Velocity model

Jacobian matrix

Angular velocity resulting of **n** joints movement

$$\overrightarrow{\omega_E} = \overrightarrow{\omega_1} + \overrightarrow{\omega_2} + \cdots + \overrightarrow{\omega_n}$$

Robotics
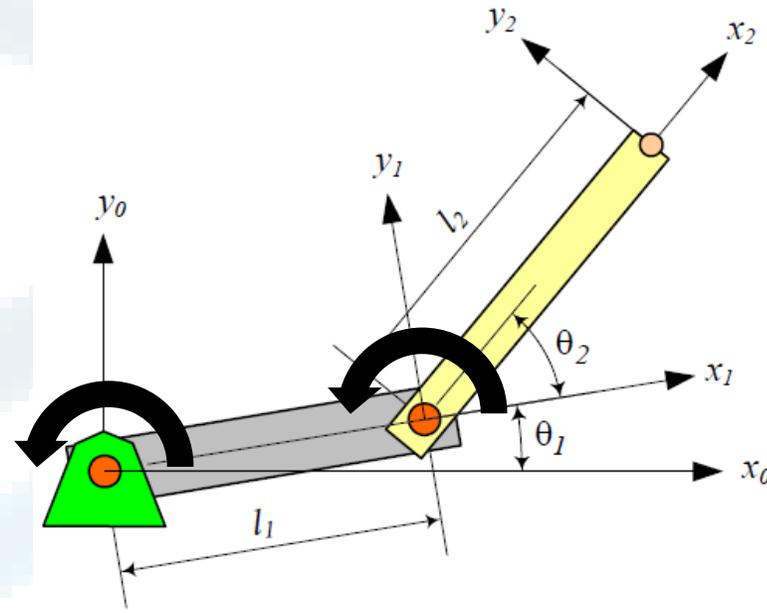
Linear velocity resulting of **n** joints movement

- $\overrightarrow{V_E} = \overrightarrow{V_{E1}} + \overrightarrow{V_{E2}} + \cdots + \overrightarrow{V_{En}}$
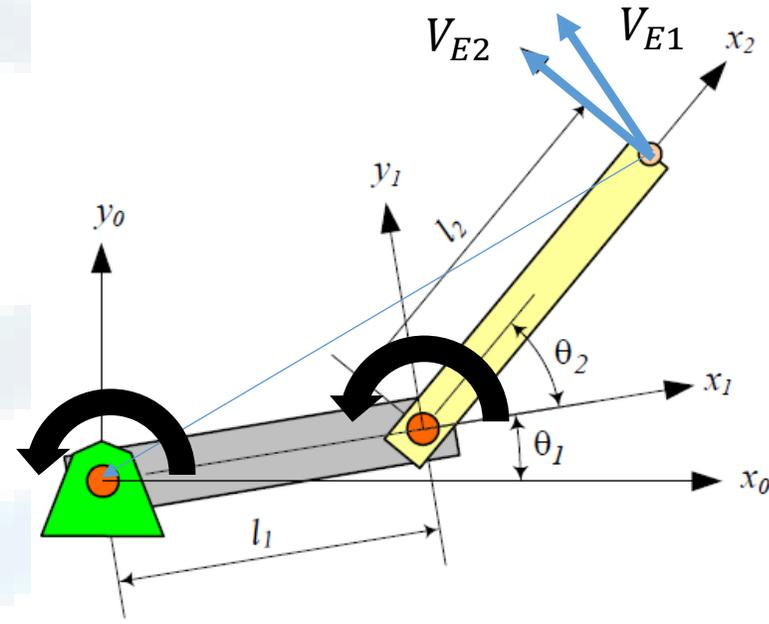
Consider a n-link robot :

w_0^n is the angular velocity of the end-effector

v_0^n is the linear velocity of the end-effector

$$
\left.\begin{array}{c}
v_0^n = J_v \dot{q} \\
w_0^n = J_w \dot{q}
\end{array}\right\}
\begin{cases}
\xi = J\dot{q} \\[4pt]
\xi = \begin{bmatrix} v_0^n \\ w_0^n \end{bmatrix} \\[10pt]
J = \begin{bmatrix} J_v \\ J_w \end{bmatrix} \\[10pt]
\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}
\end{cases}
$$
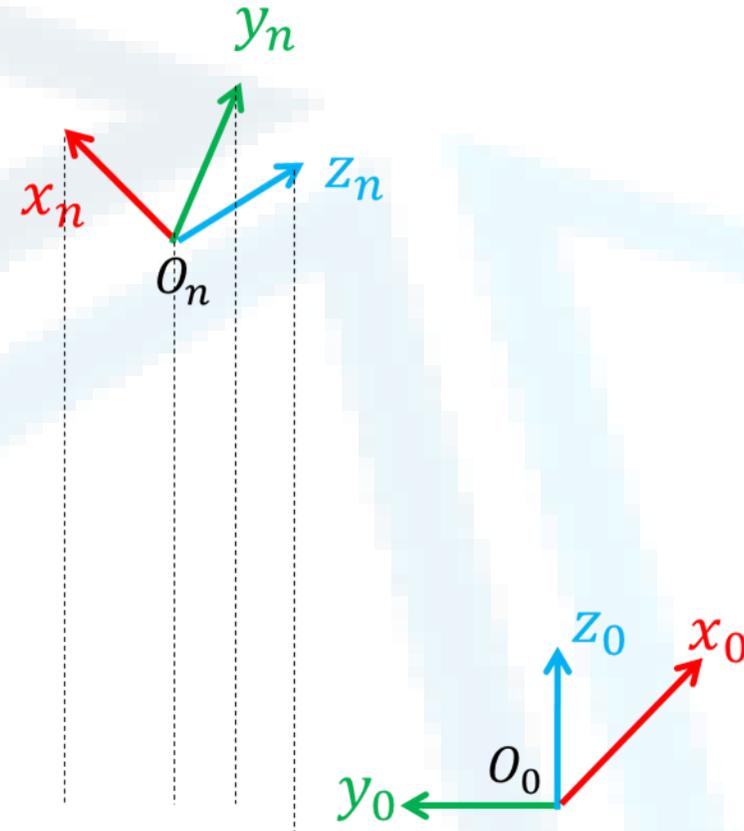
Jacobian matrix

$$J_v = \begin{bmatrix} J_{v1} & J_{v2} & \cdots & J_{vn} \end{bmatrix} : J_{vi} = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & revolute - jo\mathrm{int} - i \\ z_{i-1} & prismatic - jo\mathrm{int} - i \end{cases}$$

$$J_w = \begin{bmatrix} J_{w1} & J_{w2} & \cdots & J_{wn} \end{bmatrix} : J_{wi} = \begin{cases} z_{i-1} & revolute - jo\mathrm{int} - i \\ 0 & prismatic - jo\mathrm{int} - i \end{cases}$$

$$J = \begin{bmatrix} J_1 & J_2 & \cdots & J_n \end{bmatrix} : J_i = \begin{cases} \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix} & revolute - jo\mathrm{int} - i \\ \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & prismatic - jo\mathrm{int} - i \end{cases}$$

Example: 2-link planer robot

$$J = \begin{bmatrix} z_0 \times (o_2 - o_0) & z_1 \times (o_2 - o_1) \\ z_0 & z_1 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, o_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}, o_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{1,2} \\ l_1 s_1 + l_2 s_{1,2} \\ 0 \end{bmatrix}, z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{1,2} & -l_2 s_{1,2} \\ l_1 c_1 + l_2 c_{1,2} & l_2 c_{1,2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$
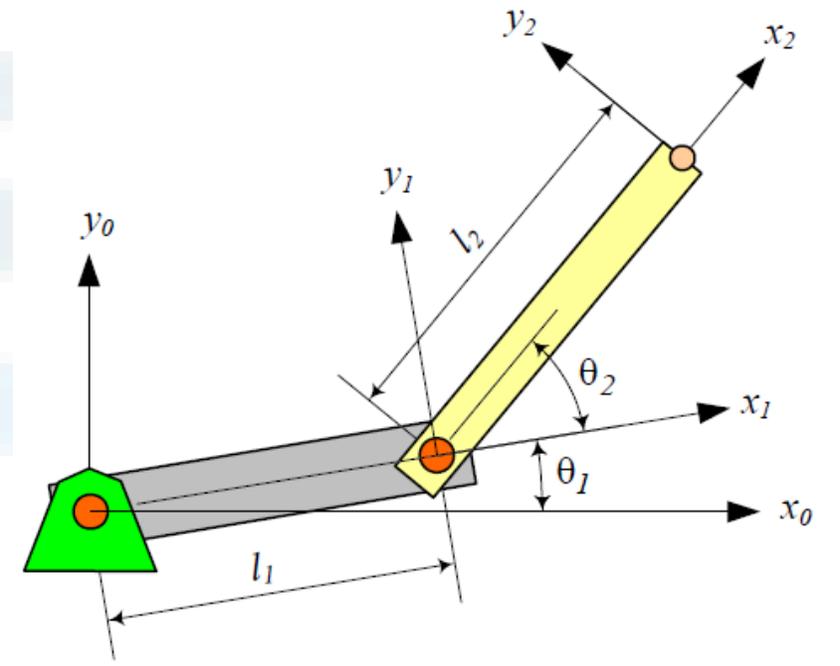
DVM of a 2R planer robot

- $\xi = \begin{bmatrix} v \\ w \end{bmatrix} = J\dot{q} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$

- $v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \dot{\theta}_1 - l_2 s_{12}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1 c_1 \dot{\theta}_1 + l_2 c_{12}(\dot{\theta}_1 + \dot{\theta}_2) \end{bmatrix}$

- $w = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$

$$\xi = J\dot{q} \Rightarrow \dot{q} = \boxed{J^{-1}}\xi$$

IVM

$Find : \dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3$

$When : \theta_1 \quad \theta_2 \quad \theta_3 \quad v_{O3} \quad w_{O3} \quad known$

IVM of a 2R planer robot

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}^{-1} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix}$$

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow T^{-1} = \frac{-1}{\det(T)} \begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \frac{-1}{l_1 l_2 s_2} \begin{bmatrix} -l_2 c_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_1 s_1 + l_2 s_{12} \end{bmatrix}^{-1} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix}$$

$$\dot{\theta}_1 = \frac{l_2 c_{12} \dot{X} + l_2 s_{12} \dot{Y}}{l_1 l_2 s_2}, \dot{\theta}_2 = -\frac{(l_1 c_1 + l_2 c_{12}) \dot{X} + (l_1 s_1 + l_2 s_{12}) \dot{Y}}{l_1 l_2 s_2}$$

Singularity (mathematical and mechanical meaning)

$$\det(J) = 0$$

$2R \_ Planer \_ Robot$

$$|J| = 0 \Rightarrow l_1 l_2 s_2 = 0 \Rightarrow \begin{cases} \theta_2 = 0 \\ \theta_2 = \pi \end{cases}$$

$$\Rightarrow \begin{cases} (a) one \_ motion \_ direction \\ \quad (b)\theta_1 = any \_ value \end{cases}$$

(a)   (b)
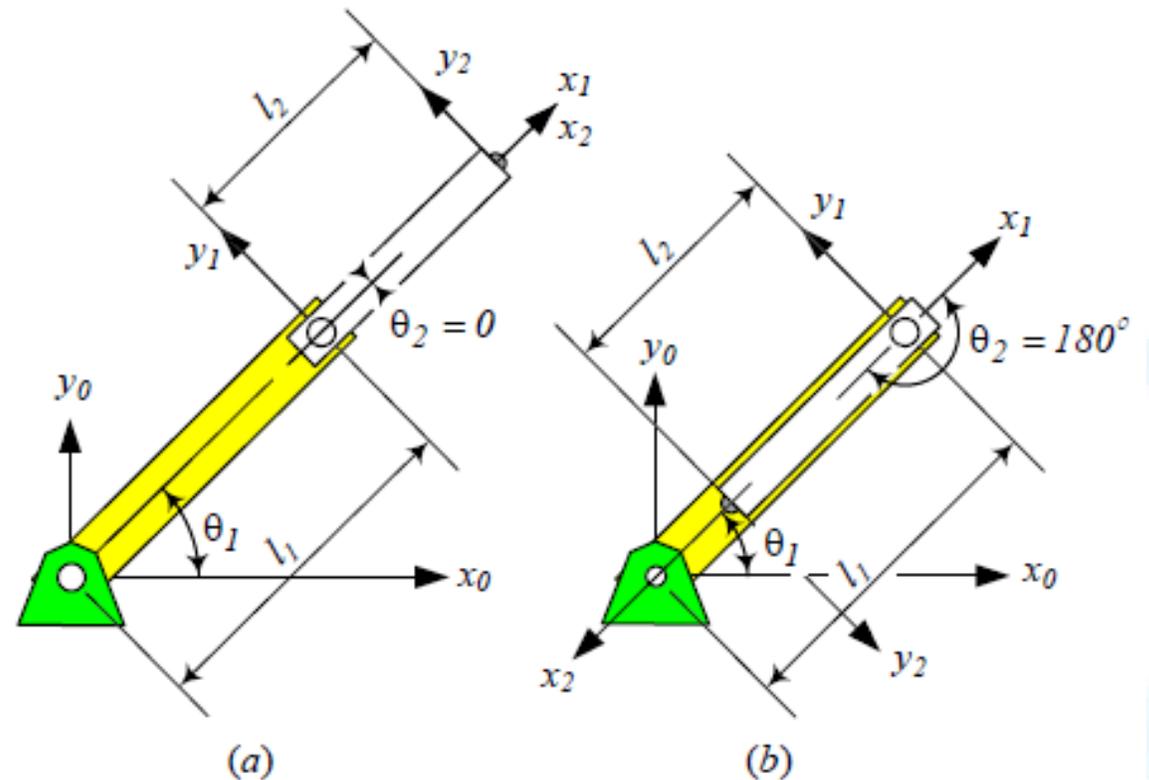
6 DOF serial robot velocity model (Paul)

$$\begin{bmatrix} V_E \\ \omega_E \end{bmatrix} = \begin{bmatrix} V_{wrist} + \omega_{wrist} \times (O_E - O_{wrist}) \\ \omega_{wrist} \end{bmatrix}$$

$$\xi_{wrist} = \begin{bmatrix} V_{wrist} \\ \omega_{wrist} \end{bmatrix} = J_{wrist}\dot{q}$$

Jacobian Matrix to the wrist

$$J_{wrist} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, A, B, C, D \in R^{3 \times 3}$$

- $A = [Z_0 \times O_{wrist} \quad Z_1 \times (O_{wrist} - O1) \quad Z_2 \times (O_{wrist} - O2)]$
- $B = [0 \quad 0 \quad 0]$
- $C = [Z_0 \quad Z_1 \quad Z_2]$
- $D = [Z_{w0} \quad Z_{w1} \quad Z_{w2}]$
- $V_{wrist} = A \times [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]^T$
- $\omega_{wrist} = C \times [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]^T + D \times [\dot{\theta}_4 \quad \dot{\theta}_5 \quad \dot{\theta}_6]^T$

Thanks