Structural Mechanics (2)

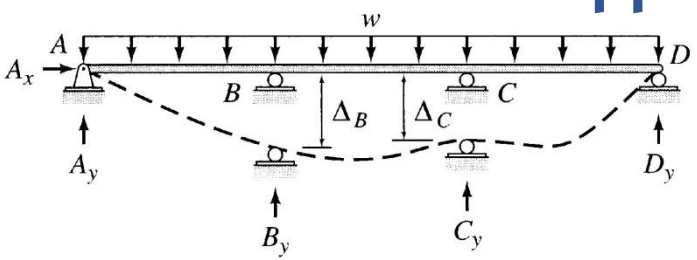
Week No-01

Part-01

Analysis of Indeterminate Structures - Force Method

- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- Structures with multiple Degrees of Indeterminacy
- Support Settlements
- **Three-Moment Equation for Continuous Beams**

Support Settlements



(a) Indeterminate Beam

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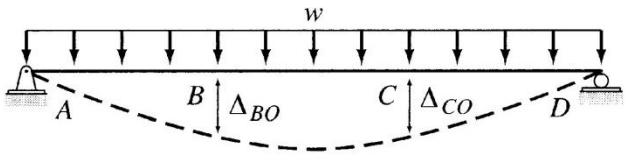

(b) Primary Beam Subjected to External Loading

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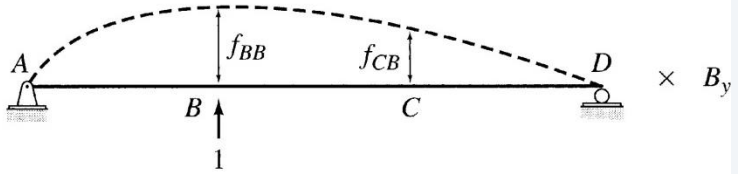

(c) Primary Beam Loaded with Redundant B_y

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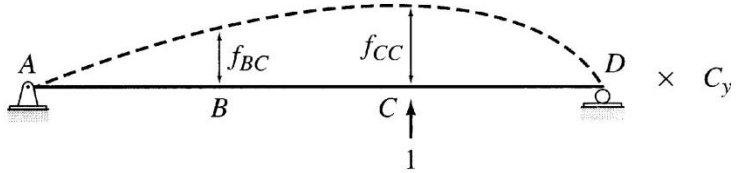
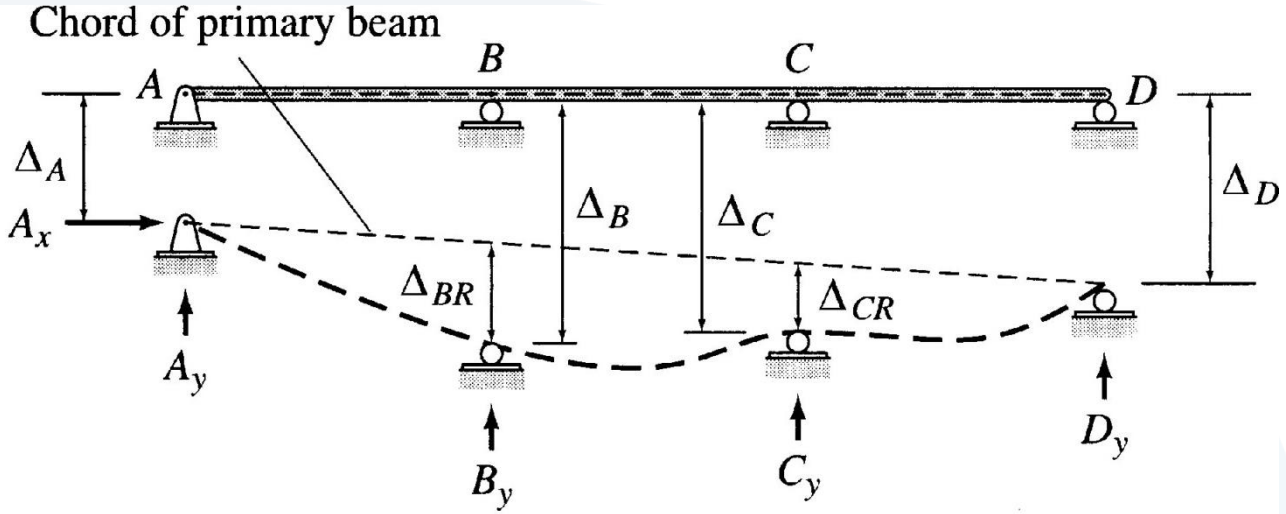


(d) Primary Beam Loaded with Redundant C_y

$$\Delta_{BO} + f_{BB}B_y + f_{BC}C_y = \Delta_B$$
$$\Delta_{CO} + f_{CB}C_y + f_{CC}C_y = \Delta_C$$

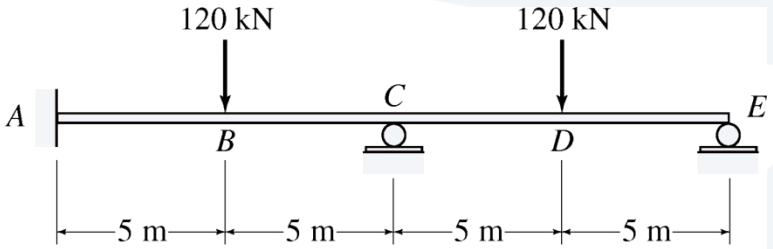
Relative Support Settlements



Compatibility equations:

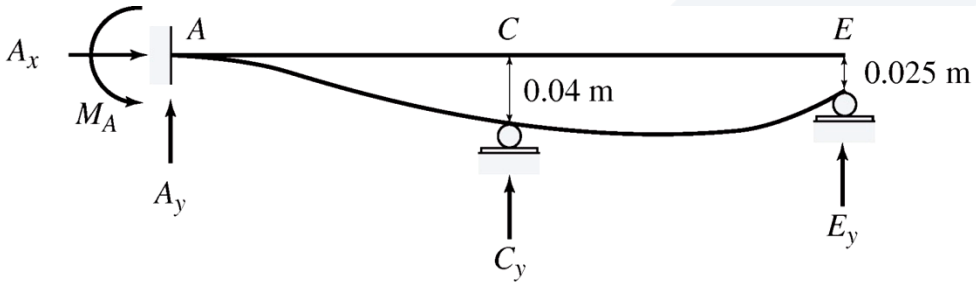
$$f_{BB}\,B_y + f_{BC}\,C_y = \Delta_{BR}$$

$$f_{CB}\,B_y + f_{CC}\,C_y = \Delta_{CR}$$

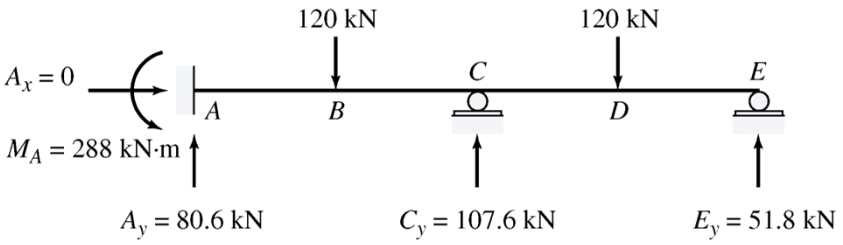


$E = 70$ GPa $\quad I = 1{,}250\ (10^6)\ \text{mm}^4$

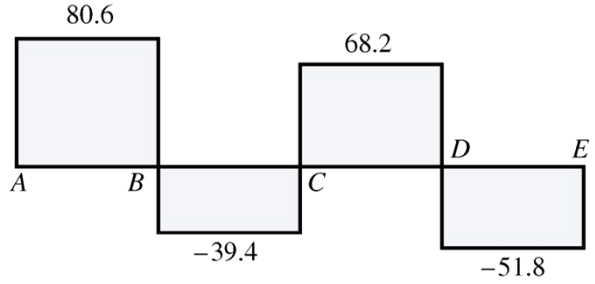
(a) Indeterminate Beam



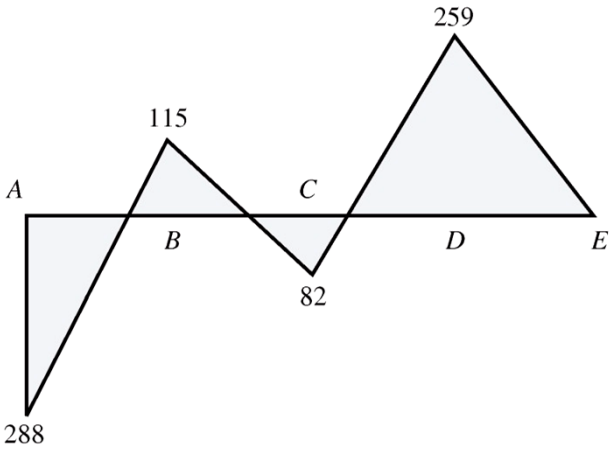
(b) Support Settlements



Reactions



Shear diagram (kN)



Bending moment diagram (kN−m)

Structural Mechanics (2)

Week No-01

Part-02

Analysis of Indeterminate Structures - Force Method

- Indeterminate Structures vs. Determinate Structures
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- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
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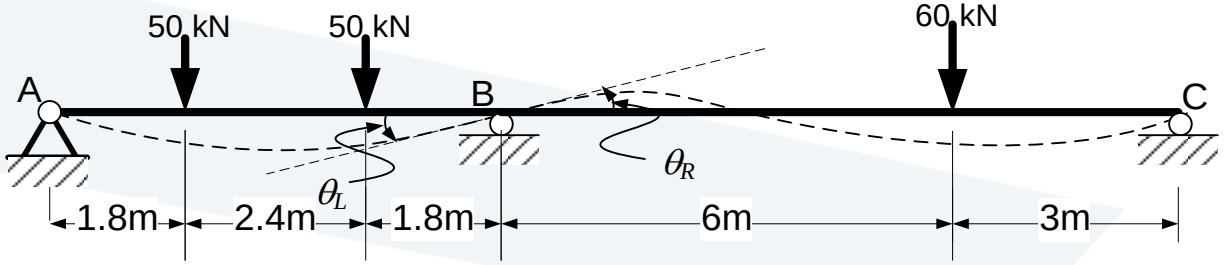
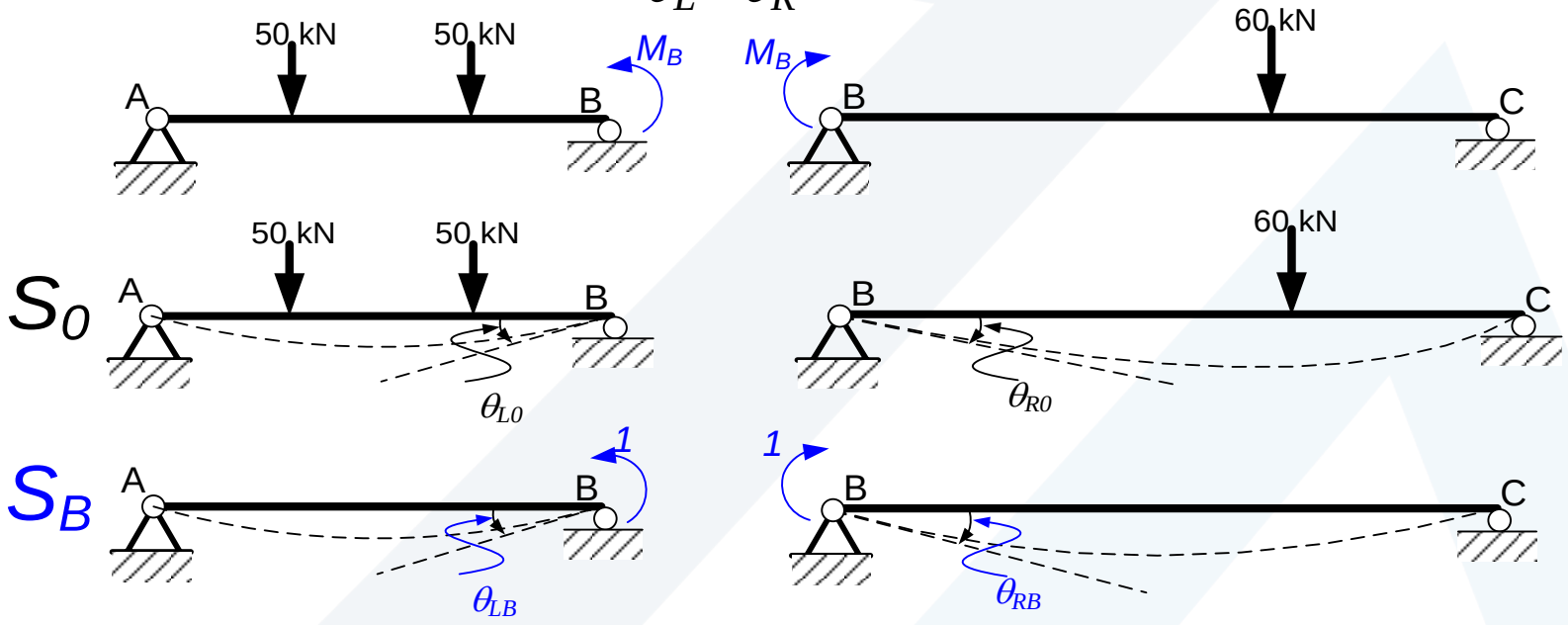
- Support Settlements
- Three-Moment Equation for Continuous Beams

The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundant

The analysis of structure with one degree of indeterminacy can also be done by choosing an internal force or moment as the redundant, provided that the released structure is determinate and stable



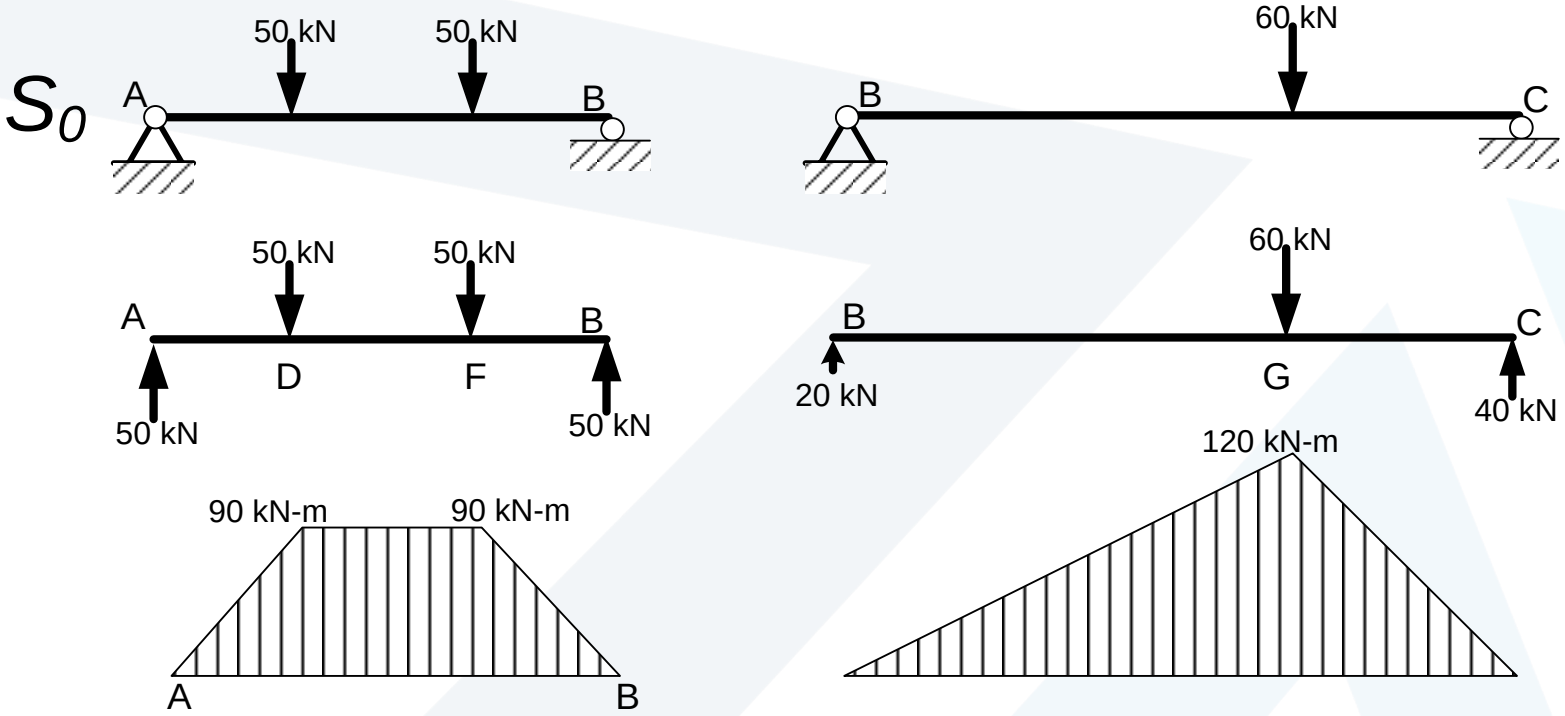
$$\theta_L = \theta_R$$



$$\theta_L = \theta_{L0} + M_B \theta_{LB} \qquad \theta_R = \theta_{R0} + M_B \theta_{RB}$$

The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundants

Analysis of S_0 Structure



M_{L0}

InAD: $M_{L0} = 50x$

InDF: $M_{L0} = 90$

InFB: $M_{L0} = 50(6 - x)$

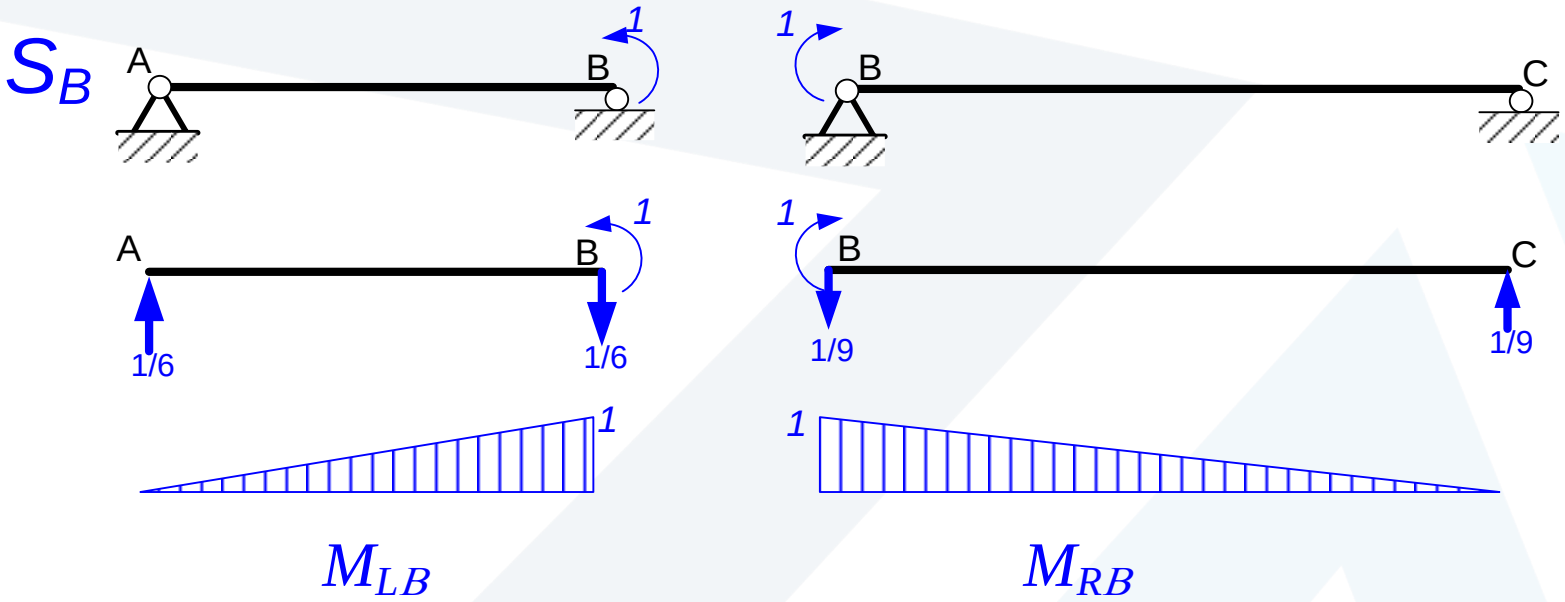
M_{R0}

InBG: $M_{R0} = 20x$

InGC: $M_{R0} = 40(9 - x)$

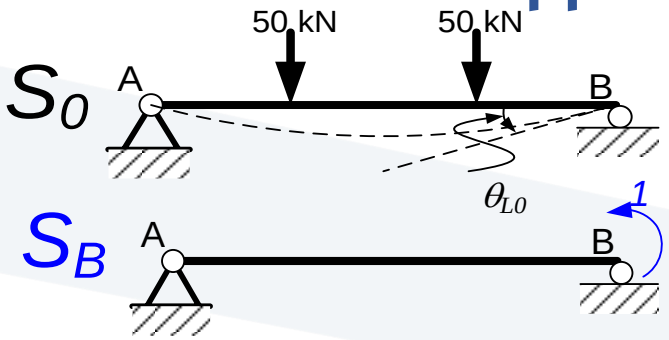
The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundants

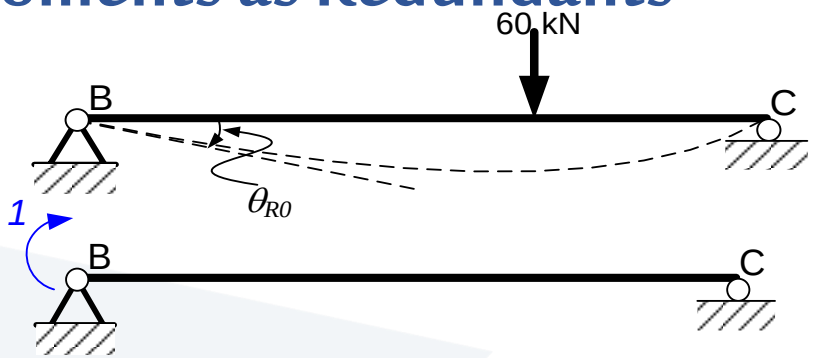
Analysis of S_B Structure



In AB: $M_{LB} = x/6$ In BC: $M_{RB} = (9-x)/9$

The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundants





$$\theta_L = \theta_{L0} + M_B \theta_{LB}$$

$$\theta_R = \theta_{R0} + M_B \theta_{RB}$$

$$EI\theta_{L0} = \int_0^6 M_{L0} M_{LB}\, dx$$

$$= \int_0^{1.8} (50x)(x/6)dx$$

$$+ \int_{1.8}^{4.2} (90)(x/6)dx$$

$$+ \int_{4.2}^{6} 50(6-x)(x/6)dx$$

$$EI\theta_{L0} = 189 \quad +$$

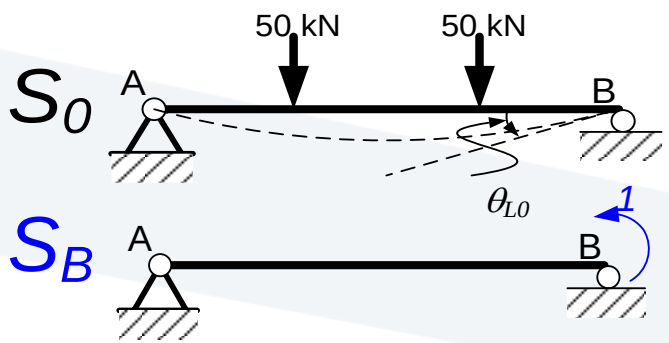
$$EI\theta_{R0} = \int_0^9 M_{R0} M_{RB}\, dx$$

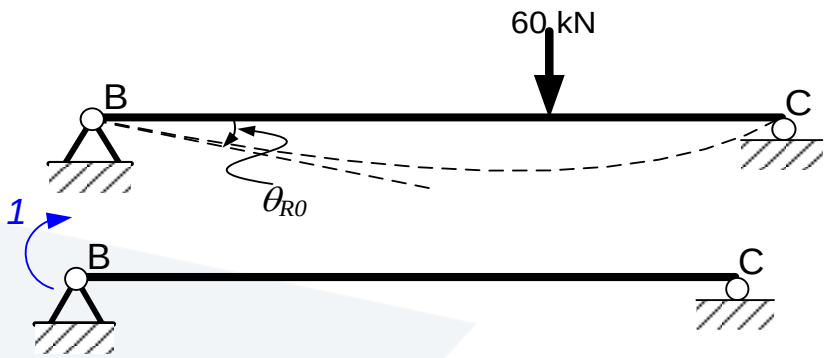
$$= \int_0^6 (20x)[(9-x)/9]dx$$

$$+ \int_6^9 [40(9-x)][(9-x)/9]dx$$

$$EI\theta_{R0} = 240 \quad +$$

The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundants





$$\theta_L = \theta_{L0} + M_B \theta_{LB}$$

$$\theta_R = \theta_{R0} + M_B \theta_{RB}$$

$$EI\theta_{LB} = \int_0^6 M_{LB} M_{LB} dx$$

$$= \int_0^6 (x/6)(x/6) dx$$

$$EI\theta_{RB} = \int_0^9 M_{RB} M_{RB} dx$$

$$= \int_0^9 [(9-x)/9][(9-x)/9] dx$$

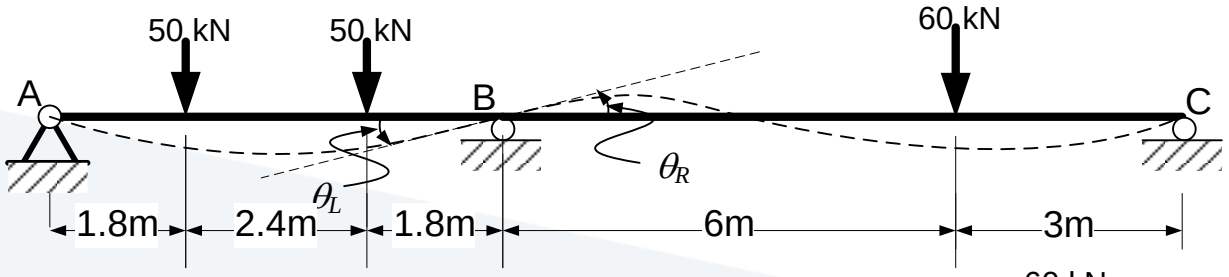
$EI\theta_{LB} = 2$ $\quad EI\theta_{L0} = 189$ $\quad EI\theta_{RB} = 3$ $\quad EI\theta_{R0} = 240$

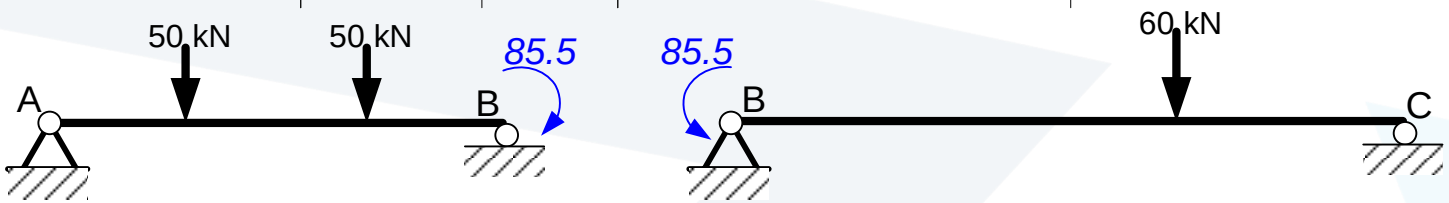
$$\theta_L = \theta_{L0} + M_B \theta_{LB} \equiv \theta_R = -(\theta_{R0} + M_B \theta_{RB})$$

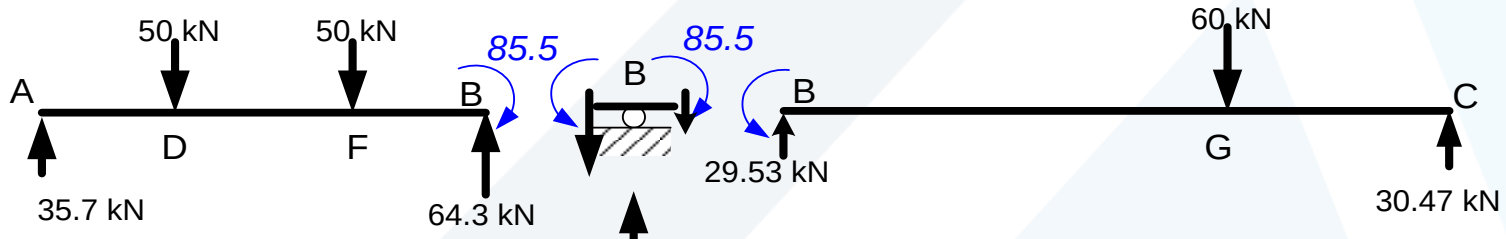
$$(\theta_{L0} + \theta_{R0}) + M_B(\theta_{LB} + \theta_{RB}) = 0$$

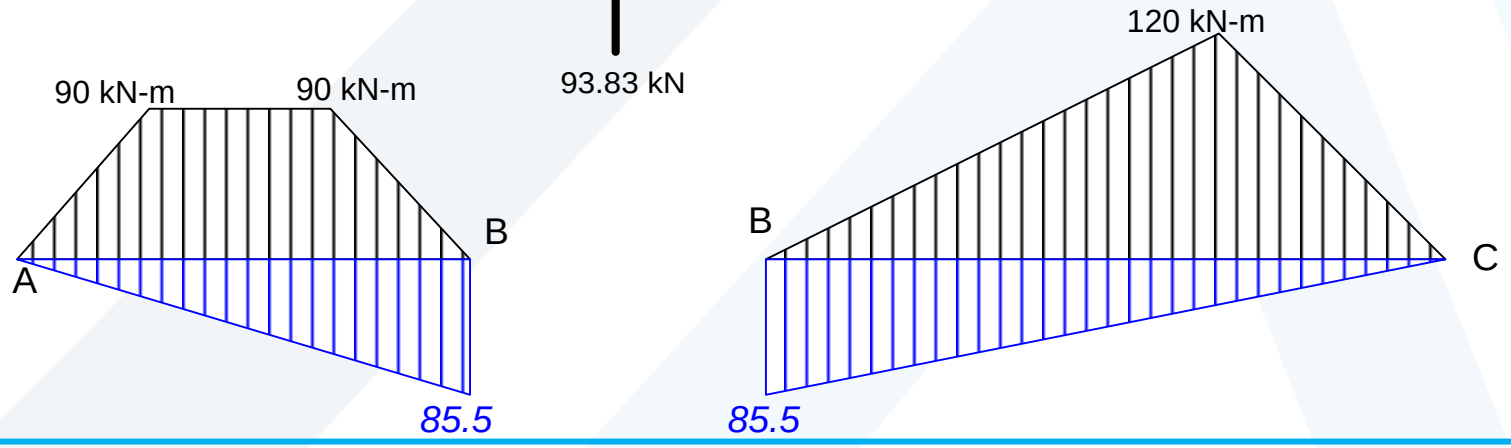
$$M_B(\theta_{LB} + \theta_{RB}) = -(\theta_{L0} + \theta_{R0}) \Rightarrow M_B = -429/5 = -85.5 \text{ kN.m}$$

The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundants

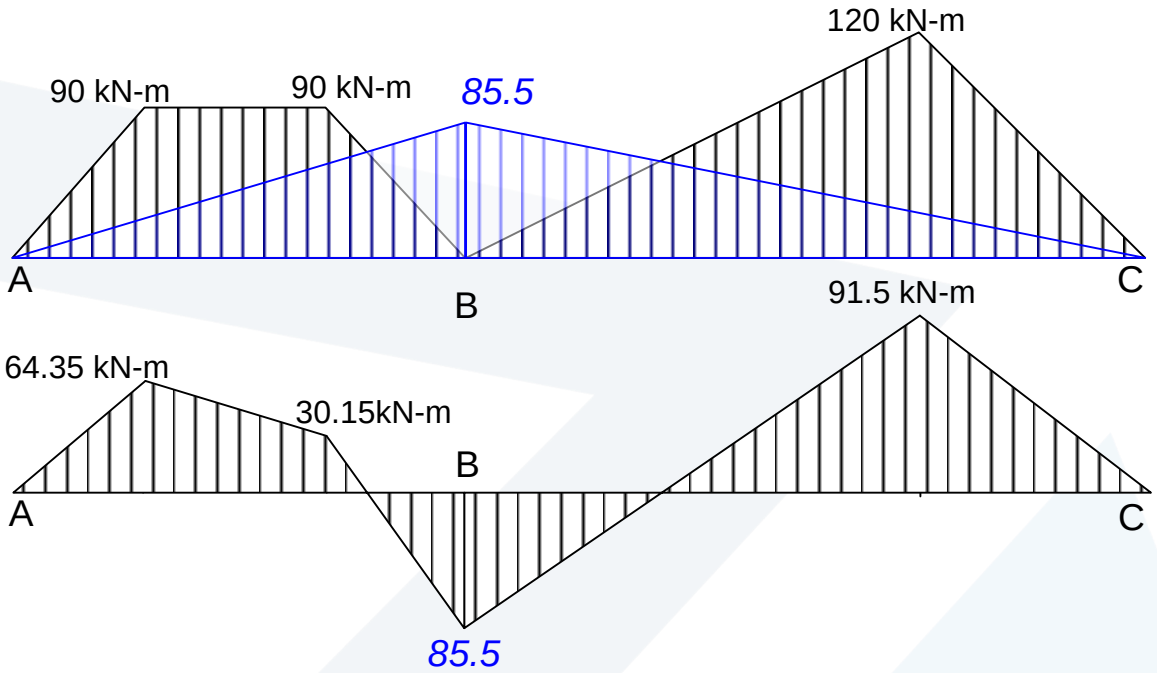




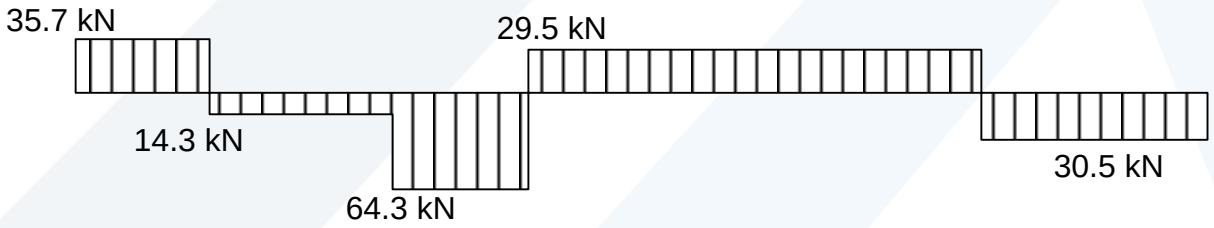




The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundants



Bending Moment



Shear Force