2 Stress

2.2.4 The Thin-Walled Pressure Vessel

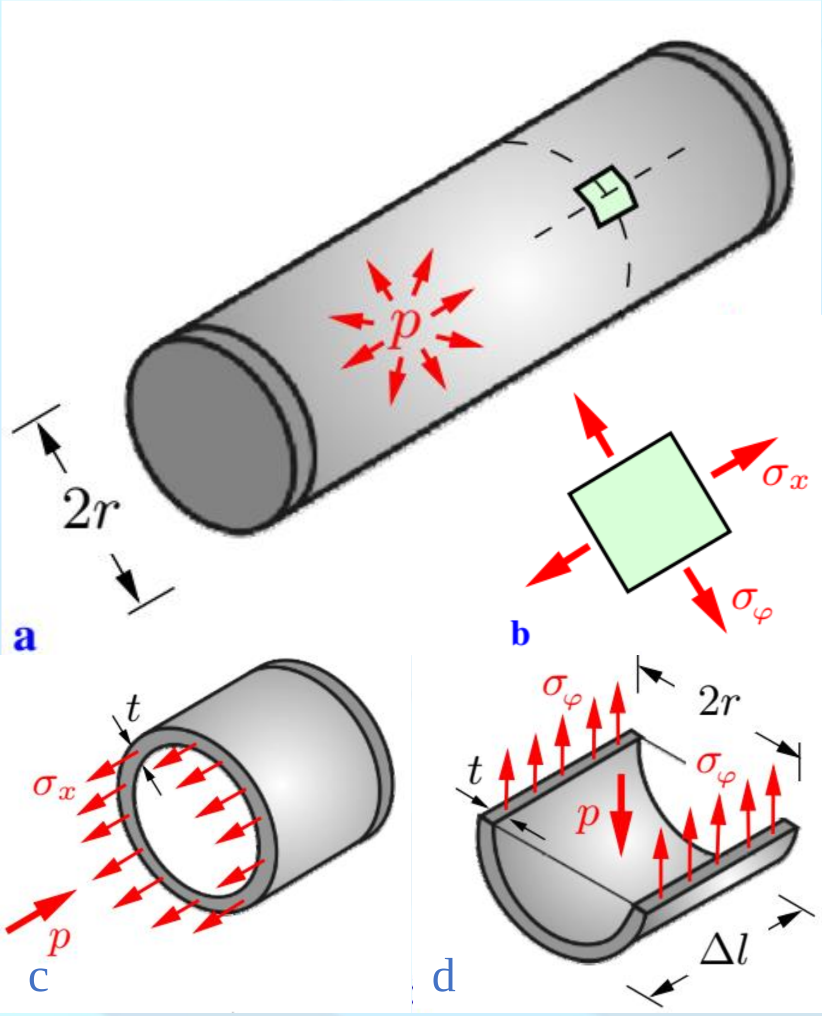
An important application of plane stress is the *thin walled* cylindrical vessel with radius r & wall thickness $t \ll r$ (Fig.a), subjected to an internal pressure p that causes stresses in its wall which need to be determined (Fig. b).

Far from the end caps of the vessel, the stress state is independent of the location (homogeneous stress state).

Given that $t \ll r$, the stresses in radial directions can be neglected.

Thus, within a good approximation a plane stress state acts locally in the wall of the vessel (note: although the element in Fig.b is curved, it is replaced by a plane element in the tangent plane).

The stress state can be described by the stresses in two sections perpendicular to each other. First, the vessel is cut perpendicularly to its longitudinal axis (Fig.c).



Since the gas or fluid pressure is independent of the location, the pressure on the section area πr^2 (of the gas or fluid) has the constant value p. Assuming that the *longitudinal stress* σ_x is constant across the wall thickness because of $t \ll r$, the equilibrium condition yields (Fig.c)

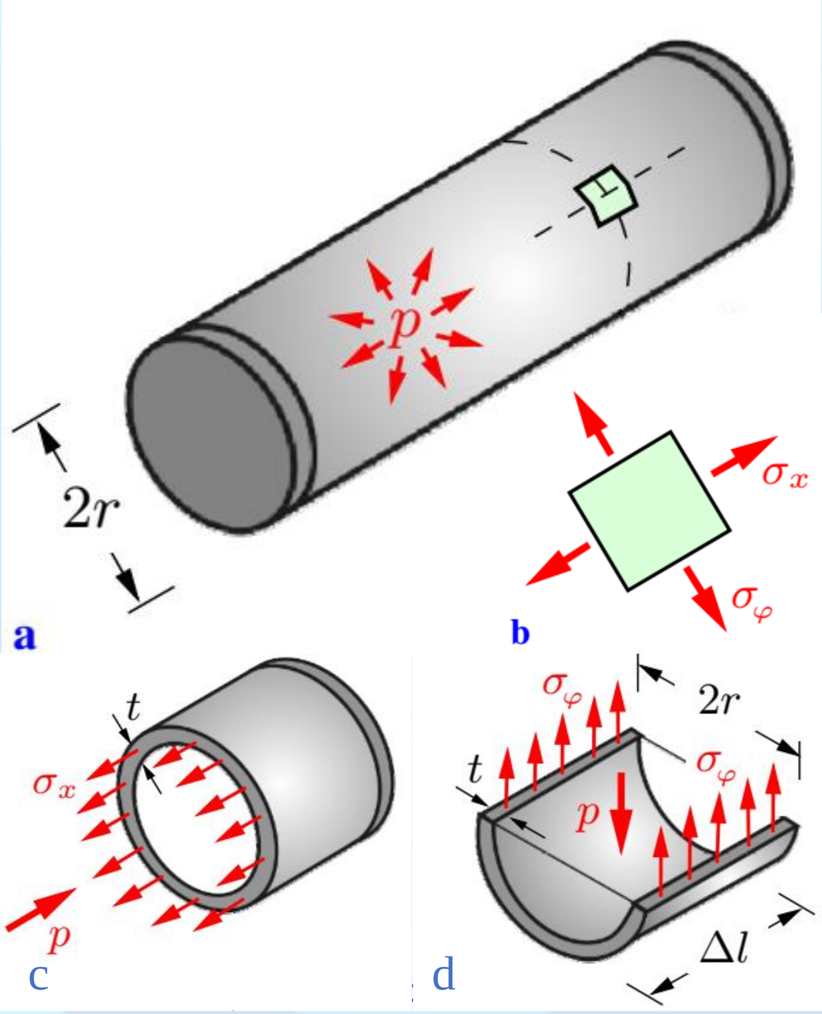
$$\sigma_x(2\pi rt) - p(\pi r^2) = 0 \;\Rightarrow\; \sigma_x = \frac{1}{2}\frac{pr}{t}.$$

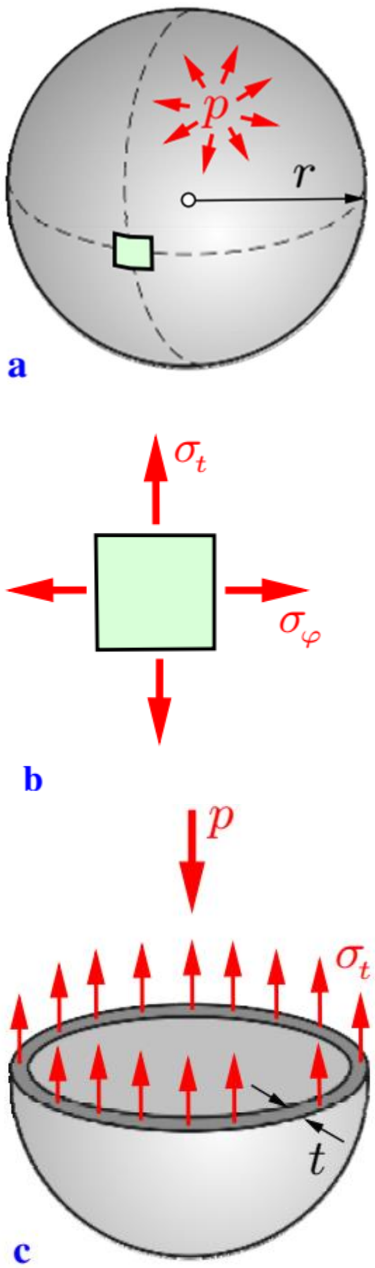
As illustrated in Fig.d a half-circular part of length Δl is separated from the vessel. The horizontal sections of the wall are subjected to the *circumferential stress* σ_{φ}, also called *hoop stress*, which again is constant across the thickness. These stresses will counteract the force $p(2\,r\Delta l)$, exerted from the gas onto the halfcircular part of the vessel. Equilibrium in the vertical direction yields

$$\sigma_{\varphi}(2t\Delta l) - p(2r\Delta l) = 0 \;\Rightarrow \sigma_{\varphi} = \frac{pr}{t} \qquad \text{notice that} \qquad \sigma_{\varphi} = 2\sigma_x$$

The two equations for σ_x and σ_{φ} sometimes are called *vessel formulas*. Because of $t \ll r$ it can be seen that $\sigma_x\,, \sigma_{\varphi} \gg p$ Therefore, the initially made assumption that the stresses σ_r in radial direction may be neglected is justified ($|\,\sigma_r\,| \leq\, p$). Generally, a vessel is called *thin-walled* when it fulfills the condition $r > 5\,t$.

The vessel formulas are also applicable to a vessel subjected to external pressure. In this case only the sign of p has to be changed, i.e. the wall is then under a compressive stress state.





Now a thin-walled spherical vessel of radius r, subjected to a gage pressure p (Fig. a) is considered. Here, the stresses σ_t and σ_φ act in the wall (Fig. b). When the vessel is cut into half (Fig. c), σ_t is obtained from the equilibrium condition:

$$\sigma_t(2\pi rt) - p\pi r^2 = 0 \;\Rightarrow\; \sigma_t = \frac{pr}{2t}$$

A cut, perpendicular to the first one, similarly leads to

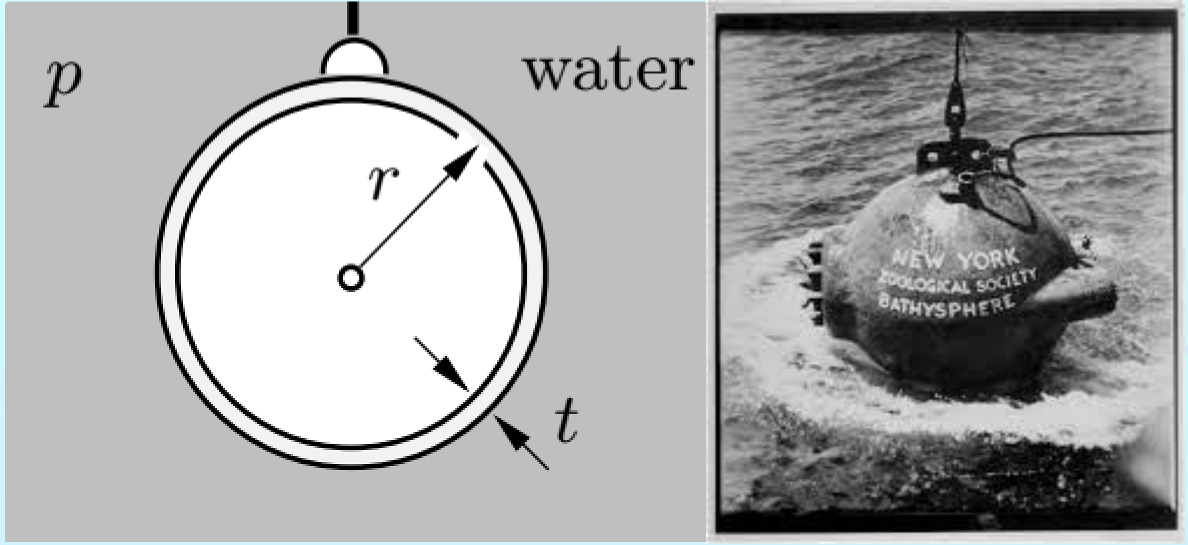
$$\sigma_\varphi(2\pi rt) - p\pi r^2 = 0 \;\Rightarrow\; \sigma_\varphi = \frac{pr}{2t}$$

Thus, $\sigma_t = \sigma_\varphi = \dfrac{pr}{2t}$

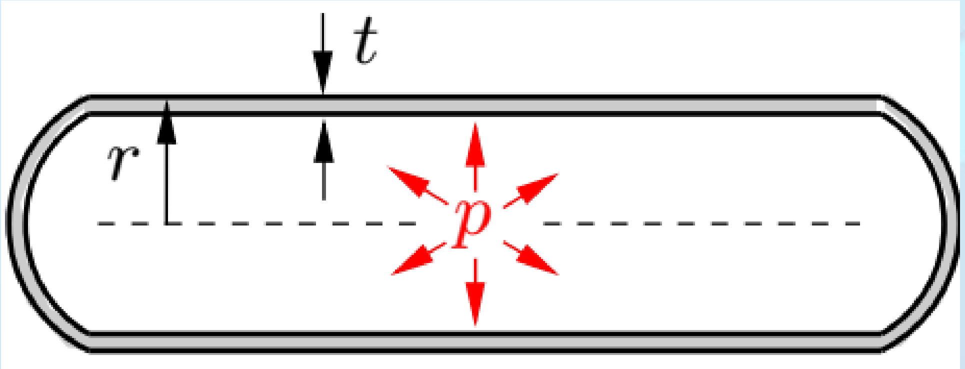
Therefore, the stress in the wall of a thin-walled spherical vessel has the value $p\,r/(2\,t)$ in any arbitrary direction. The state of stress is a hydrostatic one.

As in the foregoing case, this formula is also valid for an external pressure in which case p is negative.

Example 1. A thin-walled bathysphere (radius $r = 500$ mm, wall thickness $t = 12.5$ mm) is lowered to a depth of 500m under the water surface (pressure $p = 5$ MPa). Determine the stresses in the wall.



Example 2. A thin-walled cylindrical vessel has the radius $r = 1$ m and wall-thickness $t = 10$ mm. Determine the maximum internal pressure p_{max} so that the maximum stress in the wall does not exceed the allowable stress $\sigma_{allow} = 150$ MPa.



4. A factory uses a cylindrical pressure vessel with an inner diameter of 700 mm. The vessel operates at an internal pressure of 6.0 MPa. The steel used in the construction of the vessel has an allowable tensile stress of 280 MPa (the factor of safety has already been applied). The lid of the pressure vessel is attached to the body using 36 equally spaced bolts on a circle of diameter $D_{\text{circle}} = 800$ mm around the circumference of the pressure vessel. The allowable stress in tension for the bolts is 245 MPa.

(a) Calculate the minimum allowable wall thickness to support the load.

(b) Determine the minimum bolt diameter at the root of the threads of the bolts (*hint*: what force does each bolt need to carry)? Assume the bolt-heads and nuts are strong enough.