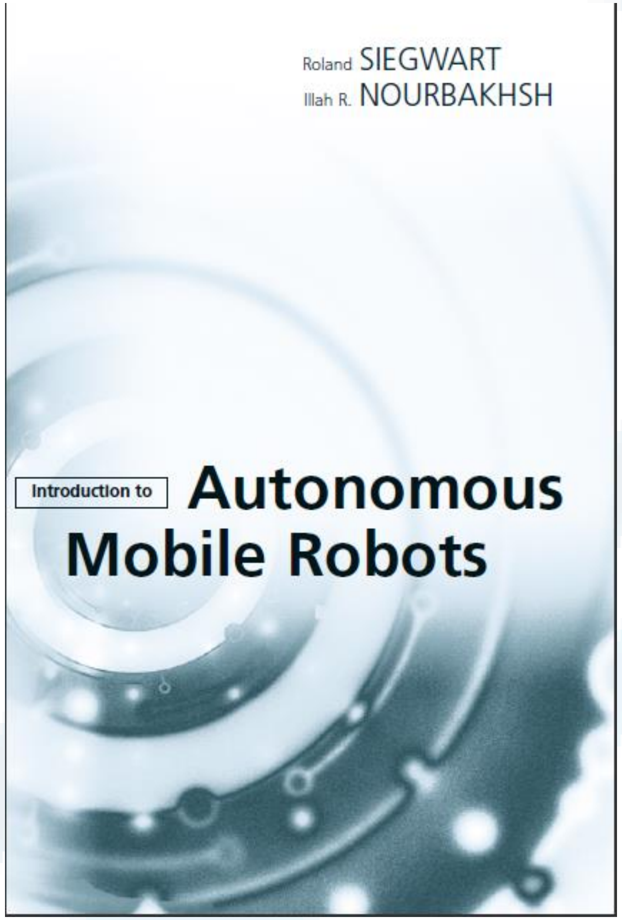
Mobile Robots Modeling

Introduction to WMR kinematic model

Books



Introduction to

Mobile Robot Control

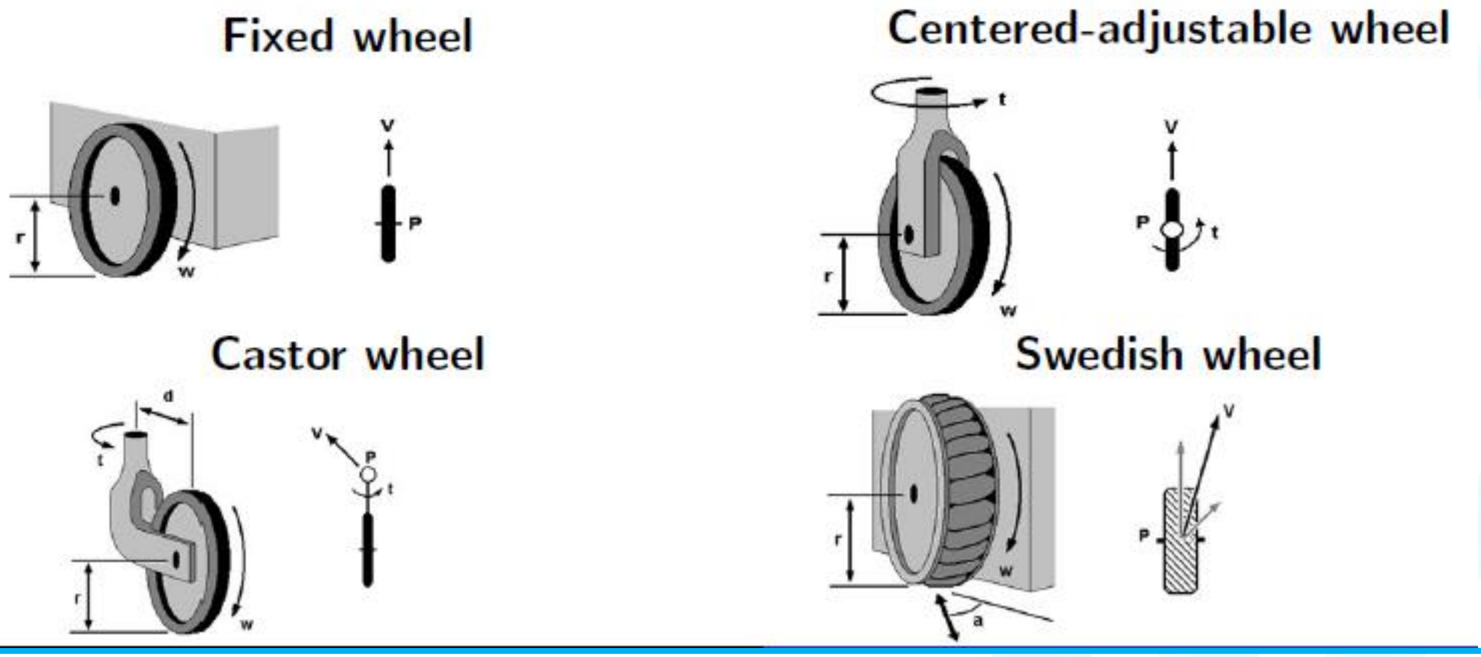
by
Spyros G. Tzafestas

National Technical University of Athens
Athens, Greece

Wheeled Mobile Robot WMR

- The Wheels is the most suitable solution for many applications from the mechanical point of view.
- Three wheels are sufficient to guarantee the static stability of the vehicle.

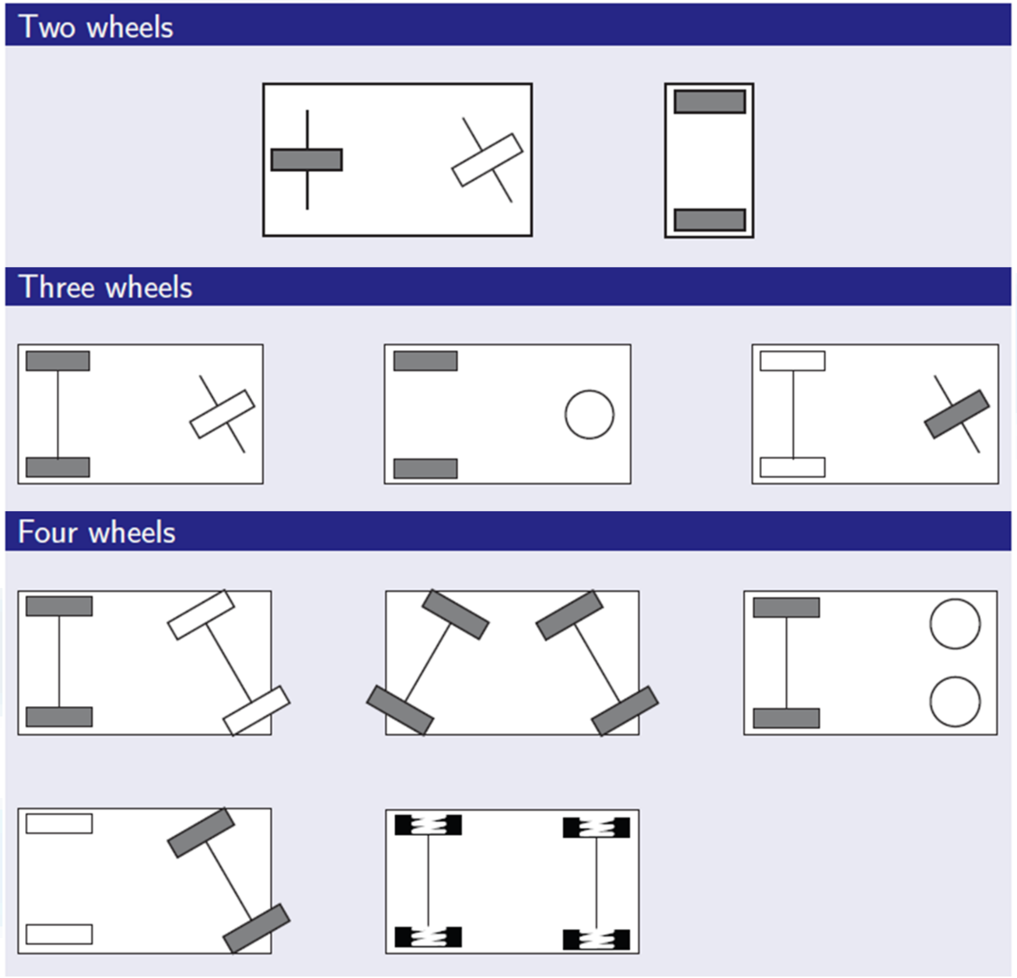
1. What wheels to use?
2. How many wheels to use?



Swedish Wheels

- They are based on the idea to take advantage of the friction also transversely to the direction of the wheel's motion.

Number of Wheels: Possible configuration

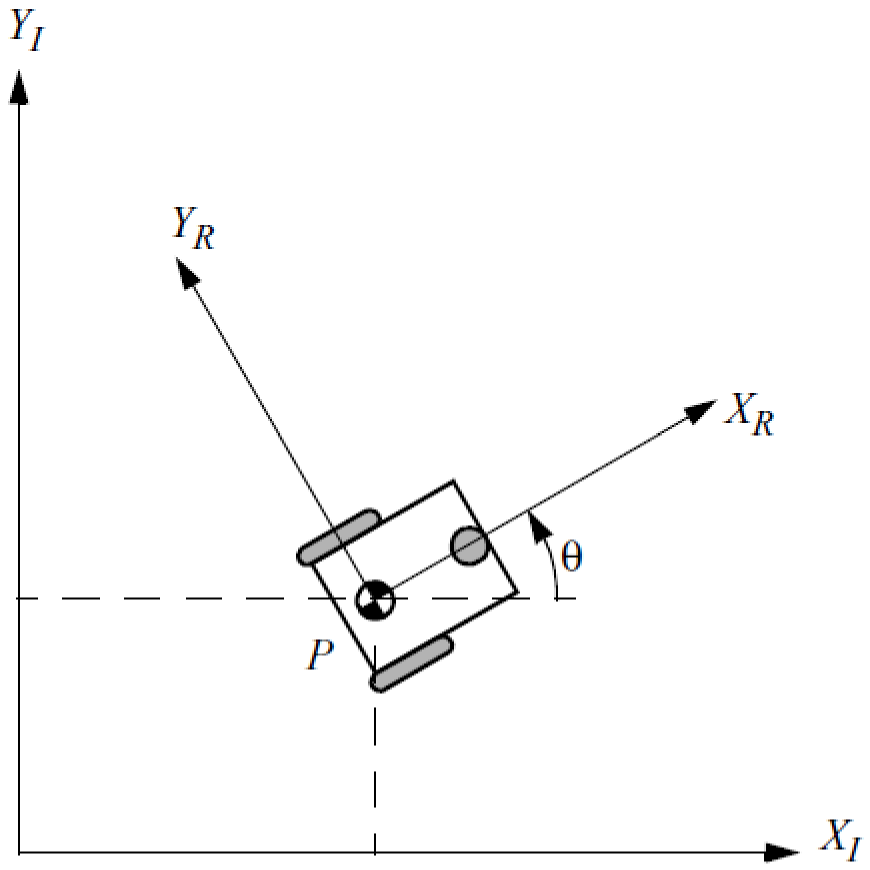


Kinematic Model : Position representation

WMR is considered as a rigid body on wheels

- Total dimensionality of WMR (shown beside) is 3
- Position in the plan (2)
- Orientation about the vertical axis (1)

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \xrightarrow{motion\ term} \dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$



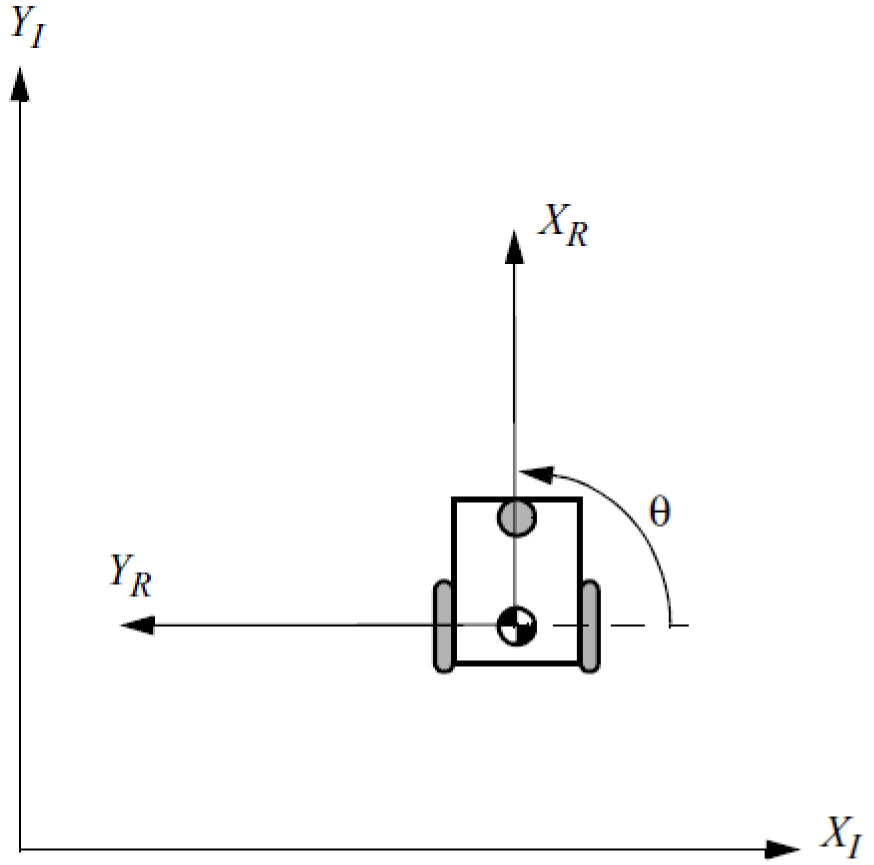
Kinematic model: velocity representation

$$\dot{\xi}_R = R(\theta)\,\dot{\xi}_I$$

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For example, when $\theta = \pi/2$

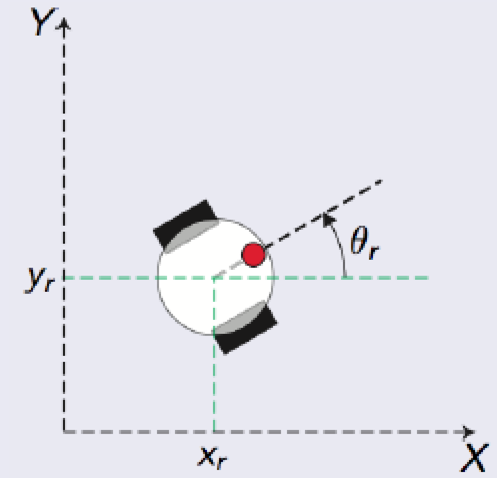
$$\dot{\xi}_R = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$



Configurations Space

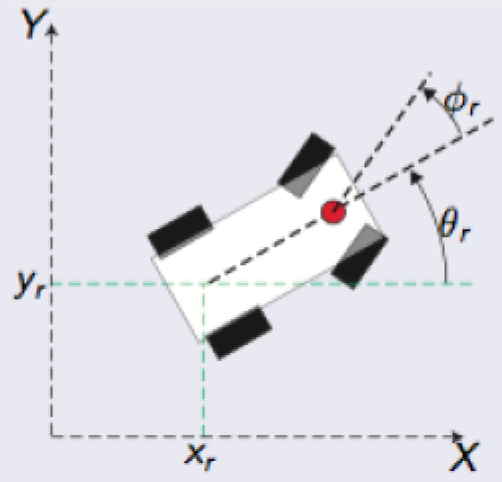
It has dimensions equal to the number of parameters needed to uniquely describe the configuration of a mobile robot. And depends on the structure of the considered robot.

Unicycle



$$q = [x_r, y_r, \theta_r]^T \in \mathbb{R}^3$$

Bicycle



$$q = [x_r, y_r, \theta_r, \phi_r]^T \in \mathbb{R}^4$$

Constraints

- **Definition 1**: a constraint is any condition imposed to a material system that prevents it to assume a generic position and/or act of motion.
- **Definition 2**: a material system is subject to a holonomic constraints if finite relation between the coordinates of the system are present, or if differentiable/integrable relations between the coordinates of the system are present.
- **Definition 3**: a constraint is said non-holonomic if a differential relation between the coordinates is not reducible to finite form.

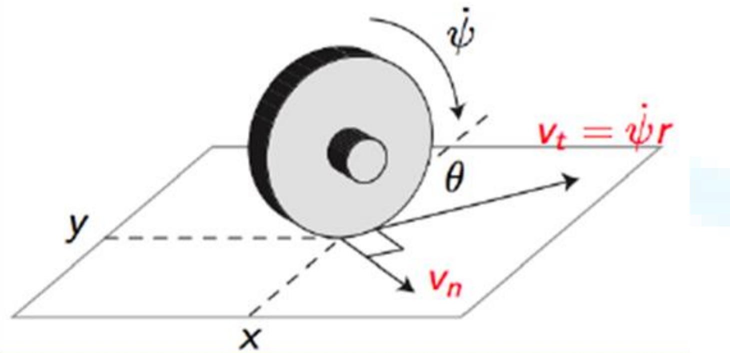
Suppose a constraint of the form $\mathrm{F}(\mathrm{q}, \dot{q}, \mathrm{t}) = 0$

this constraint is holonomic if it can be converted to the form $\mathrm{F}(\mathrm{q}, \mathrm{t}) = 0$

Non-Holonomic constraint

- A wheel rolls without slipping?
- The wheel introduce in the system a non-holonomic constraint since it does not allow normal translation to the rolling direction.
- The wheel constraints the instant robot mobility, without reducing the configuration space… (eg: parking parallel).
- With constraint Vn=0

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$



Without constraints:

$$\begin{cases} \dot{x} & = & v_t \cos\theta + v_n \cos\left(\theta + \frac{\pi}{2}\right) \\ \dot{y} & = & v_t \sin\theta + v_n \sin\left(\theta + \frac{\pi}{2}\right) \end{cases}$$

Notes.........

- All underactuated robots are non-holonomic robots
- Mobile robot with no constraint is holonomic
- Mobile robot capable of only translation is also holonomic

Suppose a robot of N degree of freedom and K actuators

If N=K the robot is holonomic

|N-K| indicates the number of non-holonomic constraints

Pfaffian constraint

- A non-holonomic constraint is called Pfaffian if it is linear in $\dot{q}$, if it is can be expressed in the form

$$a_i(q)\dot{q} = 0, i = 1,2 \dots r$$

- Constraint matrix equation (r wheels)

$$A(q)\dot{q} = 0, A(q) = \begin{bmatrix} a_1(q) \\ a_2(q) \\ \vdots \\ a_r(q) \end{bmatrix}$$

What's about this example?

$$q_1\dot{q}_1 + q_2\dot{q}_2 + q_3\dot{q}_3 = 0$$

Wheel kinematic constraints

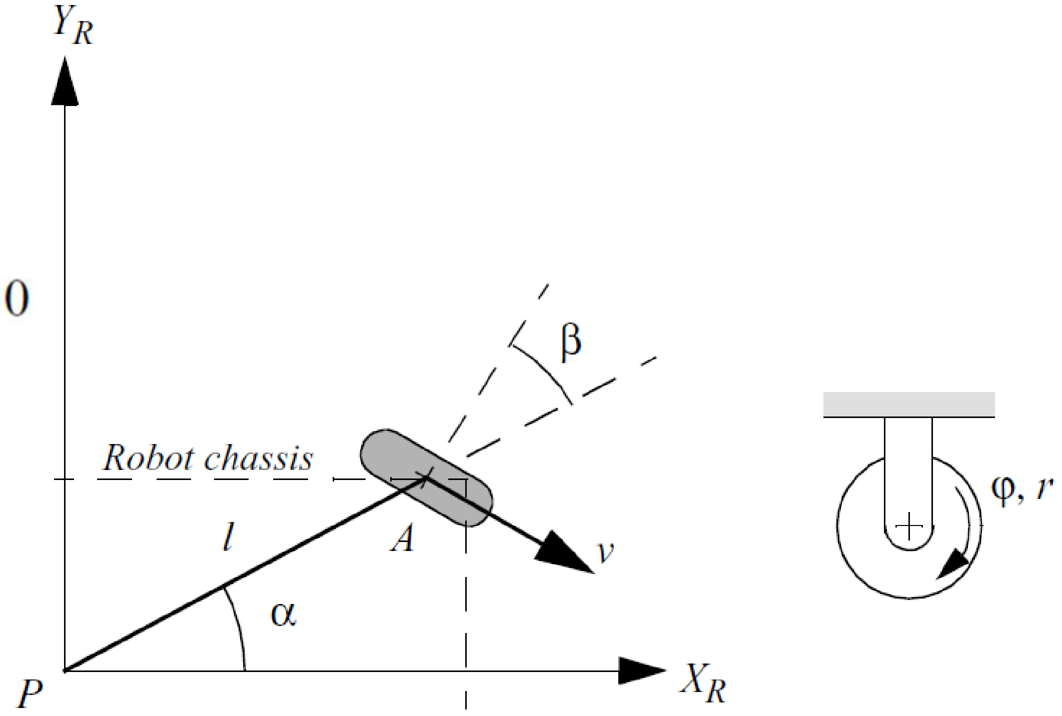
Two constraints for every wheel type, rolling and slipping.

Fixed standard wheel

β = constant

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta \end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

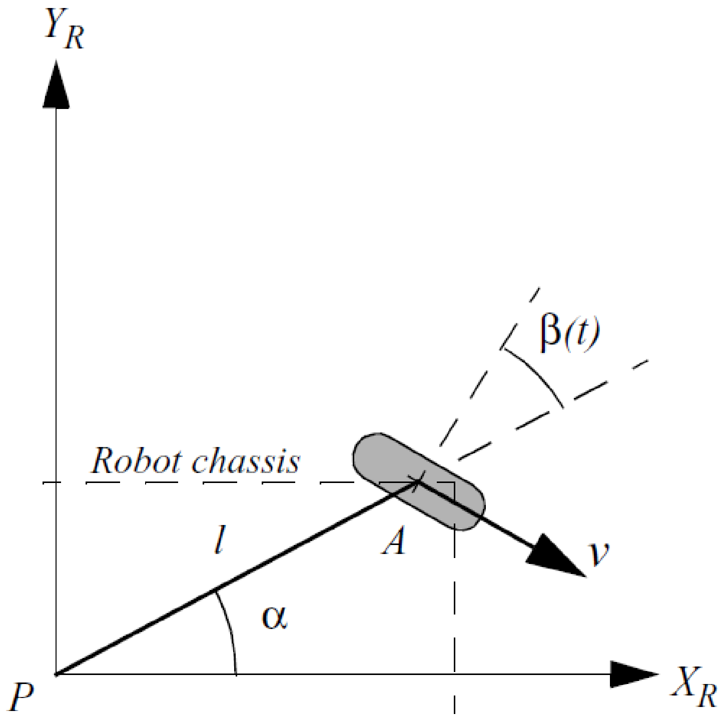


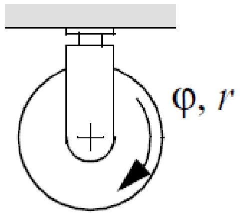
Steered standard wheel

$\beta(t) = is\ not$ constant

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l\sin\beta\end{bmatrix} R(\theta)\dot{\xi}_I = 0$$

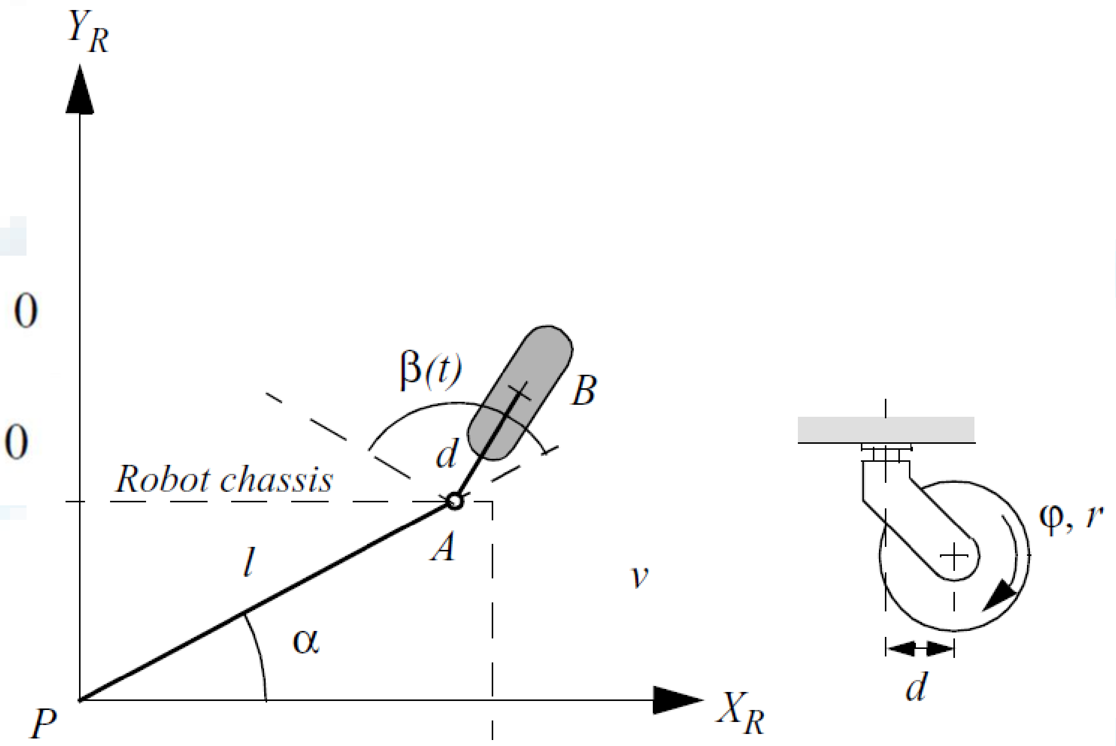




Castor Wheel

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & (-l)\cos\beta\end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & d + l\sin\beta\end{bmatrix} R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$



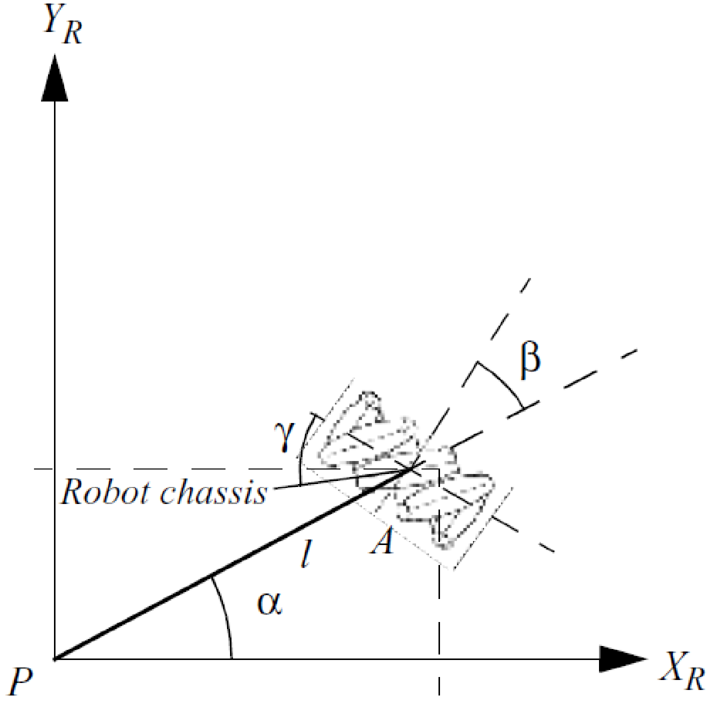
Swedish wheel

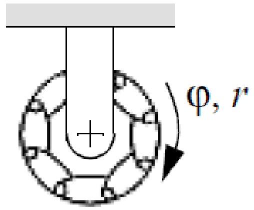
$$\left[\sin(\alpha+\beta+\gamma)\ \ -\cos(\alpha+\beta+\gamma)\ \ (-l)\cos(\beta+\gamma)\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}\cos\gamma = 0$$

$$\left[\cos(\alpha+\beta+\gamma)\ \ \sin(\alpha+\beta+\gamma)\ \ l\sin(\beta+\gamma)\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}\sin\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

$\varphi_{sw}(t) = rotation of\ small\ rollers$

r_{sw}: radius of small rollers



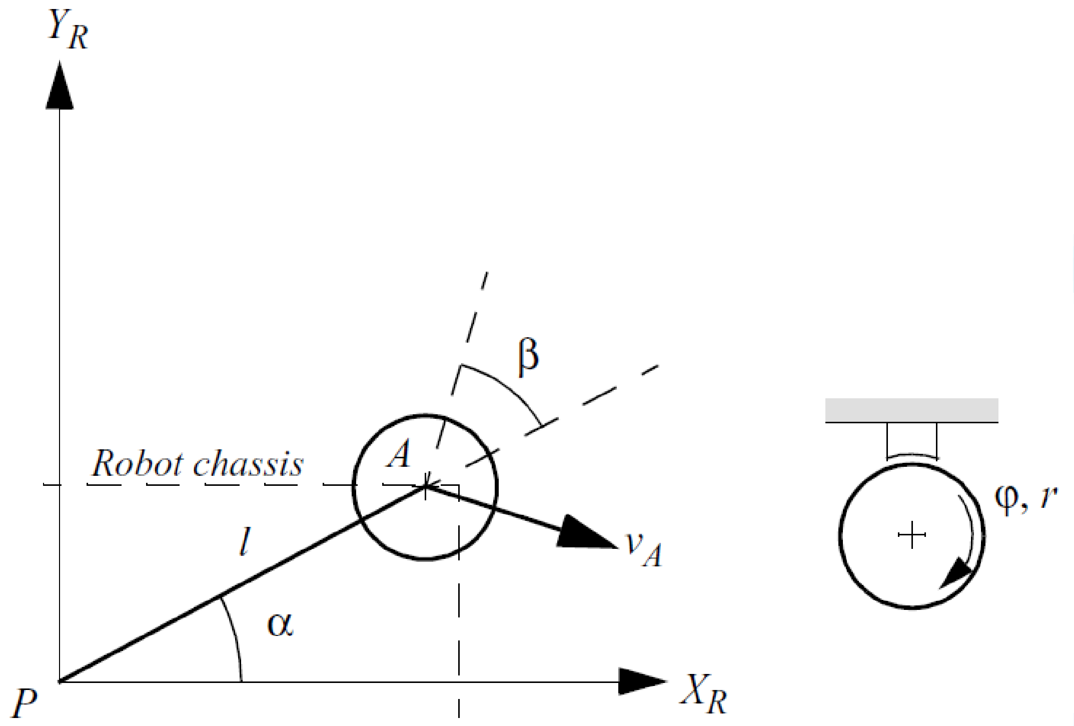


Spherical wheel

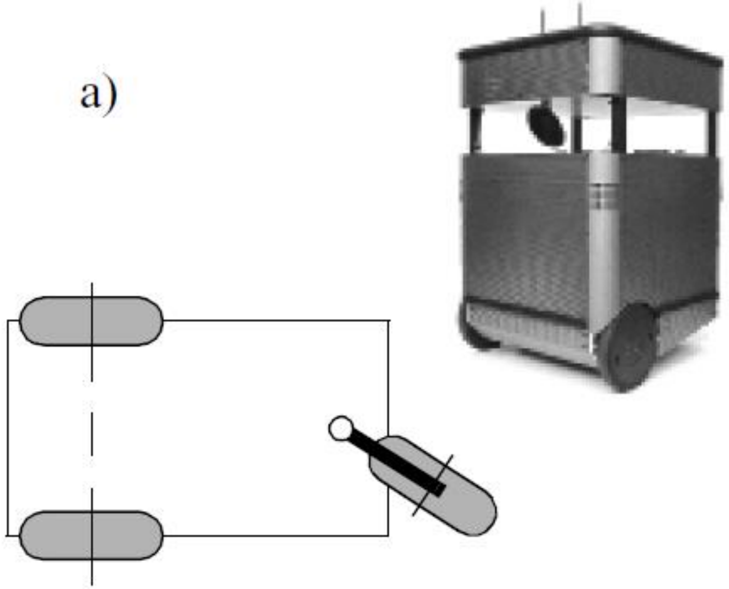
$\beta(t) = is\ a\ free\ variable$

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ (-l)\cos\beta\right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

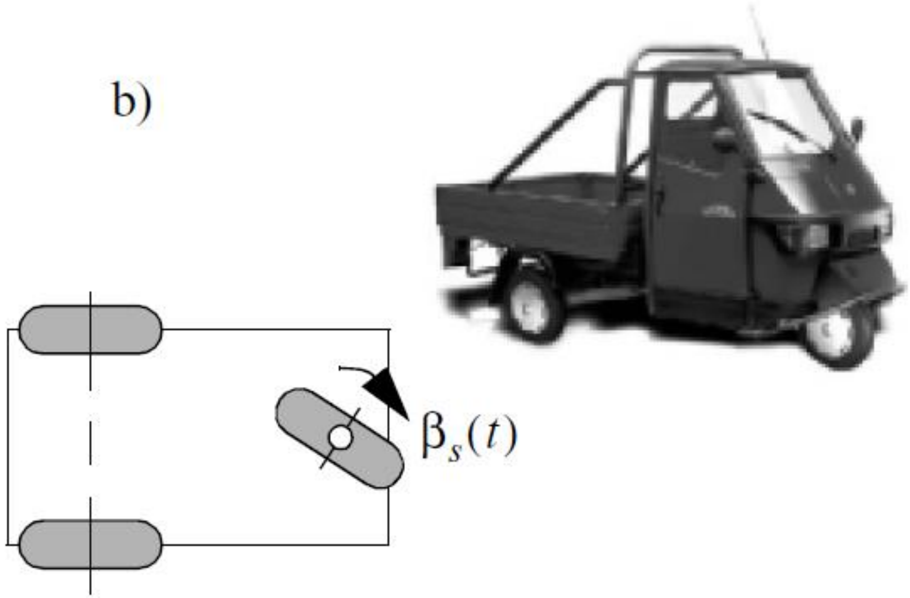
$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin\beta\right] R(\theta)\dot{\xi}_I = 0$$



a)



b)



What's a WMR kinematic constraints?

Only fixed standard wheels and steerable standard wheels have impact on robot chassis kinematics.......

Differential kinematic model

General formulation

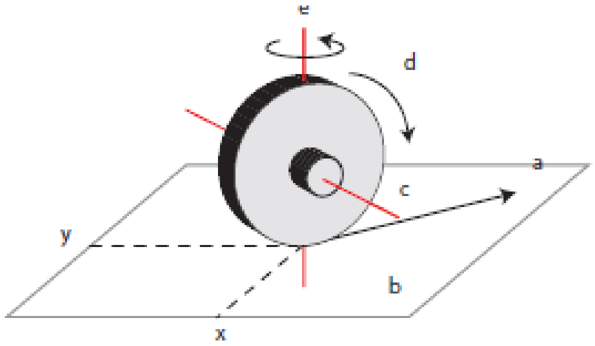
$$\dot{q} = G(q)v$$

- It represents the allowable directions of motion in the configuration space (allowable velocities)
- It binds speeds in the operational space with speeds in the configuration space

Example

Unicycle Kinematic Model

$$\dot{q} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$



- v: the linear velocity of the contact point between the wheel and the ground and is equal to the product between angular velocity of the wheel around its horizontal axis and the radius of the wheel
- ω: angular velocity of the robot, equals to the angular velocity of the wheel around the vertical axis

Control inputs

By acting on v and ω it is possible to modify the robot configuration

Thanks

Think about wheels (number and type) you want to use when designing a WMR.....