

مقرر الرياضيات المتقطعة

جلسة العلمي السادسة

Sets



Recall that \mathbf{R} denotes the set of all real numbers, \mathbf{Z} the set of all integers, and \mathbf{Z}^+ the set of all positive integers. Describe each of the following sets.

- a. $\{x \in \mathbf{R} \mid -2 < x < 5\}$
- b. $\{x \in \mathbf{Z} \mid -2 < x < 5\}$
- c. $\{x \in \mathbf{Z}^+ \mid -2 < x < 5\}$

Solution:

- a. $]-2,5[$
- b. $\{-1,0,1,2,3,4\}$
- c. $\{1,2,3,4\}$

Sets

4. Indicate the elements in each set defined in (a)–(f).
- $S = \{n \in \mathbf{Z} \mid n = (-1)^k, \text{ for some integer } k\}$.
 - $T = \{m \in \mathbf{Z} \mid m = 1 + (-1)^i, \text{ for some integer } i\}$.
 - $U = \{r \in \mathbf{Z} \mid 2 \leq r \leq -2\}$
 - $V = \{s \in \mathbf{Z} \mid s > 2 \text{ or } s < 3\}$
 - $W = \{t \in \mathbf{Z} \mid -1 < t < -3\}$
 - $X = \{u \in \mathbf{Z} \mid u \leq 4 \text{ or } u \geq 1\}$

Solution:

- $S=\{1,-1\}$
- $T=\{2,0\}$
- $U=\{\}=\emptyset$
- $V=[3,\infty[\cup]-\infty, 2] = \mathbf{Z}$
- $W=\{\}=\emptyset$
- $X=]-\infty, 4] \cup [1, \infty[= \mathbf{Z}$

Sets



Which of the following sets are equal?

$$A = \{a, b, c, d\} \quad B = \{d, e, a, c\}$$

$$C = \{d, b, a, c\} \quad D = \{a, a, d, e, c, e\}$$

Solution:

$$A=C$$

$$B=D$$

Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$. Answer each of the following questions. Give reasons for your answers.

- a. Is $B \subseteq A$? b. Is $C \subseteq A$?

Solution:

- False , $\exists j \in B$ but $j \notin A$
- True, every element in C exist in A

Sets

List the members of these sets.

- a) $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
- b) $\{x \mid x \text{ is a positive integer less than } 12\}$
- c) $\{x \mid x \text{ is the square of an integer and } x < 100\}$
- d) $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

Solution:

- a. $\{-1, 1\}$
- b. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
- c. $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$
- d. \emptyset

Sets



- a. Is the number 0 in \emptyset ?
- b. Is $\emptyset = \{\emptyset\}$?
- c. Is $\emptyset \in \{\emptyset\}$?
- d. Is $\emptyset \in \emptyset$?

Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, and $C = \{d, g\}$.

- c. Is $C \subseteq C$?
- d. Is C a proper subset of A ?

Solution:

- a. False.
- b. False
- c. True
- d. False

Solution:

- c. True
- d. True

Sets



Let $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9\}$, and $C = \{2, 4, 6, 8\}$.

Find each of the following:

- a. $A \cup B$
- b. $A \cap B$
- c. $A \cup C$
- d. $A \cap C$
- e. $A - B$
- f. $B - A$
- g. $B \cup C$
- h. $B \cap C$

Solution:

- a. $\{1, 3, 5, 7, 9, 6\}$
- b. $\{3, 9\}$
- c. $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- d. \emptyset
- e. $\{1, 5, 7\}$
- f. $\{6\}$
- g. $\{3, 6, 9, 2, 4, 8\}$
- h. $\{6\}$

Sets



Let $A = \{x, y, z, w\}$ and $B = \{a, b\}$. List the elements of each of the following sets:

- a. $A \times B$
- b. $B \times A$
- c. $A \times A$
- d. $B \times B$

Solution:

- a. $\{(x,a), (x,b), (y,a), (y,b), (z,a), (z,b), (w,a), (w,b)\}$
- b. $\{(a,x), (a,y), (a,z), (a,w), (b,x), (b,y), (b,z), (b,w)\}$
- c. $\{(x,x), (x,y), (x,z), (x,w), (y,x), (y,y), (y,z), (y,w), (z,x), (z,y), (z,z), (z,w), (w,x), (w,y), (w,z), (w,w)\}$
- d. $\{(a,a), (a,b), (b,a), (b,b)\}$

Sets



Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$

b) $C \times B \times A$

Solution:

- a. $A \times B \times C = \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$
- b. $C \times B \times A = \{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$



Relations

Relations



Let $C = \{2, 3, 4, 5\}$ and $D = \{3, 4\}$ and define a binary relation S from C to D as follows:

For all $(x, y) \in C \times D$, $(x, y) \in S \Leftrightarrow x \geq y$.

- Is $2 S 4$? Is $4 S 3$? Is $(4, 4) \in S$? Is $(3, 2) \in S$?
- Write S as a set of ordered pairs.

Solution:

a. $2 S 4$:False

$4 S 3$:True

$(4,4) \in S$

$(3,2) \notin S$

a. $S = \{(3,3), (4,3), (4,4), (5,3), (5,4)\}$

Relations



Define a binary relation R from \mathbb{R} to \mathbb{R} as follows:

For all $(x, y) \in \mathbb{R} \times \mathbb{R}$, $x R y \Leftrightarrow y = x^2$.

a. Is $(2, 4) \in R$? Is $(4, 2) \in R$? Is $(-3) R 9$? Is $9 R (-3)$?

Solution:

a. $(2,4) \in R$

$(4,2) \notin R$

$(-3) R 9$:True

$9 R (-3)$:False

Relations



Let S be the set of all strings of a 's and b 's of length 4.
Define a relation R on S as follows:

For all $s, t \in S$,

$s R t \Leftrightarrow s$ has the same first two characters as t .

- a. Is $abaa R abba$?
- b. Is $aabb R bbaa$?
- c. Is $aaaa R aaab$?

Solution:

- a. True
- b. False
- c. True

Relations



➤ Let $A=\{1,2,3,4\}$, Define a binary relation S from A to A as follows:

$$S = \{(a,b) : a \text{ divides } b\}$$

Write S as a set of ordered pairs.

Solution:

$$S=\{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

Relations



Consider the following relations on $\{1, 2, 3, 4\}$:

$$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\},$$

$$R_2 = \{(1, 1), (1, 2), (2, 1)\},$$

$$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\},$$

$$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\},$$

$$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}.$$

	reflexive	irreflexive	symmetric	<i>antisymmetric</i>	asymmetric	transitive
R_1						
R_2			\times			
R_3	\times		\times			
R_4		\times		\times	\times	\times
R_5	\times			\times		\times

Properties of Relations



Consider these relations on the set of integers:

$$R_1 = \{(a, b) \mid a \leq b\},$$

$$R_2 = \{(a, b) \mid a > b\},$$

$$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\},$$

$$R_4 = \{(a, b) \mid a = b\},$$

$$R_5 = \{(a, b) \mid a = b + 1\},$$

$$R_6 = \{(a, b) \mid a + b \leq 3\}.$$

Determine the properties of these relations

	reflexive	irreflexive	symmetric	antisymmetric	asymmetric	transitive
R_1	×			×		×
R_2		×		×	×	×
R_3	×		×			×
R_4	×		×	×		×
R_5		×		×	×	
R_6			×			

Note :

- R_5 not transitive because $\exists(2,1) \in R_3$ and $(1,0) \in R_3$ but $(2,0) \notin R_3$
- R_6 not transitive because $\exists(2,0) \in R_6 : 2+0 \leq 3$ and $(0,3) \in R_6 : 0+3 \leq 3$ but $(2,3) \notin R_6 : 2+3 \geq 3$

Represent relations using matrices

Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order).

- a) $\{(1, 1), (1, 2), (1, 3)\}$
- b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$
- c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$
- d) $\{(1, 3), (3, 1)\}$

Solution:

a)
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c)
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

b)
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

d)
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Representing relations using matrices

Represent each of these relations on $\{1, 2, 3, 4\}$ with a matrix (with the elements of this set listed in increasing order).

- a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
- b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$
- c) $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$
- d) $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$

Solution:

$$a) \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b) \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$c) \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$d) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Representing relations using matrices

List the ordered pairs in the relations on $\{1, 2, 3\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a)
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

b)
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

c)
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Solution:

a) $\{(1,1),(1,3),(2,2),(3,1),(3,3)\}$

b) $\{(1,2),(2,2),(3,2)\}$

c) $\{(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)\}$

Combining relations using matrices

Suppose that the relations R1 and R2 on a set A={1,2,3} ,

$$R_1=\{(1,1),(1,3), (2,1),(3,2)\}$$

$$R_2=\{(1,1),(1,3),(2,2),(2,3),(3,1)\}$$

Find $R_1 \cup R_2$, $R_1 \cap R_2$

Solution:

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow R_1 \cup R_2 = \{(1,1),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2)\}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow R_1 \cap R_2 = \{(1,1),(1,3)\}$$

Combining relations using matrices

Exercise: Suppose that the relations R1 and R2 on a set A={1,2,3} ,

$$R_1=\{(1,1),(1,3), (2,1),(2,2)\}$$

$$R_2=\{(1,2),(2,3),(3,1),(3,3)\}$$

Find $S \circ R$

Solution:

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow S \circ R = \{(1,1),(1,2),(1,3),(2,2),(2,3)\}$$

Combining relations using matrices

Let R_1 and R_2 be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent

- a) $R_1 \cup R_2$.
- b) $R_1 \cap R_2$.
- c) $R_2 \circ R_1$.
- d) $R_1 \circ R_1$.

Solution:

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_2 \circ R_1} = M_{R_1} \odot M_{R_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{R_1 \circ R_1} = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Properties of Relations



Represent each of these relations on $\{1,2,3,4\}$ with a matrix ,and determine its properties.

- a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
- b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- c) $\{(2, 4), (4, 2)\}$
- d) $\{(1, 2), (2, 3), (3, 4)\}$

Solution:

$$a) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad b) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad c) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$d) \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

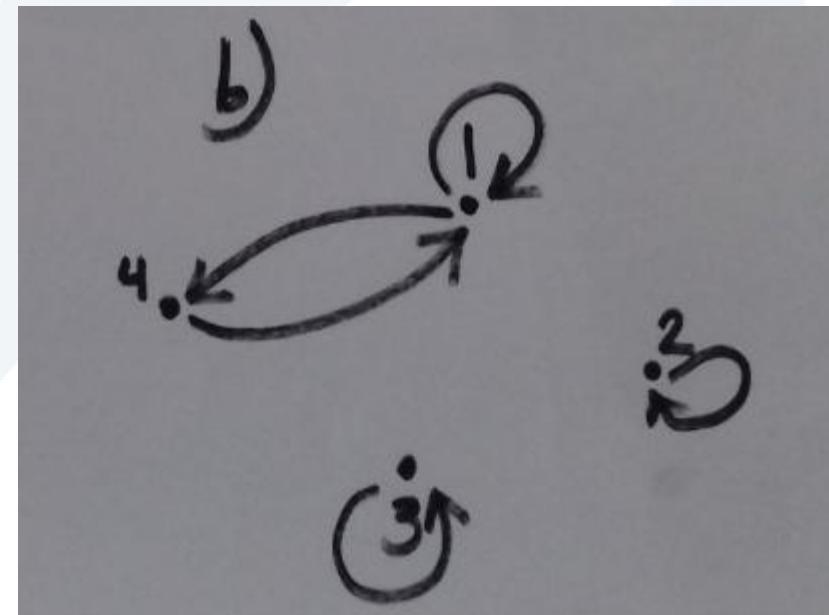
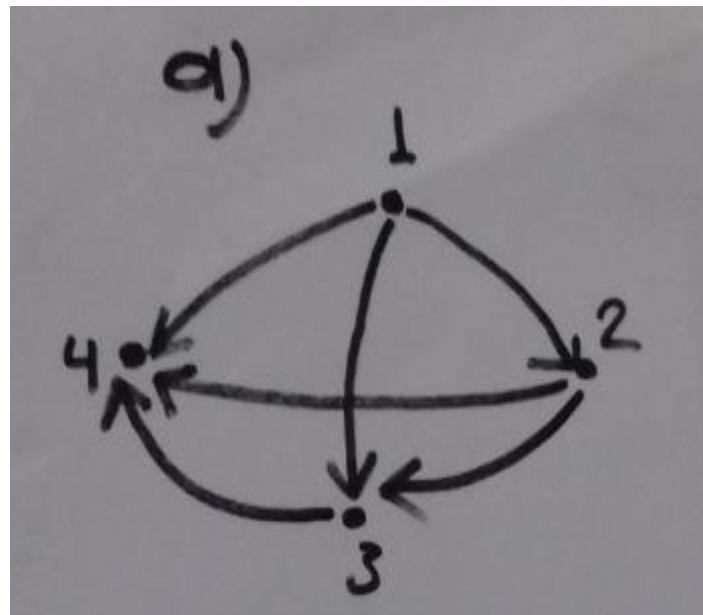
	reflexive	irreflexive	symmetric	antisymmetric	asymmetric	transitive
a)						×
b)	×		×			×
c)		×	×			
d)		×		×	×	

Representing relations with graph

represent these relations with graph.

- a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
- b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$

Solution:



Properties of relation

Let $A = \{-3, -1, 2, 4, 5\}$ and relation R defined from A to A as follows $R = \{(a, b) : ab \geq 0\}$

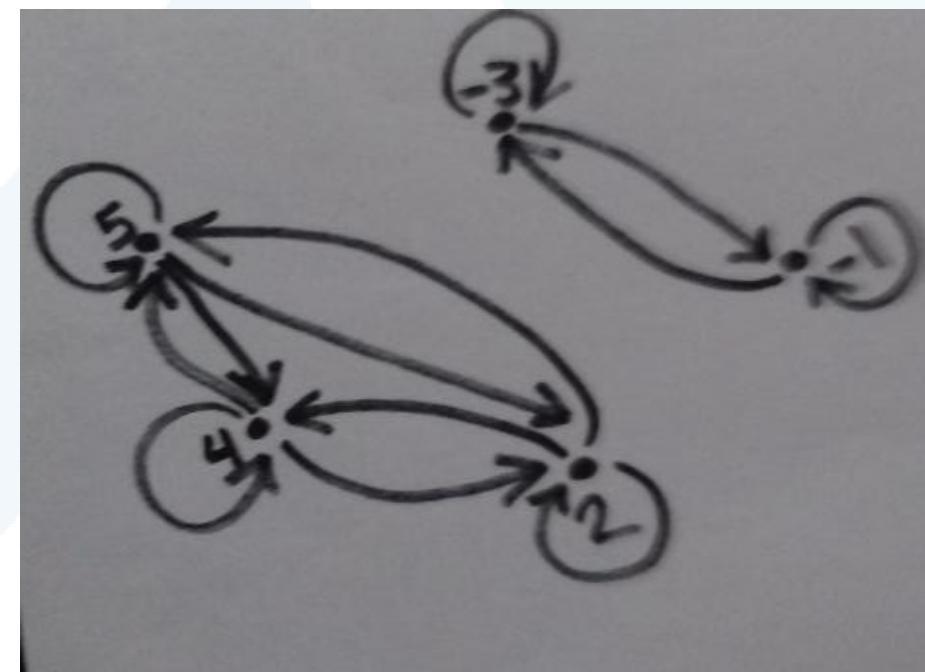
- 1- write R as a set of ordered pairs.
- 2- represent the relation with matrix.
- 3-represent the relation with graph.
- 4-determine if R is equivalence relation

Solution:

$$R = \{(-3, -3), (-3, -1), (-1, -3), (-1, -1), (2, 2), (2, 4), (2, 5), (4, 2), (4, 4), (4, 5), (5, 2), (5, 4), (5, 5)\}$$

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

R is equivalence relation because it is reflexive, symmetric,transitive .



Properties of relation

Let $A = \{-1, 1, 2, 3, 4\}$ and relation R defined from A to A as follows $R = \{(x, y) : x = y^2\}$

- 1- write R as a set of ordered pairs.
- 2- represent the relation with matrix.
- 3-represent the relation with graph.
- 4-determine if R is equivalence relation
- 5-determine if R is partial ordering relation

Solution:

$$R = \{(1, -1), (1, 1), (4, 2)\}$$

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

R is not reflexive , antisymmetric , transitive

R is **not equivalence** relation

R is **not partial ordering** relation

