

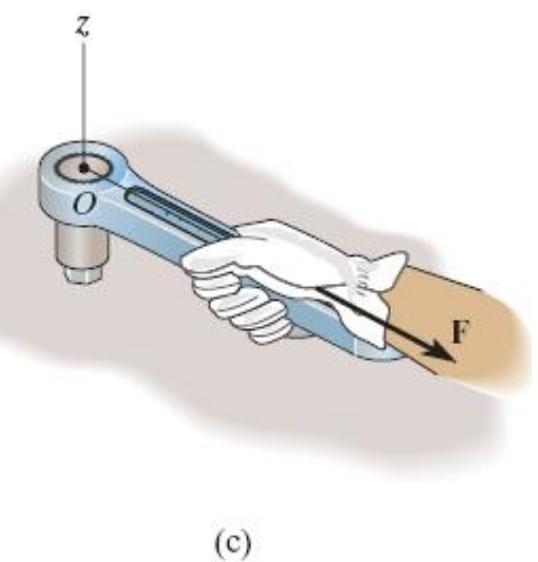
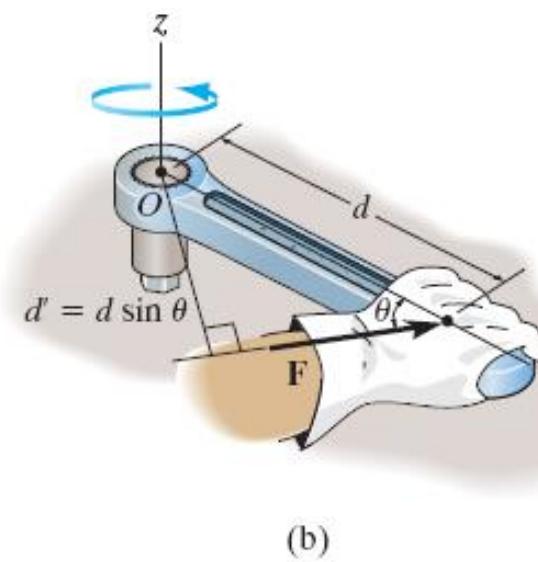
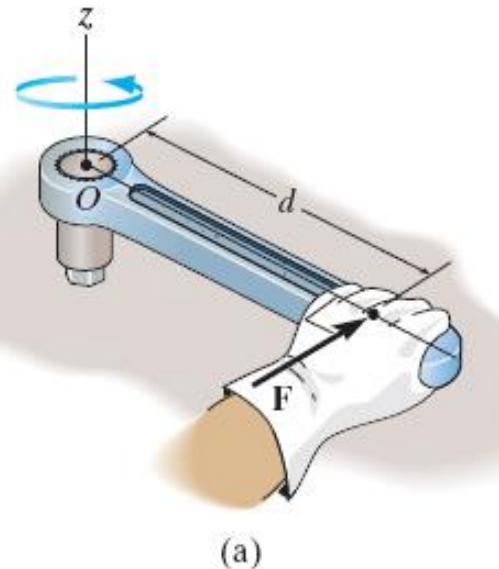
Moment of a Force – Scalar Formulation

عزم القوة – الصياغة السلمية (غير الشعاعية)

Moment of a force about a point or axis: “*Moment sometime is called Torque*”

A measure of its tendency to cause a body to rotate about the point or axis

عزم القوة : مقياس لـزعة القوة على تدوير جسم ما حول نقطة ثابتة أو حول محور



Moment of a Force – Scalar Formulation

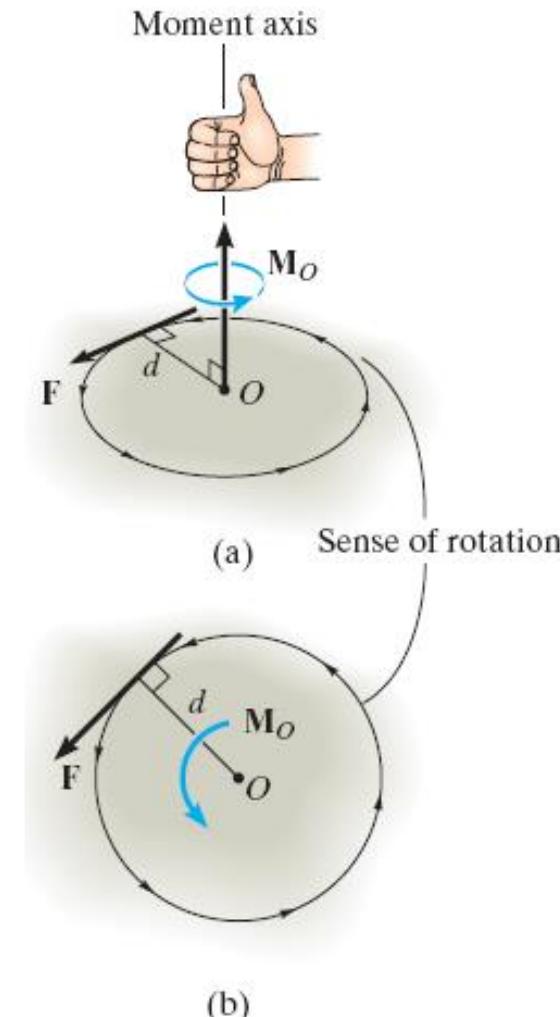
عزم القوة – الصياغة السلمية (غير الشعاعية)

Magnitude: $M_O = Fd$ (Nm)

where d “perpendicular distance” from O to its line of action of force

Direction:

Direction using “right hand rule”



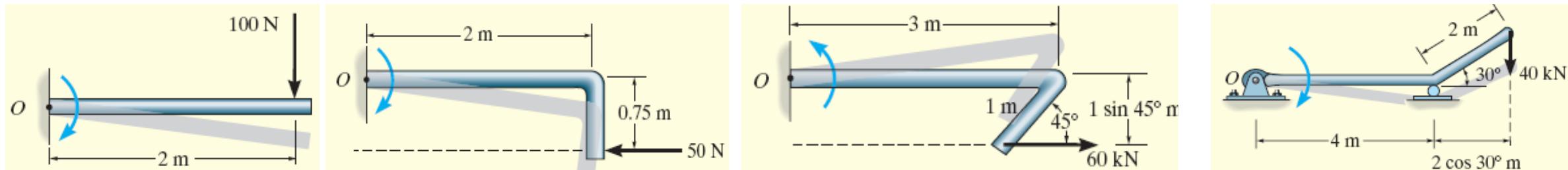
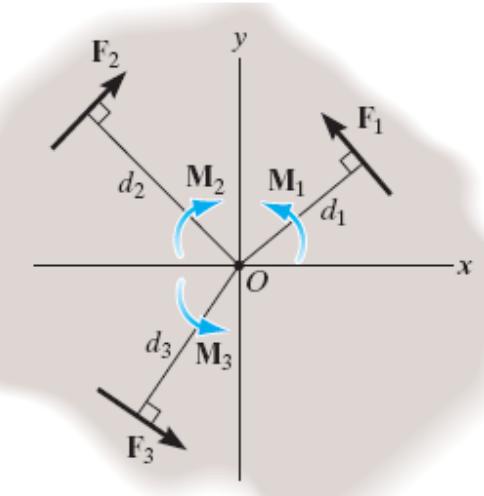
Moment of a Force – Scalar Formulation

عزم القوة – الصياغة السلمية (غير الشعاعية)

Resultant Moment:

$$M_{Ro} = \text{moments of all the forces} = \sum Fd$$

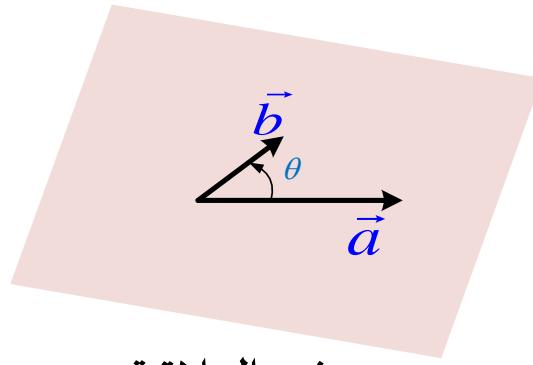
Example 1. For each case, determine the moment of the force about point O .



Solutions

$$M_o = -(100)(2) = -200 \text{ Nm} \quad M_o = -(50)(0.75) = -37.5 \text{ Nm} \quad M_o = +(60)(1\sin 45^\circ) = 42.4 \text{ kNm} \quad M_o = -(40)(4+2\cos 30^\circ) = -229 \text{ kNm}$$

الجداء السلمي (النقطي) لشعاعين



ناتجه عدد حقيقي معرف بالعلاقة

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

تبديلي، توزيعي مع جمع الأشعة. معدوم لمعامدين.
في جملة احداثيات ديكارتية متعامدة نظامية

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

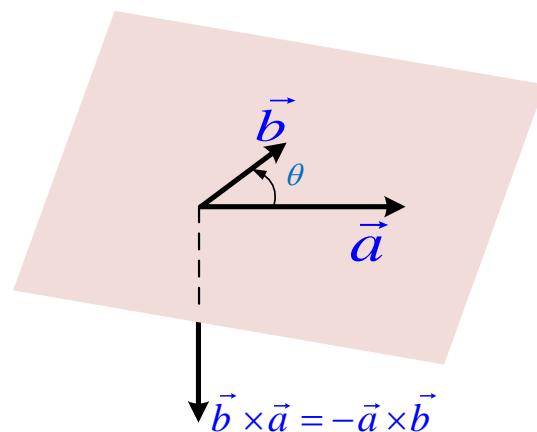
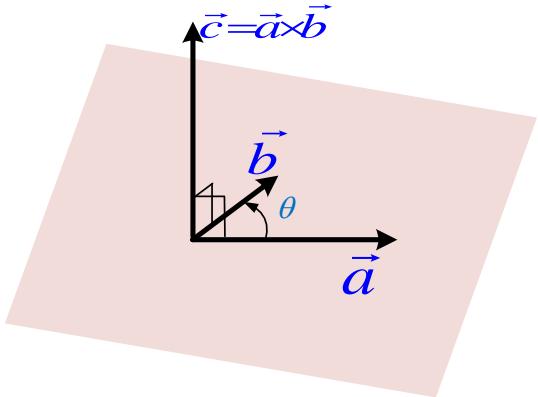
$$\vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$$

يعطى الجداء والزاوية بين الشعاعين بالعلاقتين

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\cos \theta = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \sqrt{b_x^2 + b_y^2 + b_z^2}}$$

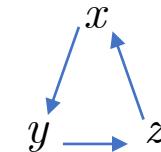
الجداء الشعاعي لشعاعين ناتجه شعاع متعامد مع مستويهما



$$|\vec{c}| = |\vec{a}| |\vec{b}| \sin \theta$$

تبديلي عكسا، توزيعي مع جمع الأشعة. معدوم لشعاعين متوازيين. وتحليلياً لدينا

$$\begin{aligned} \vec{a} \times \vec{b} &= (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k}) \\ &= a_x b_x (\vec{i} \times \vec{i}) + a_x b_y (\vec{i} \times \vec{j}) + a_x b_z (\vec{i} \times \vec{k}) \\ &\quad + a_y b_x (\vec{j} \times \vec{i}) + a_y b_y (\vec{j} \times \vec{j}) + a_y b_z (\vec{j} \times \vec{k}) \\ &\quad + a_z b_x (\vec{k} \times \vec{i}) + a_z b_y (\vec{k} \times \vec{j}) + a_z b_z (\vec{k} \times \vec{k}) \\ &= \vec{i} (a_y b_z - a_z b_y) + \vec{j} (a_z b_x - a_x b_z) + \vec{k} (a_x b_y - a_y b_x) \end{aligned}$$



Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

- Moment of force \mathbf{F} about point O can be expressed using cross (vector) product

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \quad \text{Hand writing: } \vec{M}_o = \vec{r} \times \vec{F}$$

- Magnitude: For magnitude of cross product: $M_O = rF\sin\theta$.
- Treat \mathbf{r} as a sliding vector. Since $d = r\sin\theta$, $\Rightarrow M_O = rF\sin\theta = F(r\sin\theta) = Fd$.
- Direction and sense of \mathbf{M}_O are determined by right-hand rule.

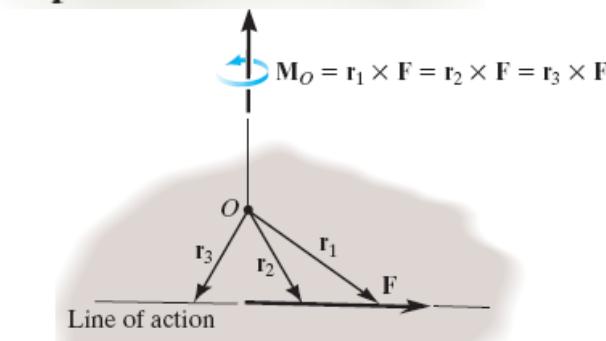
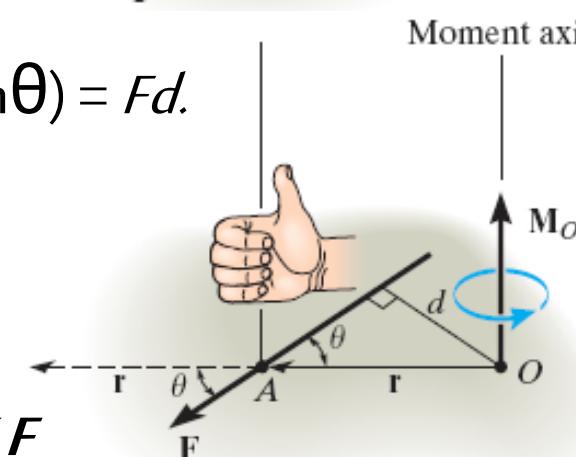
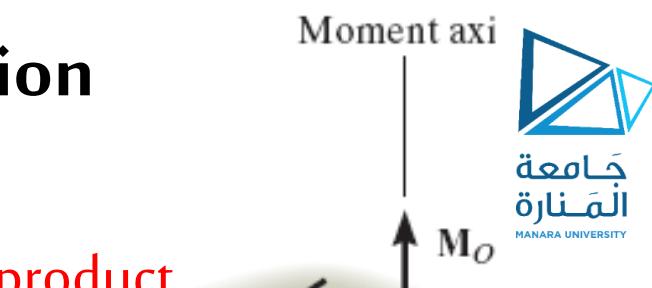
Principle of Transmissibility:

For force \mathbf{F} applied at any point A , moment created about O is: $\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F}$

The force \mathbf{F} acting on rigid body can slide and the cross product properties

allow to write: $\mathbf{M}_O = \mathbf{r}_1 \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} = \mathbf{r}_3 \times \mathbf{F}$

$$\mathbf{r}_1 = \mathbf{r}_2 + \lambda \mathbf{u}_F, \mathbf{r}_1 \times \mathbf{F} = (\mathbf{r}_2 + \lambda \mathbf{u}_F) \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} + \lambda \mathbf{u}_F \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} + \mathbf{0} = \mathbf{r}_2 \times \mathbf{F}$$



Moment of a Force – Vector Formulation

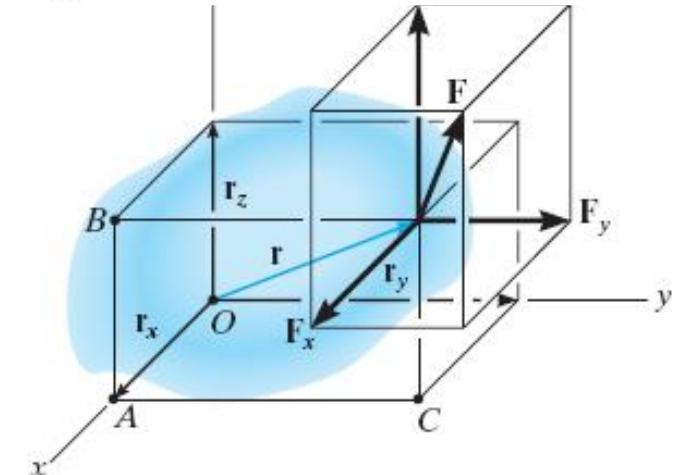
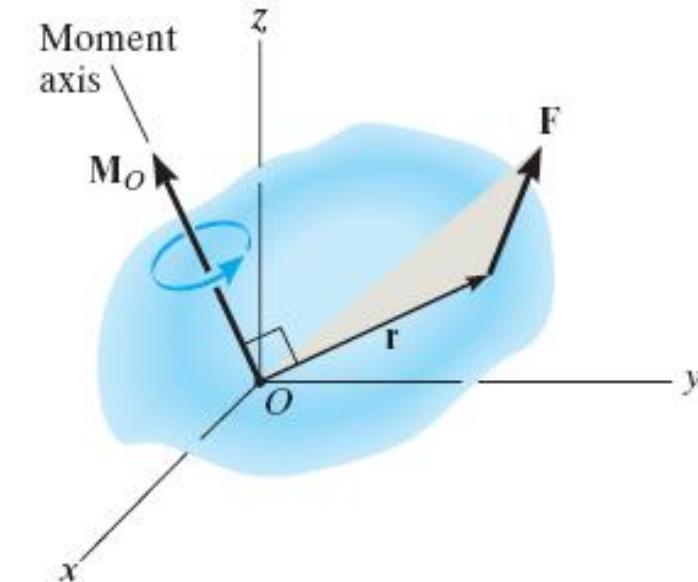
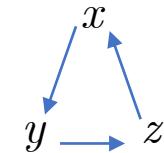
عزم القوة – الصياغة الشعاعية

Cartesian Vector Formulation: For force expressed in Cartesian form,

$\vec{r} = r_x \vec{i} + r_y \vec{j} + r_z \vec{k}$ and $\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$, the vector moment is:

In terms of the components,

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{i} (r_y F_z - r_z F_y) + \mathbf{j} (r_z F_x - r_x F_z) + \mathbf{k} (r_x F_y - r_y F_x)$$



Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

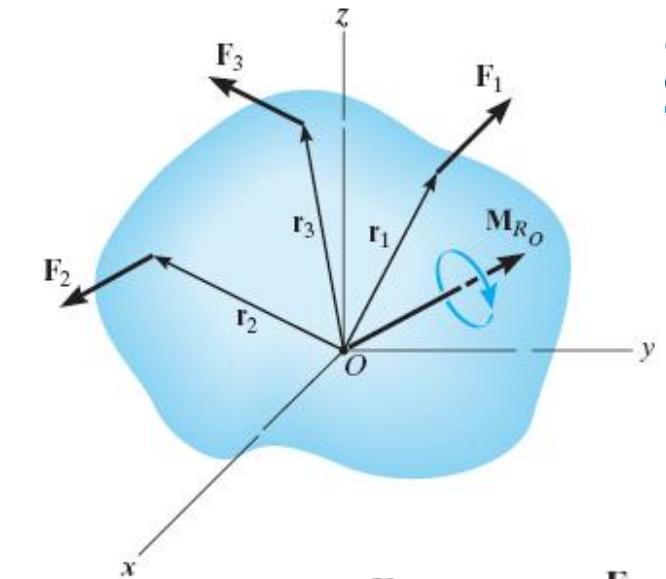
Resultant Moment of a System of Forces

Resultant moment of forces about point O can be determined by vector addition

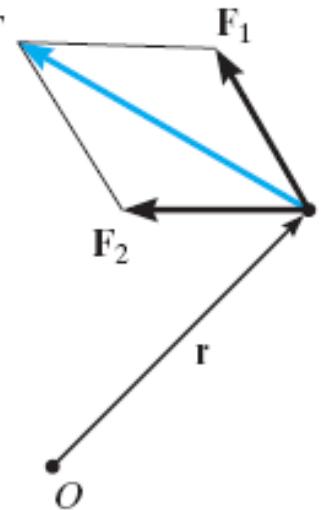
$$M_{Ro} = \sum(r \times F)$$

Varignon's Principle

“Moment of a force about a point is equal to the sum of the moments of the forces' components about the point”



Since $F = F_1 + F_2$, then $M_O = r \times F = r \times (F_1 + F_2) = r \times F_1 + r \times F_2$



Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

Example 2:

Two forces act on the pipe system. Determine the resultant moment they create about the flange at O . Express the result as a Cartesian vector.

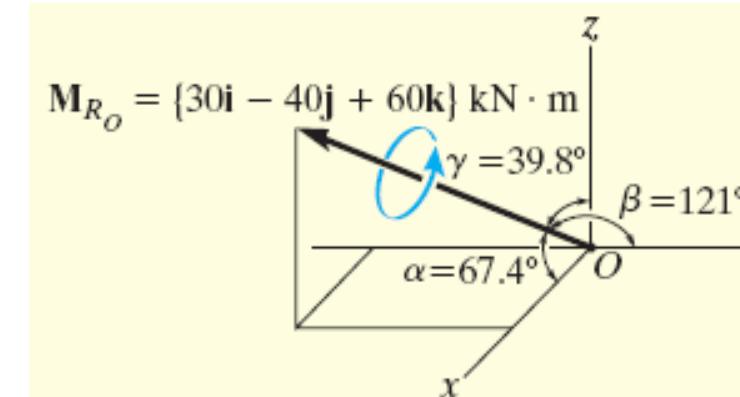
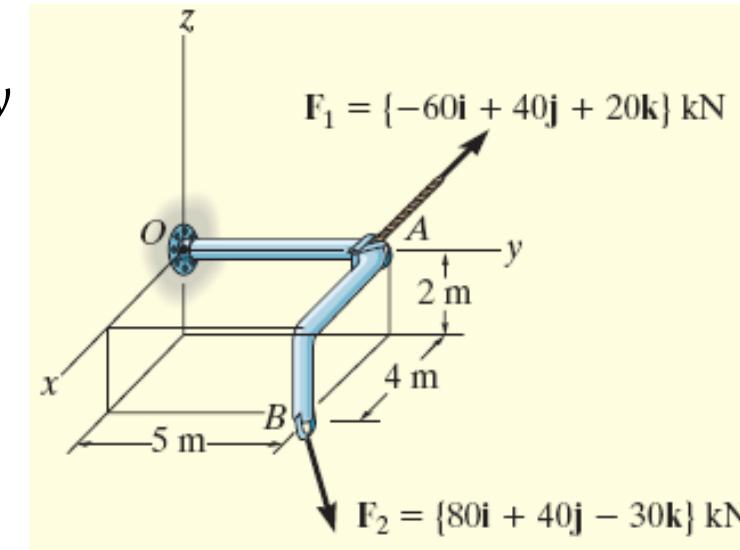
Solution:

Position vectors are directed from O to each force as shown.

$$\vec{r}_A = \{5\vec{j}\} \text{ m} \quad \text{and} \quad \vec{r}_B = \{4\vec{i} + 5\vec{j} - 2\vec{k}\} \text{ m}$$

The resultant moment about O is:

$$\vec{M}_O = \sum(\vec{r} \times \vec{F}) = \vec{r}_A \times \vec{F}_1 + \vec{r}_B \times \vec{F}_2 = \{30\vec{i} - 40\vec{j} + 60\vec{k}\} \text{ kN} \cdot \text{m}$$



Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

Example 3:

Determine the moment of the force about point O .

Solution:

The moment arm d can be found from trigonometry,

$$d = (3) \sin 75^\circ = 2.898 \text{ m} \quad \text{Thus, } M_O = Fd = -(5)(2.898) = -14.5 \text{ kN}\cdot\text{m}$$

Without using the arm d , the moment can be found from the components by

$$\begin{aligned} M_O &= |d_x F_y| + |d_y F_x| = -(3 \cos 30^\circ)(5 \sin 45^\circ) - (3 \sin 30^\circ)(5 \cos 45^\circ) = -14.5 \text{ kN}\cdot\text{m} \\ &= -15(\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ) = -15 \sin 75^\circ \end{aligned}$$

