جامعة المنارة
MANARA UNIVERSITY

تحليل رياضي 2

المحاضرة 9

ميكاترونيكس

أ.د. سامي انجرو

الاشتقاق الجزئي من المرتبة الثانية

إذا كان f تابع لمتحولين، عندئذ يمكن اعتبار مشتقاته الجزئية f_x و f_y كتوابع لمتحولين أيضاً،

وعليه فإن $(f_x)_x$ و $(f_y)_x$ و $(f_x)_y$ و $(f_y)_y$ المشتقات الجزئية من المرتبة الثانية للتابع

$$\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)=f_{xx} \qquad \frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)=f_{yy}$$

$$\frac{\partial^2 f}{\partial y\,\partial x}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)=f_{xy} \qquad \frac{\partial^2 f}{\partial x\,\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)=f_{yx}$$

مبرهنة: (مبرهنة كليرو)

بفرض أن f تابع معرّف على القرص D، الذي يحوي النقطة (a,b)، وإذا كان f_{xy} و f_{yx} مستمرين على D، عندئذ فإن:

$$f_{xy}(a,b)=f_{yx}(a,b)$$

مثال أوجد المشتقات الجزئية للتابع $f(x, y) = x\cos y + ye^x$

الحل

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x\cos y + ye^x) = \cos y + ye^x$$

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = -\sin y + e^x$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = ye^x.$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x\cos y + ye^x) = -x\sin y + e^x$$

$$\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = -\sin y + e^x$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = -x\cos y$$

الاشتقاق الجزئي من مراتب عليا

$$\frac{\partial^3 f}{\partial x\, \partial y^2} = f_{yyx}$$

$$\frac{\partial^4 f}{\partial x^2\, \partial y^2} = f_{yyxx}$$

مثال أوجد f_{yxyz} للتابع $f(x, y, z) = 1 - 2xy^2z + x^2y$

الحل

$$f_y = -4xyz + x^2 \qquad f_{yx} = -4yz + 2x \qquad f_{yxy} = -4z$$

$$f_{yxyz} = -4$$

تمارين

1 أوجد كل من $\frac{\partial f}{\partial x}$ و $\frac{\partial f}{\partial y}$ للتوابع الآتية

$$f(x,y) = x^2 - xy + y^2 \qquad f(x,y) = \sqrt{x^2 + y^2} \qquad f(x,y) = e^{xy}\ln y$$

$$f(x,y) = \cos^2(3x - y^2)$$

الحل:

$$f(x,y) = x^2 - xy + y^2$$

$$\frac{\partial f}{\partial x} = 2x - y, \frac{\partial f}{\partial y} = -x + 2y$$

$$f(x,y) = \sqrt{x^2 + y^2}$$

$$\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}, \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

تمارين

$$f(x,y) = e^{xy} \ln y$$

$$\frac{\partial f}{\partial x} = e^{xy} \cdot \frac{\partial}{\partial x}(xy) \cdot \ln y = ye^{xy} \ln y$$

$$\frac{\partial f}{\partial y} = e^{xy} \cdot \frac{\partial}{\partial y}(xy) \cdot \ln y + e^{xy} \cdot \frac{1}{y} = xe^{xy} \ln y + \frac{e^{xy}}{y}$$

$$f(x,y) = \cos^2(3x - y^2)$$

$$\frac{\partial f}{\partial x} = 2\cos\left(3x - y^2\right) \cdot \frac{\partial}{\partial x}\cos\left(3x - y^2\right) = -2\cos\left(3x - y^2\right)\sin\left(3x - y^2\right) \cdot \frac{\partial}{\partial x}\left(3x - y^2\right)$$

$$= -6\cos\left(3x - y^2\right)\sin\left(3x - y^2\right)$$

$$\frac{\partial f}{\partial y} = 2\cos\left(3x - y^2\right) \cdot \frac{\partial}{\partial y}\cos\left(3x - y^2\right) = -2\cos\left(3x - y^2\right)\sin\left(3x - y^2\right) \cdot \frac{\partial}{\partial y}\left(3x - y^2\right)$$

$$= 4y\cos\left(3x - y^2\right)\sin\left(3x - y^2\right)$$

تمارين

2 أوجد جميع المشتقات الجزئية من المرتبة الثانية للتوابع الآتية

$$g(x,y) = x^2y + \cos y + y\sin x \qquad w = ye^{x^2-y} \qquad w = x\sin(x^2y)$$

الحل:

$$g(x,y) = x^2y + \cos y + y\sin x$$

$$\frac{\partial g}{\partial x} = 2xy + y\cos x, \quad \frac{\partial g}{\partial y} = x^2 - \sin y + \sin x, \qquad \frac{\partial^2 g}{\partial x^2} = 2y - y\sin x, \quad \frac{\partial^2 g}{\partial y^2} = -\cos y, \quad \frac{\partial^2 g}{\partial y\partial x} = \frac{\partial^2 g}{\partial x\partial y} = 2x + \cos x$$

$$w = ye^{x^2-y}$$

$$\frac{\partial w}{\partial x} = ye^{x^2-y}\cdot 2x = 2xye^{x^2-y}, \quad \frac{\partial w}{\partial y} = (1)e^{x^2-y} + ye^{x^2-y}\cdot(-1) = e^{x^2-y}(1-y),$$

$$\frac{\partial^2 w}{\partial x^2} = 2ye^{x^2-y} + 2xy\left(e^{x^2-y}\cdot 2x\right) = 2ye^{x^2-y}\left(1+2x^2\right) \qquad \frac{\partial^2 w}{\partial y^2} = \left(e^{x^2-y}\cdot(-1)\right)(1-y) + e^{x^2-y}(-1) = e^{x^2-y}(y-2),$$

$$\frac{\partial^2 w}{\partial y\partial x} = \frac{\partial^2 w}{\partial x\partial y} = \left(e^{x^2-y}\cdot 2x\right)(1-y) = 2xe^{x^2-y}(1-y)$$

$$w = x\sin(x^2y)$$

$$\frac{\partial w}{\partial x} = \sin(x^2y) + x\cos(x^2y)\cdot 2xy = \sin(x^2y) + 2x^2y\cos(x^2y),\quad \frac{\partial w}{\partial y} = x\cos(x^2y)\cdot x^2 = x^3\cos(x^2y),$$

$$\frac{\partial^2 w}{\partial x^2} = \cos(x^2y)\cdot 2xy + 4xy\cos(x^2y) - 2x^2y\sin(x^2y)\cdot 2xy = 6xy\cos(x^2y) - 4x^3y^2\sin(x^2y)$$

$$\frac{\partial^2 w}{\partial y^2} = -x^3\sin(x^2y)\cdot x^2 = -x^5\sin(x^2y)$$

$$\frac{\partial^2 w}{\partial y\partial x} = \frac{\partial^2 w}{\partial x\partial y} = 3x^2\cos(x^2y) - x^3\sin(x^2y)\cdot 2xy = 3x^2\cos(x^2y) - 2x^4y\sin(x^2y)$$

تمارين

3 ليكن لدينا $f(x,y) = 2x + 3y - 4$ ، أوجد ميل المماس لهذا السطح في النقطة$(2,-1)$ والواقع في المستوي $x = 2$ ، ثم الواقع في المستوي $y = -1$.

الحل:

$$y = -1 \Rightarrow f_x(x,y) = 2 \Rightarrow f_y(2,-1) = 2 \Rightarrow m = 2$$

$$x = 2 \Rightarrow f_y(x,y) = 3 \Rightarrow f_y(2,-1) = 3 \Rightarrow m = 3$$

4 أوجد تابع $z = f(x,y)$ الذي مشتقاته الجزئية معطاة وفق الآتي، وفي عدم الامكانية اشرح لماذا؟

- $\dfrac{\partial f}{\partial x} = \dfrac{2y}{(x+y)^2}, \quad \dfrac{\partial f}{\partial y} = \dfrac{2x}{(x+y)^2}$
- $\dfrac{\partial f}{\partial x} = 3x^2y^2 - 2x, \quad \dfrac{\partial f}{\partial y} = 2x^3y + 6y$

تمارين

الحل:

- $$\frac{\partial f}{\partial x} = 3x^2y^2 - 2x, \quad \frac{\partial f}{\partial y} = 2x^3y + 6y$$

$$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 6x^2y = \frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

بالتالي يمكن إيجاد تابع وفق الآتي

$$\frac{\partial f}{\partial x} = 3x^2y^2 - 2x \Rightarrow f(x,y) = x^3y^2 - x^2 + g(y) \qquad \Rightarrow \frac{\partial f}{\partial y} = 2x^3y + g'(x) = 2x^3y + 6y$$

$$\Rightarrow g'(y) = 6y \Rightarrow g(y) = 3y^2 \quad \Rightarrow f(x,y) = x^3y^2 - x^2 + 3y^2$$

- $$\frac{\partial f}{\partial x} = \frac{2y}{(x+y)^2}, \quad \frac{\partial f}{\partial y} = \frac{2x}{(x+y)^2} \quad \longrightarrow \quad \frac{\partial^2 f}{\partial y\partial x} = \frac{2x-2y}{(x+y)^3} \neq \frac{\partial^2 f}{\partial x\partial y} = \frac{2y-2x}{(x+y)^3}$$

بالتالي لا يمكن إيجاد تابع.

مبرهنة: إذا كان $w=f(x,y)$ تابع قابل للمفاضلة، وإذا كان $x=x(t)$ و $y=y(t)$ تابعين قابلين للمفاضلة بالنسبة لـ t عندئذ فإن $w=f(x(t),y(t))$ تابع قابل للمفاضلة بالنسبة لـ t و:

$$\frac{dw}{dt}=\frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}$$

أو

$$\frac{dw}{dt}=\frac{\partial w}{\partial x}\frac{dx}{dt}+\frac{\partial w}{\partial y}\frac{dy}{dt}$$

مثال استخدم قاعدة السلسلة لإيجاد مشتق التابع الآتي بالنسبة لـ t وفق المسار $x = \cos t , y = \sin t$

احسب قيمة المشتق عندما $t = \pi / 2$. $w = xy$

الحل

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} = \frac{\partial(xy)}{\partial x}\frac{d}{dt}(\cos t) + \frac{\partial(xy)}{\partial y}\frac{d}{dt}(\sin t) = (y)(-\sin t) + (x)(\cos t)$$

$$= (\sin t)(-\sin t) + (\cos t)(\cos t) = -\sin^2 t + \cos^2 t = \cos 2t.$$

$$\longrightarrow \left.\frac{dw}{dt}\right|_{t=\pi/2} = \cos\left(2\frac{\pi}{2}\right) = \cos \pi = -1.$$

مبرهنة: إذا كان $w = f(x, y, z)$ تابع قابل للمفاضلة، وإذا كان $x = x(t)$ و $y = y(t)$ و $z = z(t)$ توابع قابلة للمفاضلة بالنسبة لـ t عندئذ فإن $w = f(x(t), y(t), z(t))$ تابع قابل للمفاضلة بالنسبة لـ t و:

$$\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

أو

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

مثال أوجد dw/dt ، حيث: $w = xy + z\,, \quad x = \cos t\,, y = \sin t\,, z = t$

احسب قيمة المشتق عندما $t = 0$.

الحل

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} = (y)(-\sin t) + (x)(\cos t) + (1)(1) = (\sin t)(-\sin t) + (\cos t)(\cos t) + 1$$

$$= -\sin^2 t + \cos^2 t + 1 = 1 + \cos 2t,$$

$$\left.\frac{dw}{dt}\right|_{t=0} = 1 + \cos(0) = 2.$$

مبرهنة: إذا كان $w = f(x, y, z)$ و $x = g(r, s)$ و $y = h(r, s)$ و $z = k(r, s)$ توابع قابلة للمفاضلة عندئذ تعطى المشتقات الجزئية لـ r و s وفق الآتي:

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

مثال أوجد $\partial w / \partial r$ و، $\partial w / \partial s$ حيث:

$$w = x + 2y + z^2, \quad x = \frac{r}{s}, y = r + \ln s, z = 2r$$

الحل

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r} = 1(\frac{r}{s}) + 2(1) + 2z(2) = \frac{r}{s} + 2 + 8r$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} = 1(\frac{-r}{s^2}) + 2(\frac{1}{s}) + 2z(0) = \frac{-r}{s^2} + \frac{2}{s}$$

مثال أوجد $\partial w / \partial r$ و، $\partial w / \partial s$ حيث:

$$w = x^2 + y^2, \quad x = r - s, y = r + s$$

الحل

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} = (2x)(1) + (2y)(1) = 2(r - s) + 2(r + s) = 4r$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} = (2x)(-1) + (2y)(1) = -2(r - s) + 2(r + s) = 4s$$

اشتقاق التوابع الضمنية

مبرهنة: إذا كان $F(x,y)$ تابع قابل للمفاضلة، وإذا كانت المعادلة $F(x,y)=0$ تعرف y كتابع قابل للمفاضلة بالنسبة لـ x عندئذ من أجل كل النقاط التي تحقق $F_y \neq 0$، يكون لدينا:

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

مثال أوجد dy/dx، حيث: $y^2 - x^2 + \sin xy = 0$

الحل

بوضع $F(x,y) = y^2 - x^2 - \sin xy$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{-2x - y\cos xy}{2y - x\cos xy} = \frac{2x + y\cos xy}{2y - x\cos xy}$$

اشتقاق التوابع الضمنية

مبرهنة: إذا كان $F(x,y,z)$ تابع قابل للمفاضلة، وإذا كانت المعادلة $F(x,y,z)=0$ تعرف z كتابع قابل للمفاضلة بالنسبة لـ x و y عندئذ من أجل كل النقاط التي تحقق $F_z \neq 0$ ، يكون لدينا:

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}\ , \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

مثال أوجد $\partial z/\partial x$ و $\partial z/\partial y$ في النقطة $(0,0,0)$ حيث: $x^3+z^2+ye^{xz}+z\cos y=0$

الحل بوضع $F(x,y,z) = x^3 + z^2 + ye^{xz} + z\cos y$

$$F_x = 3x^2 + zye^{xz}, \quad F_y = e^{xz} - z\sin y, \quad F_z = 2z + xye^{xz} + \cos y.$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{3x^2 + zye^{xz}}{2z + xye^{xz} + \cos y} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{e^{xz} - z\sin y}{2z + xye^{xz} + \cos y}$$

$(0,0,0) \Rightarrow$

$$\frac{\partial z}{\partial x} = -\frac{0}{1} = 0 \qquad \frac{\partial z}{\partial y} = -\frac{1}{1} = -1$$

تمارين

1 أوجد dw/dt ، لما يأتي

$$w = x^2 + y^2, \quad x = \cos t + \sin t, \quad y = \cos t - \sin t; \quad t = 0$$

$$w = \ln(x^2 + y^2 + z^2), \quad x = \cos t, \quad y = \sin t, \quad z = 4\sqrt{t}; \quad t = 3$$

الحل

$$w = x^2 + y^2, \quad x = \cos t + \sin t, \quad y = \cos t - \sin t; \quad t = 0$$

$$\frac{\partial w}{\partial x} = 2x, \quad \frac{\partial w}{\partial y} = 2y, \quad \frac{dx}{dt} = -\sin t + \cos t, \quad \frac{dy}{dt} = -\sin t - \cos t$$

$$\frac{dw}{dt} = (2x)(-\sin t + \cos t) + (2y)(-\sin t - \cos t) = 2(\cos t + \sin t)(\cos t - \sin t) - 2(\cos t - \sin t)(\sin t + \cos t)$$

$$= \left(2\cos^2 t - 2\sin^2 t\right) - \left(2\cos^2 t - 2\sin^2 t\right) = 0$$

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$$w = \ln(x^2 + y^2 + z^2), \quad x = \cos t, \quad y = \sin t, \quad z = 4\sqrt{t}; \quad t = 3$$

$$\frac{\partial w}{\partial x} = \frac{2x}{x^2+y^2+z^2}, \quad \frac{\partial w}{\partial y} = \frac{2y}{x^2+y^2+z^2}, \quad \frac{\partial w}{\partial z} = \frac{2z}{x^2+y^2+z^2} \qquad \frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t, \quad \frac{dz}{dt} = 2t^{-1/2}$$

$$\frac{dw}{dt} = \frac{-2x\sin t}{x^2+y^2+z^2} + \frac{2y\cos t}{x^2+y^2+z^2} + \frac{4zt^{-1/2}}{x^2+y^2+z^2} = \frac{-2\cos t\sin t + 2\sin t\cos t + 4\left(4t^{1/2}\right)t^{-1/2}}{\cos^2 t + \sin^2 t + 16t} = \frac{16}{1+16t} \qquad \frac{dw}{dt}(3) = \frac{16}{49}$$

2 أوجد $\partial z / \partial u$ و، $\partial z / \partial v$ في النقطة المعطاة، حيث:

$$z = 4e^x \ln y, \quad x = \ln(u\cos v), \quad y = u\sin v; \; (u, v) = (2, \pi/4)$$

الحل

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = \left(4e^x \ln y\right)\left(\frac{\cos v}{u\cos v}\right) + \left(\frac{4e^x}{y}\right)(\sin v) = \frac{4e^x \ln y}{u} + \frac{4e^x \sin v}{y}$$

$$= \frac{4(u\cos v)\ln(u\sin v)}{u} + \frac{4(u\cos v)(\sin v)}{u\sin v} = (4\cos v)\ln(u\sin v) + 4\cos v;$$

تمارين

$$\frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}=\left(4e^x\ln y\right)\left(\frac{-u\sin v}{u\cos v}\right)+\left(\frac{4e^x}{y}\right)(u\cos v)=-\left(4e^x\ \ln y\right)(\tan v)+\frac{4e^x u\cos v}{y}$$

$$=\left[-4(u\cos v)\ln(u\sin v)\right](\tan v)+\frac{4(u\cos v)(u\cos v)}{u\sin v}=(-4u\sin v)\ln(u\sin v)+\frac{4u\cos^2 v}{\sin v}$$

$\left(2,\frac{\pi}{4}\right)$ →

$$\frac{\partial z}{\partial u}=4\cos\frac{\pi}{4}\ln\left(2\sin\frac{\pi}{4}\right)+4\cos\frac{\pi}{4}=2\sqrt{2}\ \ln\sqrt{2}+2\sqrt{2}=\sqrt{2}(\ln 2+2)$$

$$\frac{\partial z}{\partial v}=(-4)(2)\sin\frac{\pi}{4}\ln\left(2\sin\frac{\pi}{4}\right)+\frac{(4)(2)\left(\cos^2\frac{\pi}{4}\right)}{\left(\sin\frac{\pi}{4}\right)}=-4\sqrt{2}\ \ln\sqrt{2}+4\sqrt{2}=-2\sqrt{2}\ \ln 2+4\sqrt{2}$$

3 أوجد $dy\ /dx$ ، في النقاط المعطاة:

$x^3-2y^2+xy=0,\quad(1,1)$ $\qquad xe^y+\sin xy+y-\ln 2=0,\quad(0,\ln 2)$

الحل

$x^3-2y^2+xy=0,\quad(1,1)$

بوضع $F(x,y)=x^3-2y^2+xy=0$

$F_x(x,y)=3x^2+y \qquad F_y(x,y)=-4y+x \quad\Rightarrow\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{3x^2+y}{(-4y+x)} \quad\Rightarrow\frac{dy}{dx}(1,1)=\frac{4}{3}$

تمارين

$$xe^y + \sin xy + y - \ln 2 = 0, \quad (0, \ln 2)$$

بوضع $F(x, y) = xe^y + \sin xy + y - \ln 2 = 0$

$$F_x(x, y) = e^y + y\cos xy \qquad F_y(x, y) = xe^y + x\sin xy + 1 \qquad \Rightarrow \frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{e^y + y\cos xy}{xe^y + x\sin xy + 1}$$

$$\Rightarrow \frac{dy}{dx}(0, \ln 2) = -(2 + \ln 2)$$

4 أوجد $\partial z / \partial x$ و $\partial z / \partial y$ في النقطة المعطاة:

$$z^3 - xy + yz + y^3 - 2 = 0, \quad (1, 1, 1)$$

$$\sin(x + y) + \sin(y + z) + \sin(x + z) = 0, \quad (\pi, \pi, \pi)$$

الحل

$$z^3 - xy + yz + y^3 - 2 = 0, \quad (1, 1, 1)$$

بوضع $F(x, y, z) = z^3 - xy + yz + y^3 - 2 = 0$

$$F_x(x, y, z) = -y, \quad F_y(x, y, z) = -x + z + 3y^2, F_z(x, y, z) = 3z^2 + y$$

تمارين

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{-y}{3z^2+y} = \frac{y}{3z^2+y} \Rightarrow \frac{\partial z}{\partial x}(1,1,1) = \frac{1}{4}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-x+z+3y^2}{3z^2+y} = \frac{x-z-3y^2}{3z^2+y} \Rightarrow \frac{\partial z}{\partial y}(1,1,1) = -\frac{3}{4}$$

$$\sin(x+y) + \sin(y+z) + \sin(x+z) = 0, \quad (\pi, \pi, \pi)$$

بوضع $F(x,y,z) = \sin(x+y) + \sin(y+z) + \sin(x+z) = 0$

$$F_x(x,y,z) = \cos(x+y) + \cos(x+z) \quad F_y(x,y,z) = \cos(x+y) + \cos(y+z), \quad F_z(x,y,z) = \cos(y+z) + \cos(x+z)$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{\cos(x+y)+\cos(x+z)}{\cos(y+z)+\cos(x+z)} \Rightarrow \frac{\partial z}{\partial x}(\pi,\pi,\pi) = -1$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{\cos(x+y)+\cos(y+z)}{\cos(y+z)+\cos(x+z)} \Rightarrow \frac{\partial z}{\partial y}(\pi,\pi,\pi) = -1$$