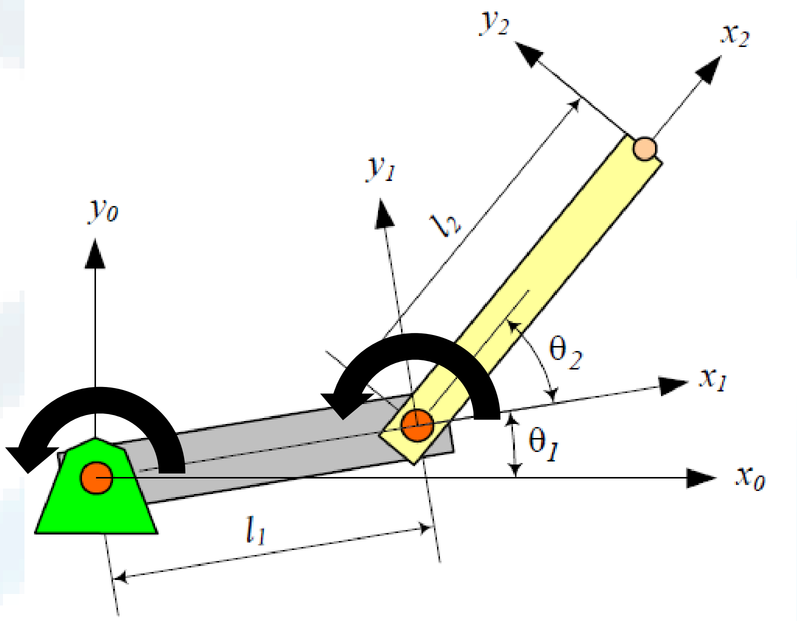
Velocity model

Direct and inverse

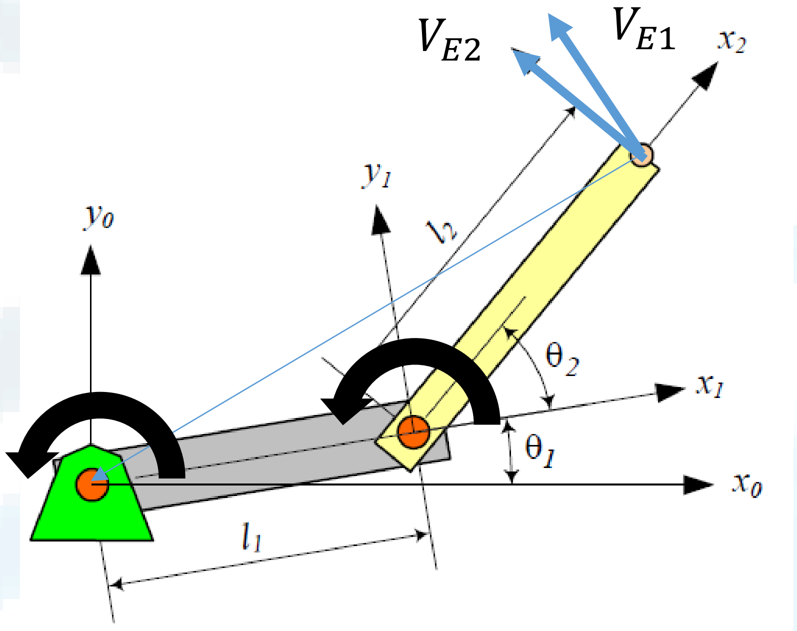
Angular velocity resulting of **n** joints movement

$$\overrightarrow{\omega_E} = \overrightarrow{\omega_1} + \overrightarrow{\omega_2} + \cdots + \overrightarrow{\omega_n}$$



Linear velocity resulting of **n** joints movement

- $\overrightarrow{V_E} = \overrightarrow{V_{E1}} + \overrightarrow{V_{E2}} + \cdots + \overrightarrow{V_{En}}$

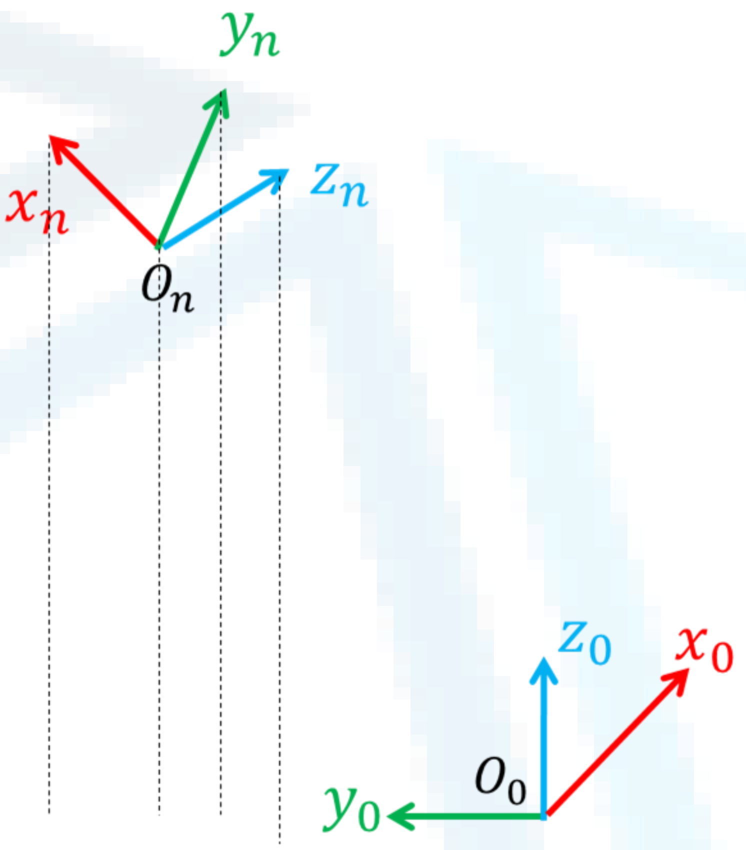


Consider a n-link robot :

w_0^n is the angular velocity of the end-effector

v_0^n is the linear velocity of the end-effector

$$\left.\begin{matrix} v_0^n = J_v \dot{q} \\ w_0^n = J_w \dot{q} \end{matrix}\right\} \begin{cases} \xi = J\dot{q} \\ \xi = \begin{bmatrix} v_0^n \\ w_0^n \end{bmatrix} \\ J = \begin{bmatrix} J_v \\ J_w \end{bmatrix} \\ \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix} \end{cases}$$



Jacobian matrix

$$J_v = [J_{v1} \quad J_{v2} \quad \cdots \quad J_{vn}] : J_{vi} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & revolute - joint - i \\ z_{i-1} & prismatic - joint - i \end{cases}$$

$$J_w = [J_{w1} \quad J_{w2} \quad \cdots \quad J_{wn}] : J_{wi} = \begin{cases} z_{i-1} & revolute - joint - i \\ 0 & prismatic - joint - i \end{cases}$$

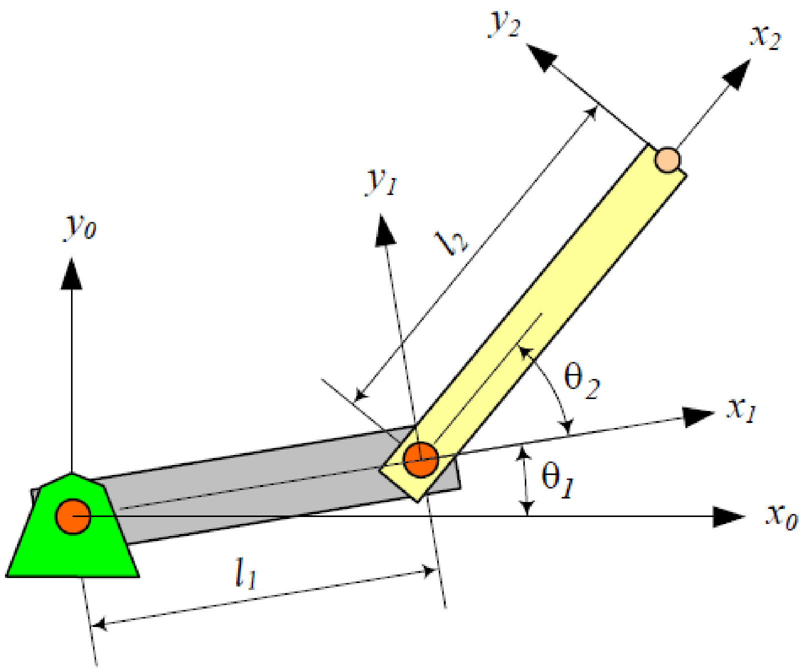
$$J = [J_1 \quad J_2 \quad \cdots \quad J_n] : J_i = \begin{cases} \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix} & revolute - joint - i \\ \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & prismatic - joint - i \end{cases}$$

Example: 2-link planer robot

$$J = \begin{bmatrix} z_0 \times (o_2 - o_0) & z_1 \times (o_2 - o_1) \\ z_0 & z_1 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, o_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{1,2} \\ l_1 s_1 + l_2 s_{1,2} \\ 0 \end{bmatrix}, z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{1,2} & -l_2 s_{1,2} \\ l_1 c_1 + l_2 c_{1,2} & l_2 c_{1,2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$



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DVM of a 2R planer robot

- $\xi = \begin{bmatrix} v \\ w \end{bmatrix} = J\dot{q} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$
- $v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \end{bmatrix}$
- $w = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$

IVM

$$\xi = J\dot{q} \Rightarrow \dot{q} = J^{-1}\xi$$

$$IVM$$

$$Find : \dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3$$

$$When : \theta_1 \quad \theta_2 \quad \theta_3 \quad v_{O3} \quad w_{O3} \quad known$$

IVM of a 2R planer robot

$$\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}=\begin{bmatrix}-l_1s_1-l_2s_{12} & -l_2s_{12}\\ l_1c_1+l_2c_{12} & l_2c_{12}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}\Rightarrow\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}=\begin{bmatrix}-l_1s_1-l_2s_{12} & -l_2s_{12}\\ l_1c_1+l_2c_{12} & l_2c_{12}\end{bmatrix}^{-1}\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}$$

$$T=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\Rightarrow T^{-1}=\frac{-1}{\det(T)}\begin{bmatrix}-d & b\\ c & -a\end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}=\frac{-1}{l_1l_2s_2}\begin{bmatrix}-l_2c_{12} & -l_2s_{12}\\ l_1c_1+l_2c_{12} & l_1s_1+l_2s_{12}\end{bmatrix}^{-1}\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}$$

$$\dot{\theta}_1=\frac{l_2c_{12}\dot{X}+l_2s_{12}\dot{Y}}{l_1l_2s_2},\dot{\theta}_2=-\frac{(l_1c_1+l_2c_{12})\dot{X}+(l_1s_1+l_2s_{12})\dot{Y}}{l_1l_2s_2}$$

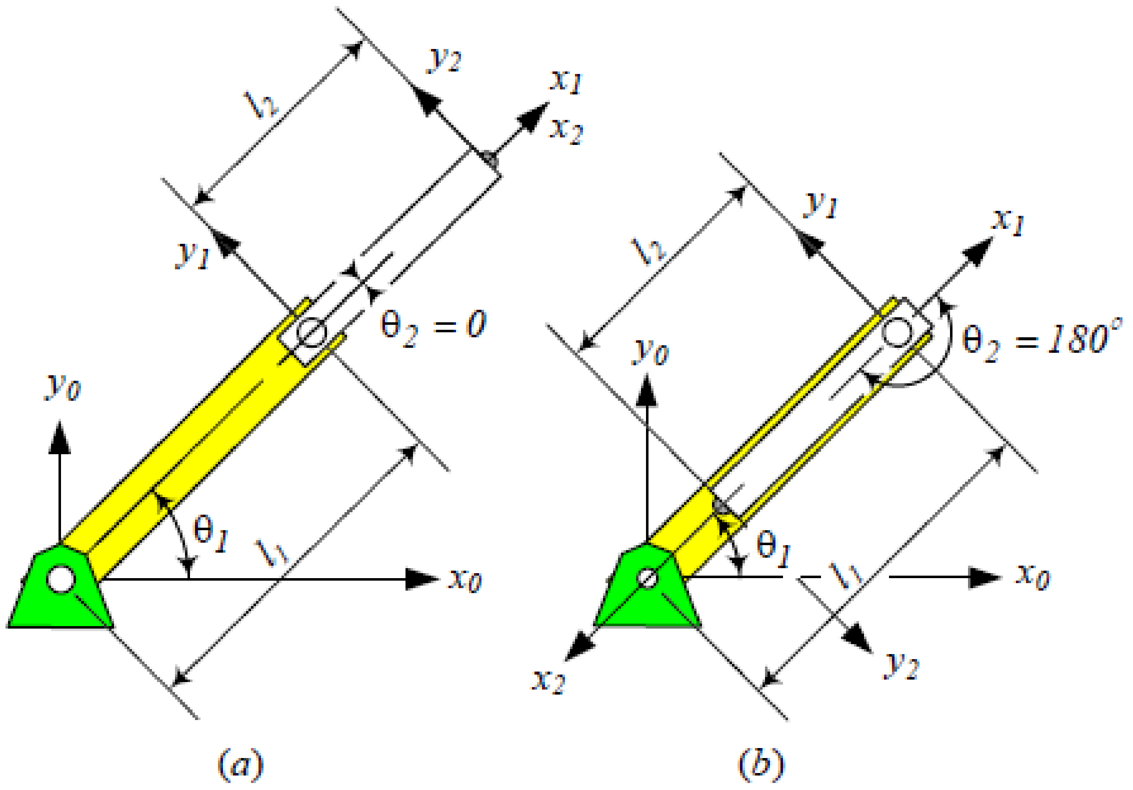
$\det(J) = 0$

Singularity (mathematical and mechanical meaning)

$$2R\_Planer\_Robot$$

$$|J| = 0 \Rightarrow l_1 l_2 s_2 = 0 \Rightarrow \begin{cases} \theta_2 = 0 \\ \theta_2 = \pi \end{cases}$$

$$\Rightarrow \begin{cases} (a) one\_motion\_direction \\ (b) \theta_1 = any\_value \end{cases}$$



6 DOF serial robot velocity model (Paul)

$$\begin{bmatrix} V_E \\ \omega_E \end{bmatrix} = \begin{bmatrix} V_{wrist} + \omega_{wrist} \times (O_E - O_{wrist}) \\ \omega_{wrist} \end{bmatrix}$$

$$\xi_{wrist} = \begin{bmatrix} V_{wrist} \\ \omega_{wrist} \end{bmatrix} = J_{wrist}\dot{q}$$

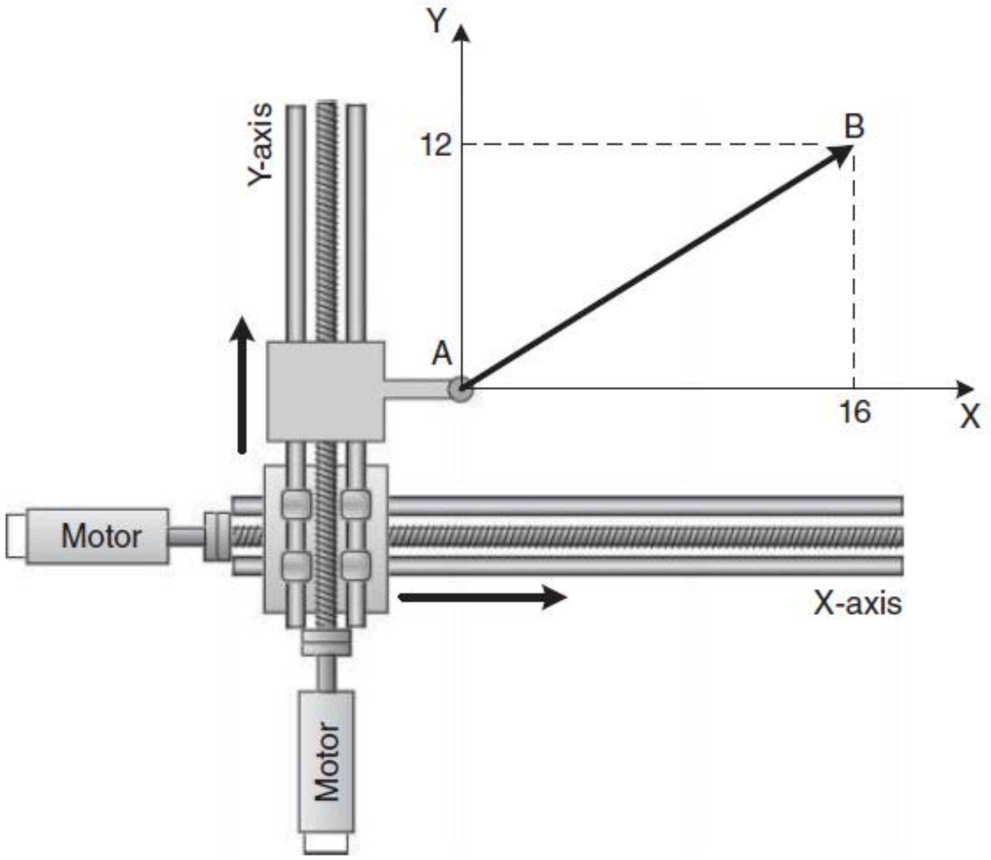
Jacobian Matrix to the wrist

$$J_{wrist} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, A, B, C, D \in R^{3\times 3}$$

- $A = [Z_0 \times O_{wrist} \quad Z_1 \times (O_{wrist} - O1) \quad Z_2 \times (O_{wrist} - O2)]$
- $B = [0 \quad 0 \quad 0]$
- $C = [Z_0 \quad Z_1 \quad Z_2]$
- $D = [Z_{w0} \quad Z_{w1} \quad Z_{w2}]$
- $V_{wrist} = A \times [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]^T$
- $\omega_{wrist} = C \times [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]^T + D \times [\dot{\theta}_4 \quad \dot{\theta}_5 \quad \dot{\theta}_6]^T$

Interpolated motion

- To make the tool tip follow the straight line between points "A" and "B", we can tell the controller to interpolate the motion.
- In this case, it will execute the motion of the longer move as programmed (*X*-axis) and slow down the shorter move (*Y*-axis) so that they both finish their moves at the same time.

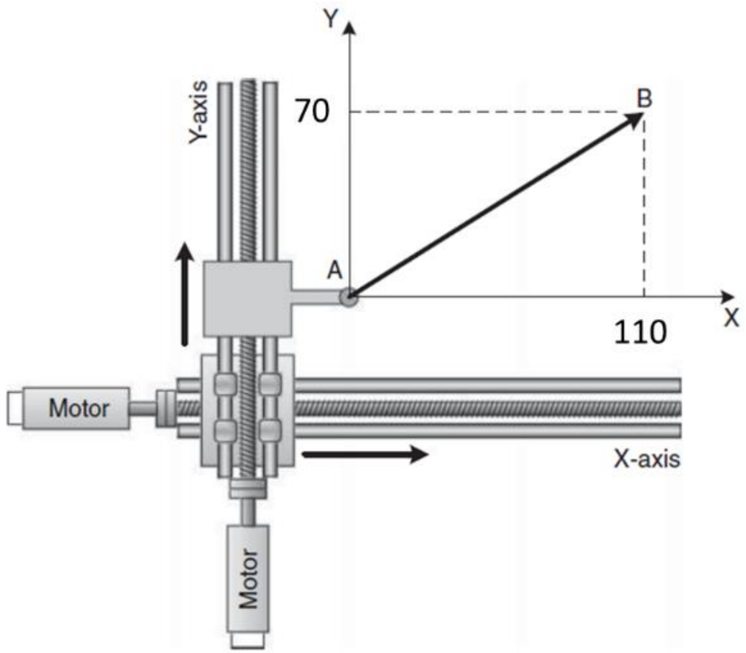


Numerical Example

The machine tool **A** is moving in **interpolated** motion using trapezoidal velocity profile.

1. Given: $v_x = 11\frac{cm}{sec}, t_{ax} = 2\ sec$, $t_{dx} = 3\ sec$
2. What should be v_y, so that both axes finish their moves at the same time?
3. Plot the velocity profile for the two axis.

$$t_{ay} = 1\ sec, t_{dy} = 1.5\ sec$$



Solution X axis

- The X-axis move time is:

$$L_x = \frac{t_{ax} \times v_x}{2} + t_{mx} \times v_x + \frac{t_{dx} \times v_x}{2}$$

$$110 = \frac{2 \times 11}{2} + t_{mx} \times 11 + \frac{3 \times 11}{2} \Rightarrow t_{m/x} = 7.5\ sec$$

The total time for the *X*-axis to complete its motion

$$t_{total/x} = t_{m/x} + t_{ax} + t_{dx} = 12.5\ sec$$

Solution Y Axis

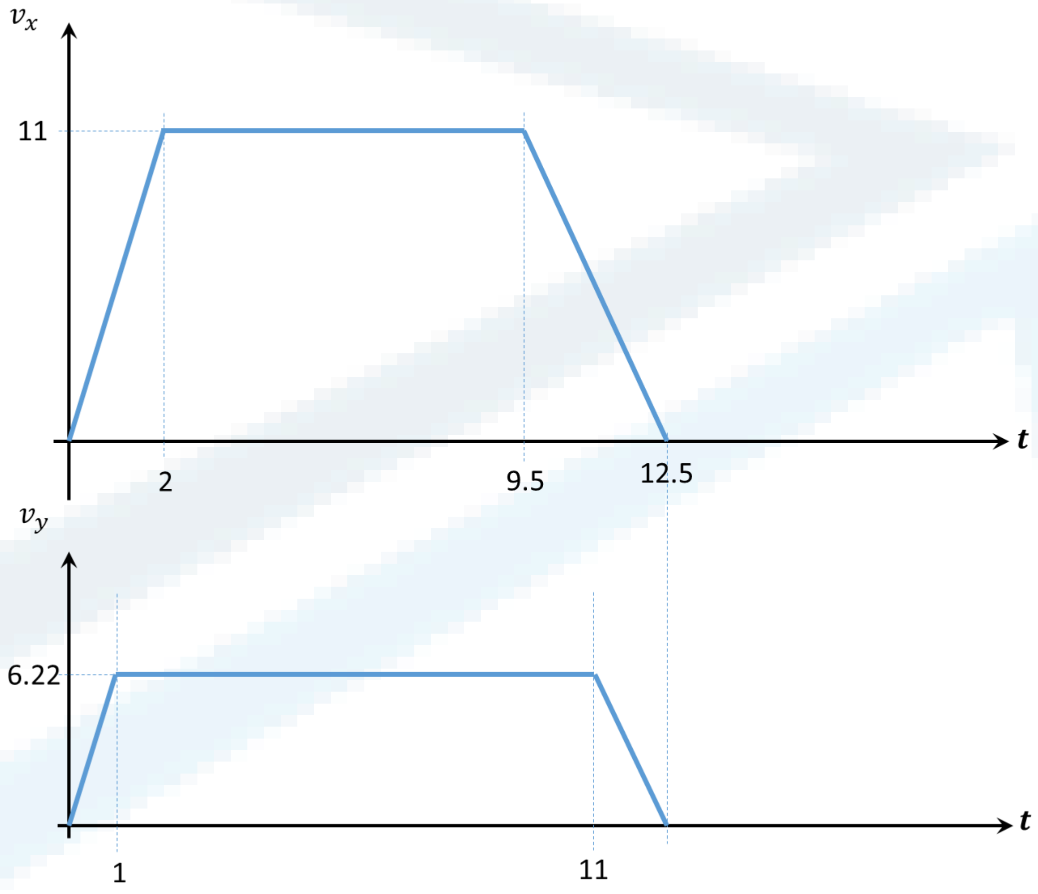
- The Y-axis move time is: $t_{total/x} = t_{total/y}$

$$t_{total/y} = t_{my} + t_{ay} + t_{dy}$$

$$\Rightarrow t_{my} = 10\ sec$$

$$v_y = \frac{L_y}{t_{m/y} + \frac{t_{ay}}{2} + \frac{t_{dy}}{2}} = \frac{70}{0.5 + 0.75 + 10} = 6.222\frac{cm}{sec}$$

Velocity Profiles



Thanks