Structural Mechanics (1)

Week No-07

Part-01

Analysis of Indeterminate Structures - Force Method

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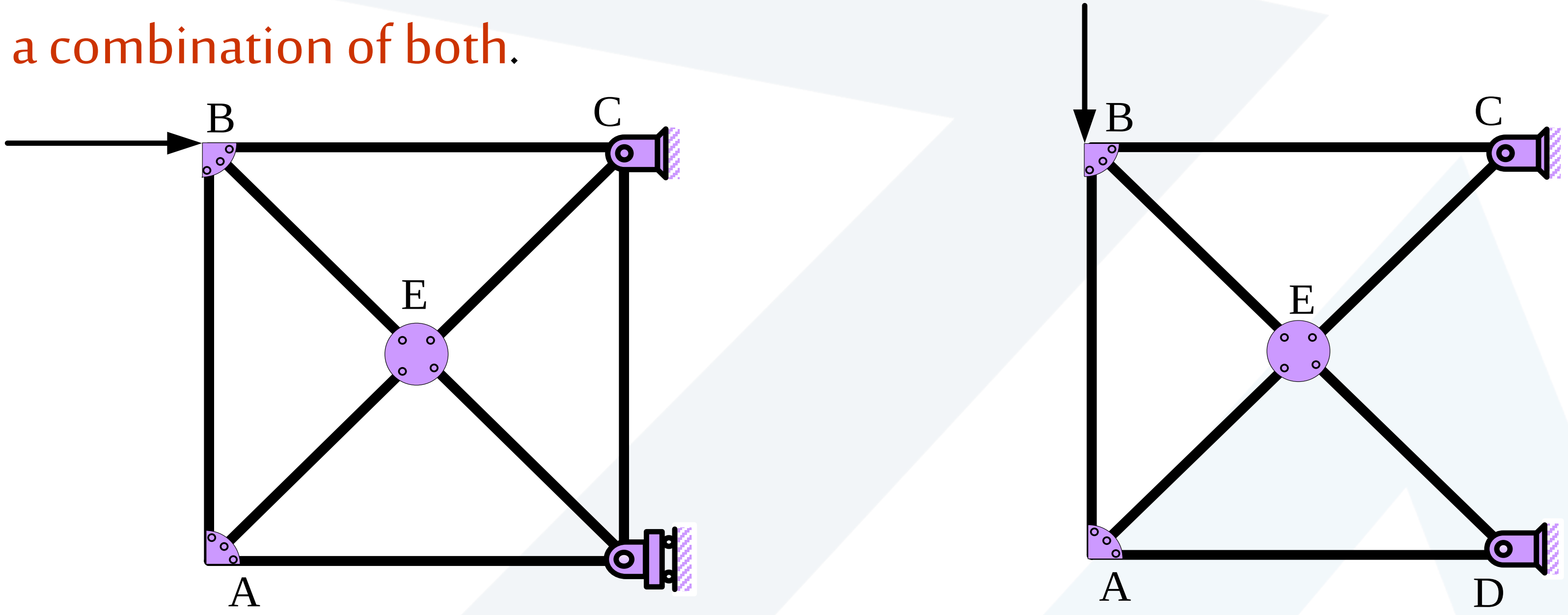
- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- **Structures with multiple Degrees of Indeterminacy**
- **Support Settlements**
- **Three-Moment Equation for Continuous Beams**

Force Method for Trusses that are statically indeterminate to degree one

Trusses can be statically indeterminate due to a variety of reasons:

- redundant support reactions (externally indeterminate),
- redundant members (internally indeterminate),
- or a combination of both.

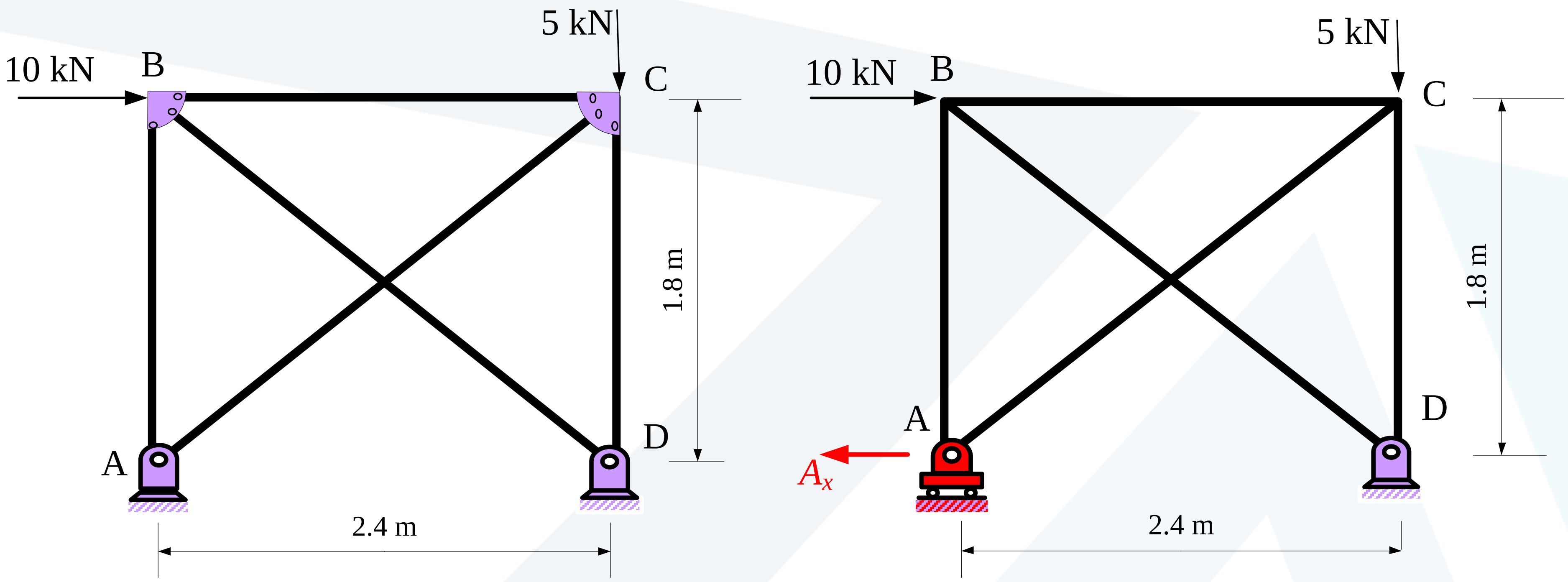


At the right, the truss is externally indeterminate $(m + r) - 2j = 7 + 4 - 2(5) = 1.$

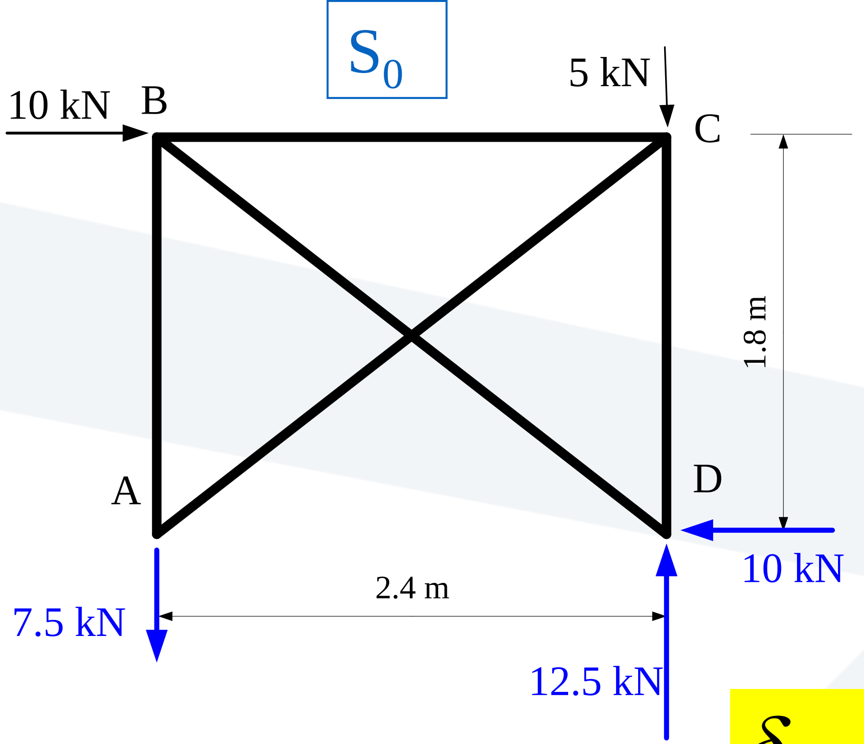
At the left, the truss is internally indeterminate $(m + r) - 2j = 8 + 3 - 2(5) = 1.$

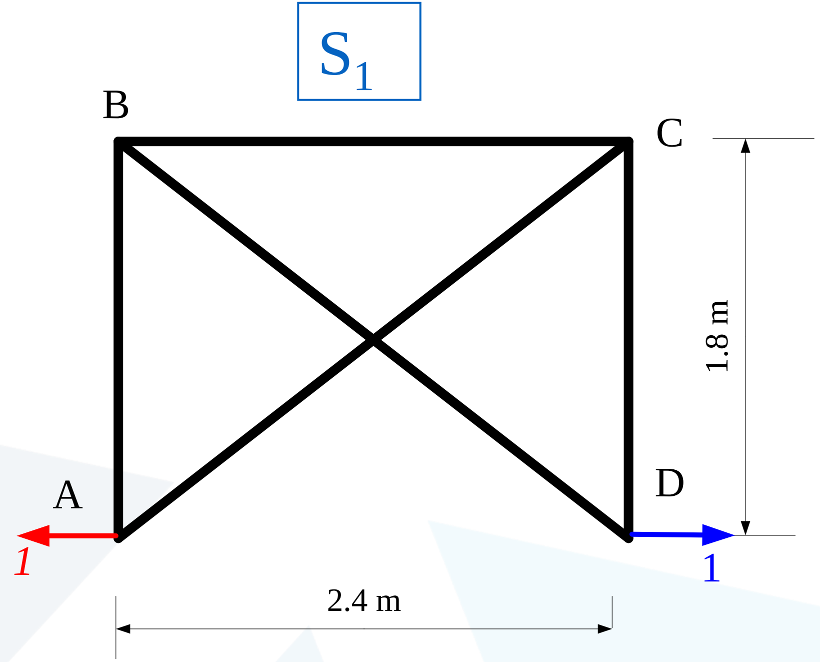
Example: Externally Indeterminate Planar Truss

Compute the support reactions and the member forces for the truss in the next figure. Take E = 200 E+6 kN/m^2 and A = 400E-6 m^2. for all the members.



A word about the crossing members: If we do not label the joint, we mean that the members do not intersect and move independently





$$\delta_{0A} + A_x \delta_{AA} = 0$$

$$\delta_{0A} = \sum \frac{NnL}{EA} \quad \& \quad \delta_{AA} = \sum \frac{n^2 L}{EA}$$

Eqm. of joint A $\Rightarrow N_{AC}=0$ & $N_{AB}= +7.5$ kN

Eqm. of joint C $\Rightarrow N_{BC}=0$ & $N_{CD}= -5$ kN

Eqm. of joint B $\Rightarrow N_{BD}= -12.5$ kN

Eqm. of joint A $\Rightarrow n_{AC}=+1.25$ & $n_{AB}= -0.75$

Eqm. of joint C $\Rightarrow n_{BC}= -1$ & $n_{CD}= -0.75$

Eqm. of joint B $\Rightarrow n_{BD}= 1.25$

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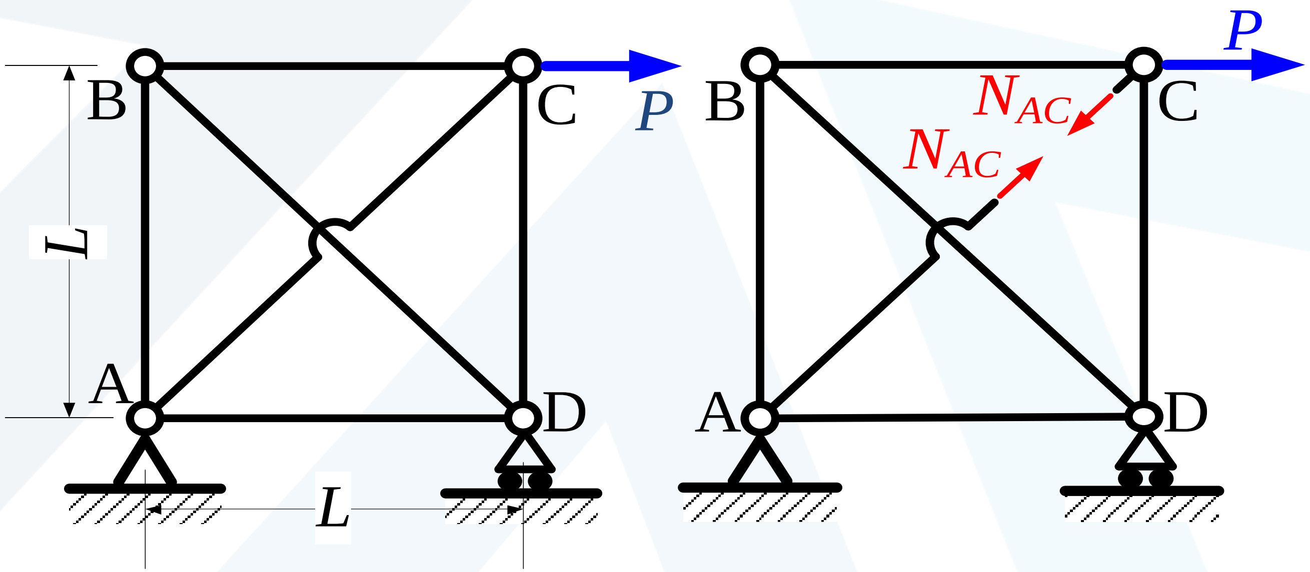
Computing Reactions and Axial Forces in the Indetrminate Truss								
Member	***L*** **[m]**	***A*** [m²]	***N*** [kN]	***n*** [--]	***L/A*** [1/m]	***NnL/A*** [kn/m]	***n²L/A*** [1/m]	$F = N + A_x n$ [kN]
AB	1.8	4.00E-04	7.5	-0.750	4.50E+03	-25313	2531	***4.7693***
AC	3.0	4.00E-04	0.0	1.250	7.50E+03	0	11719	***4.5513***
BC	2.4	4.00E-04	0.0	-1.000	6.00E+03	0	6000	***-3.6410***
BD	3.0	4.00E-04	-12.5	1.250	7.50E+03	-117188	11719	***-7.9488***
CD	1.8	4.00E-04	-5.0	-0.750	4.50E+03	16875	2531	***-7.7308***
					Sum	-125625	34500	
						$A_x = 125625/34500 = 3.64$ kN		
Reaction			R_N [kN]	R_n [--]				$R = R_N + A_x R_n$ [kN]
A_x			0	-1				***-3.6410***
A_y			-7.5	0.0				***-7.5000***
D_x			-10	1				***-6.3590***
D_y			12.5	0.0				***12.5000***

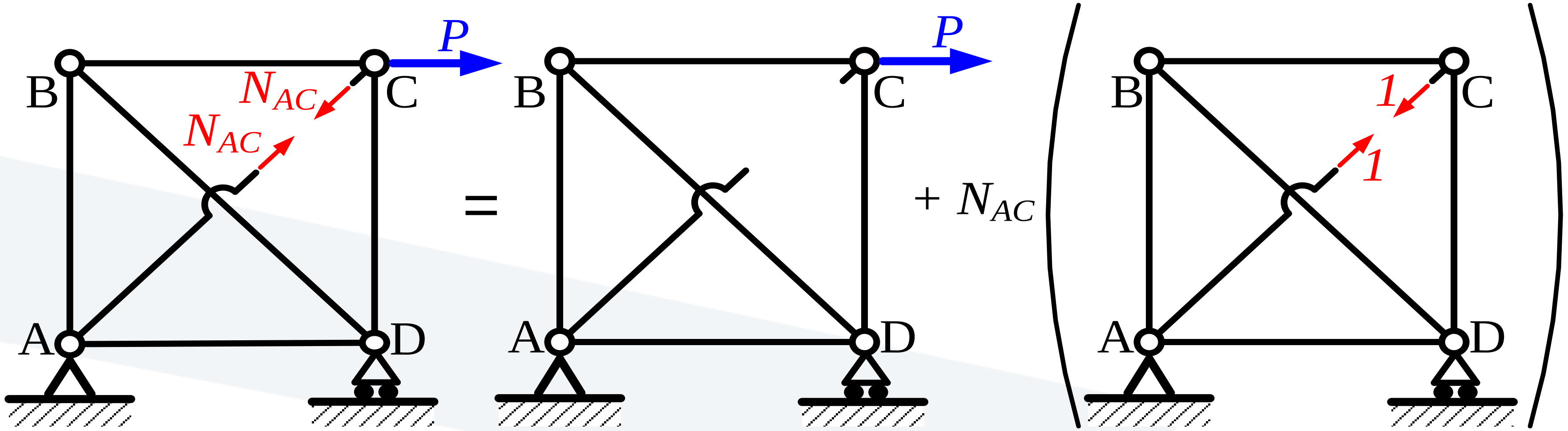
Example: Internally Indeterminate Planar Truss

Determine the forces in the members of the truss shown in Fig. The cross-sectional area, *A*, and, Young modulus, *E*, are the same for all members.

Introduction:

- Their is no intersection between AC and BD.
- There are 6 members, 3 reactions and 4 joints.
- The truss is internally indeterminate to the first order.
- There is one redundant member, like AC. The truss and the corresponding compatibility equation are





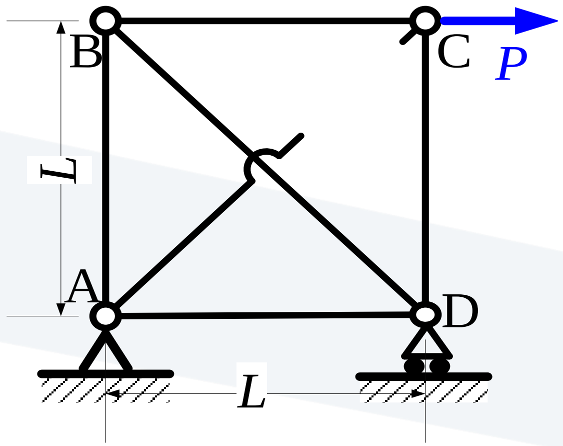
$$\Delta_{AC} = \delta^{0}{}_{AC} + N_{AC}\left(\delta^{1}{}_{AC}\right)$$

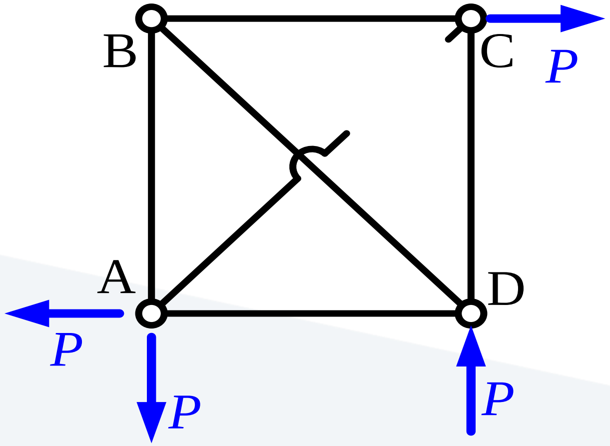
$$\Delta_{AC} = -\frac{N_{AC}L_{AC}}{E_{AC}A_{AC}}, \quad \delta^{0}_{AC} = \sum_{m=1}^{M-1}\frac{N^{0}_{m}n_{m}L_{m}}{E_{m}A_{m}}, \quad \delta^{1}_{AC} = \sum_{m=1}^{M-1}\frac{n_{m}n_{m}L_{m}}{E_{m}A_{m}}$$

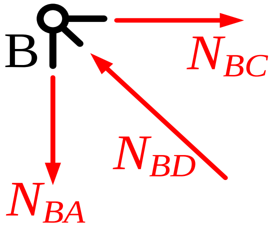
As $N^{0}_{AC} = 0$, and as: $n_{AC} = 1$, $\Delta_{AC} = -\frac{N_{AC}L_{AC}}{E_{AC}A_{AC}} = -N_{AC}\left(\frac{n_{AC}n_{AC}L_{AC}}{E_{AC}A_{AC}}\right)$

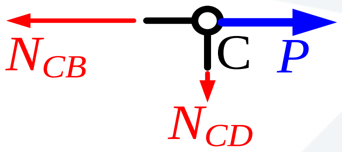
The Compatibility equation becomes

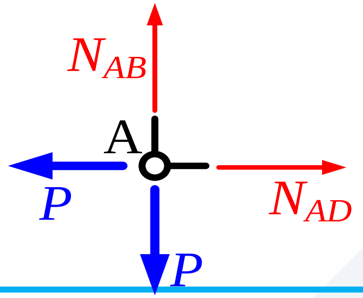
$$\left(\sum_{m=1}^{M}\frac{N^{0}_{m}n_{m}L_{m}}{E_{m}A_{m}}\right) + N_{AC}\left(\sum_{m=1}^{M}\frac{n_{m}n_{m}L_{m}}{E_{m}A_{m}}\right) = 0$$

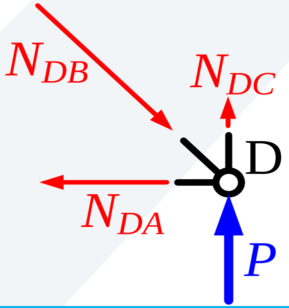












Analyzing S_0:

Eq. Eqs. of S^0:

$\Sigma F_x=0 \Rightarrow A_x=P(\leftarrow)$

$\Sigma M_A=0 \Rightarrow D_y=P(\uparrow)$

$\Sigma M_D=0 \Rightarrow A_y=P(\downarrow)$

Eq. Eqs. of C:

$\Sigma F_x=0 \Rightarrow N_{CB}=P(\mathrm{T})$

$\Sigma F_y=0 \Rightarrow N_{CD}=0$

Eq. Eqs. of B:

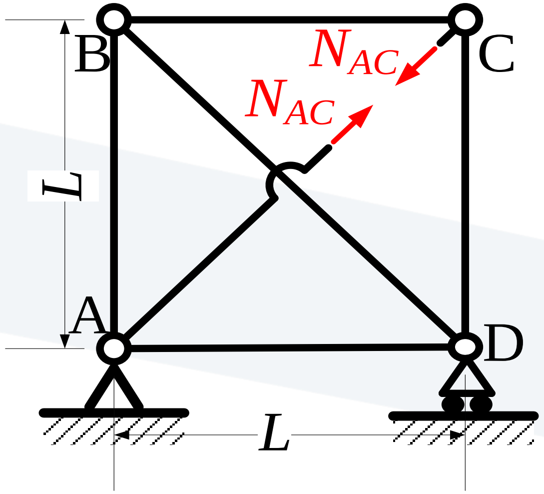
$\Sigma F_x=0 \Rightarrow N_{BD}=P\sqrt{2}$ (C)

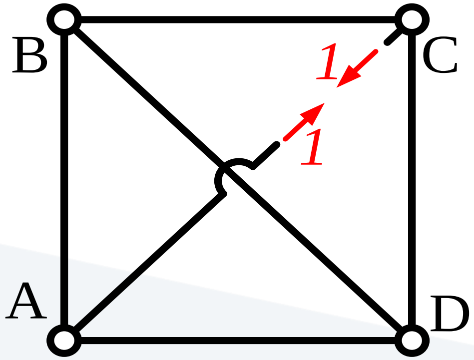
$\Sigma F_y=0 \Rightarrow N_{BA}=P(\mathrm{T})$

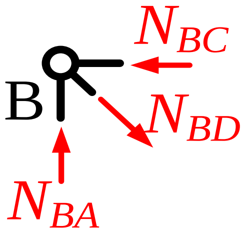
Eq. Eqs. of A:

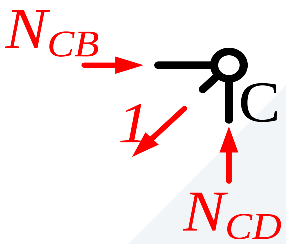
$\Sigma F_x=0 \Rightarrow N_{AD}=P(\mathrm{T})$

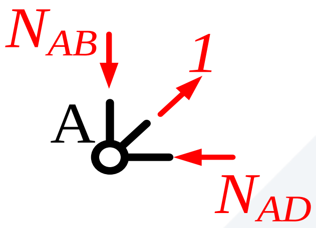
$\Sigma F_y=0 \Rightarrow N_{AB}=P(\mathrm{T})$

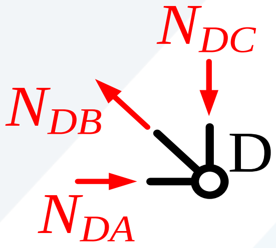












Analyzing S_1:

Eq. Eqs. of S^1:

$\Sigma F_x=0 \Rightarrow A_x=0$

$\Sigma M_A=0 \Rightarrow D_y=0$

$\Sigma M_D=0 \Rightarrow A_y=0$

Eq. Eqs. of C:

$\Sigma F_x=0 \Rightarrow n_{CB}=0.707$ (C)

$\Sigma F_y=0 \Rightarrow n_{CD}=0.707$ (C)

Eq. Eqs. of A:

$\Sigma F_x=0 \Rightarrow n_{AB}=0.707$ (C)

$\Sigma F_y=0 \Rightarrow n_{AD}=0.707$ (C)

Eq. Eqs. of B:

$\Sigma F_x=0 \Rightarrow n_{BD}=1$(T)

$\Sigma F_y=0 \Rightarrow n_{BA}=0.707$ (C)

Example: Internally Indeterminate Planar Truss

Computing Axial Forces in the Indetrminate Truss							
Member	*Length*	*area*	N^0	n	N^0nL/A	n^2L/A	$N=N^0+N_{AC}n$
AB	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4\ P$
BC	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4\ P$
AD	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4\ P$
DC	L	A	0	$-1/\sqrt{2}$	0	$L/2A$	$-0.6\ P$
DB	$L\sqrt{2}$	A	$-P\sqrt{2}$	$+1$	$-2PL/A$	$L\sqrt{2}/A$	$-0.56\ P$
AC	$L\sqrt{2}$	A	0	$+1$	0	$L\sqrt{2}/A$	$0.85\ P$
Σ					$-4.121PL/A$	$4.828\ L/A$	
$N_{AC}=4.121P/4.828=0.85P$							

Structural Mechanics (1)

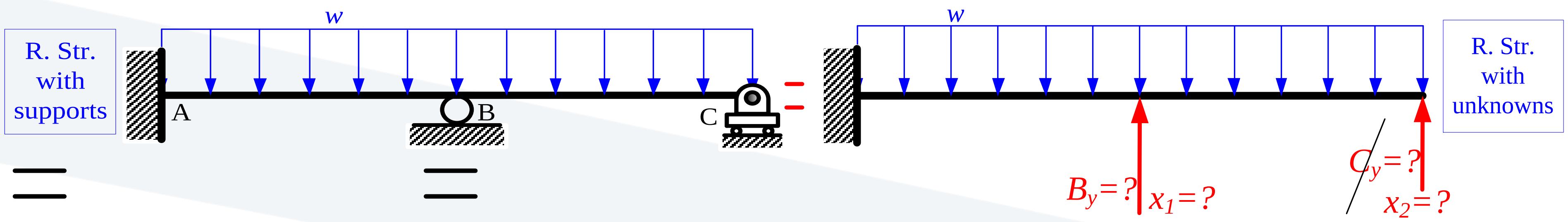
Week No-07

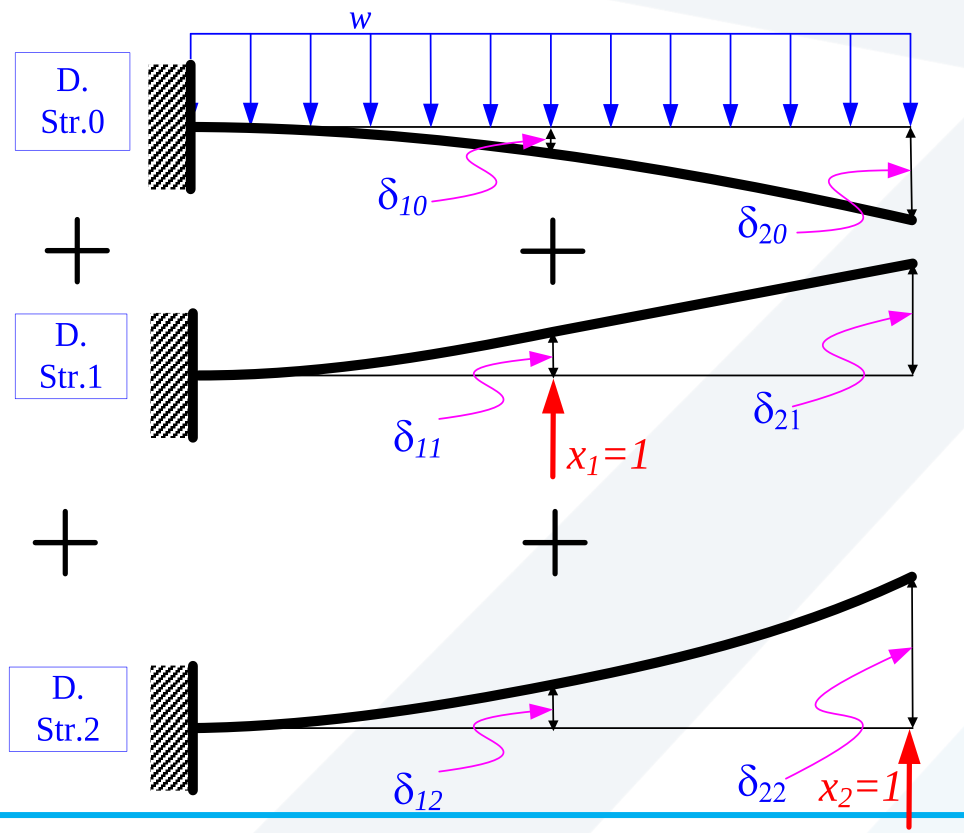
Part-02

Analysis of Indeterminate Structures - Force Method

- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- Structures with multiple Degrees of Indeterminacy
- **Support Settlements**
- **Three-Moment Equation for Continuous Beams**

Force Method for Higher degrees of indeterminacy



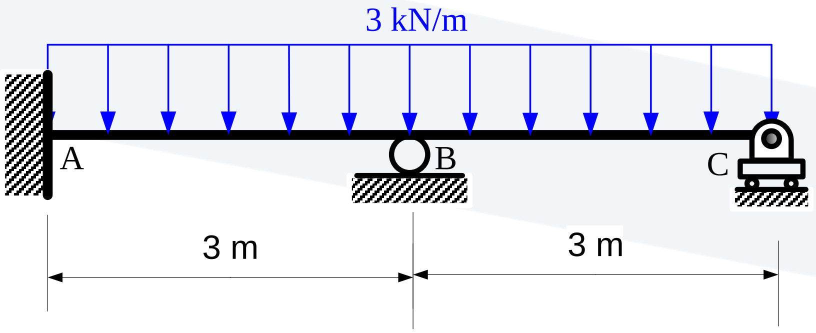


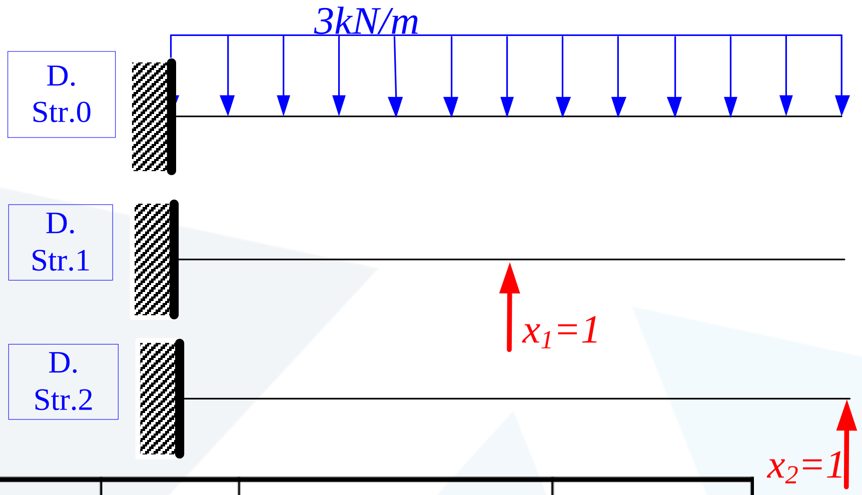
Two Compatibility Equations

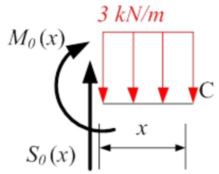
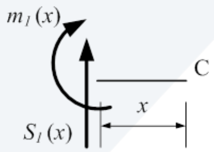
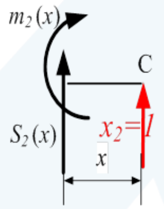
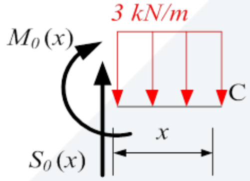
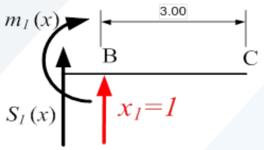
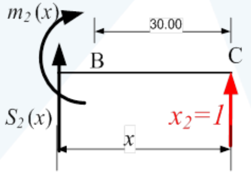
$$\delta_{10} + \delta_{11} x_1 + \delta_{12} x_2 = 0$$

$$\delta_{20} + \delta_{21} x_1 + \delta_{22} x_2 = 0$$

Ex1. Compute the supports reactions, then draw the shear force & bending moment diagrams, EI is constant





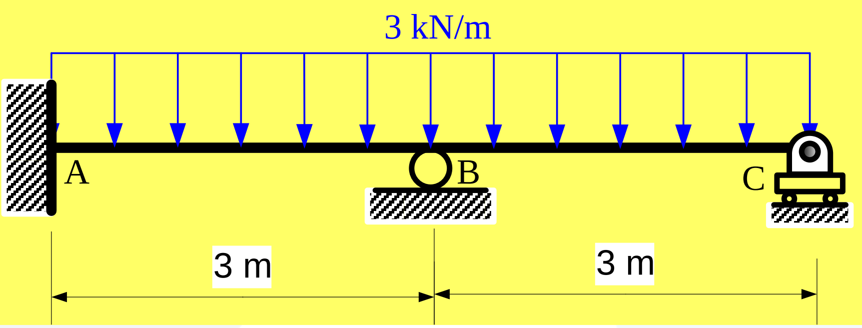
Segment	FBD in D.S.0	$M_0(x)$	FBD in D.S.1	$m_1(x)$	FBD in D.S.2	$m_2(x)$
CB $0\le x\le 3$ EI		$-3x^2/2$		0		x
BA $3\le x\le 6$ EI		$-3x^2/2$		$x-3$		x

$$\delta_{10} = -172.125, \delta_{11} = 9, \delta_{12} = 22.5, \delta_{20} = -486, \delta_{21} = 22.5, \delta_{22} = 72.$$

Ex1. Compute the supports reactions, then draw the shear force & bending moment diagrams, EI is constant

$$\left.\begin{aligned} 9x_1 + 22.5x_2 &= 172.125 \\ 22.5x_1 + 72x_2 &= 486 \end{aligned}\right\}$$

$$\Rightarrow x_1 = 10.29\,\text{kN} \;\&\; x_2 = 3.54\,\text{kN}$$



Segment	$M_0(x)$	$m_1(x)$	$m_2(x)$	$M(x)$ [kN-m]	$S(x)$ [kN]
CB $0 \le x \le 3$ EI	$-3x^2/2$	0	x	$-1.5x^2 + 3.54x$; $M_C=0$, $M(1.5)= 1.935$, $M_B= -2.88$; $M_{max}=2.09$	$-3x + 3.54$, $S_C= +3.54$, $S=0$ at $x = 1.18$m
BA $3 \le x \le 6$ EI	$-3x^2/2$	$x-3$	x	$-1.5x^2 + 13.83x - 30.87$ $M_B= -2.88$, $M(4.5)=0.99$, $M_A= -1.89$; $M_{max}=1.$	$-3x + 13.83$, $S_B= +4.93$, $S=0$ at $x = 4.61$m

Ex2. Compute the supports reactions, and the member internal forces in the next truss.

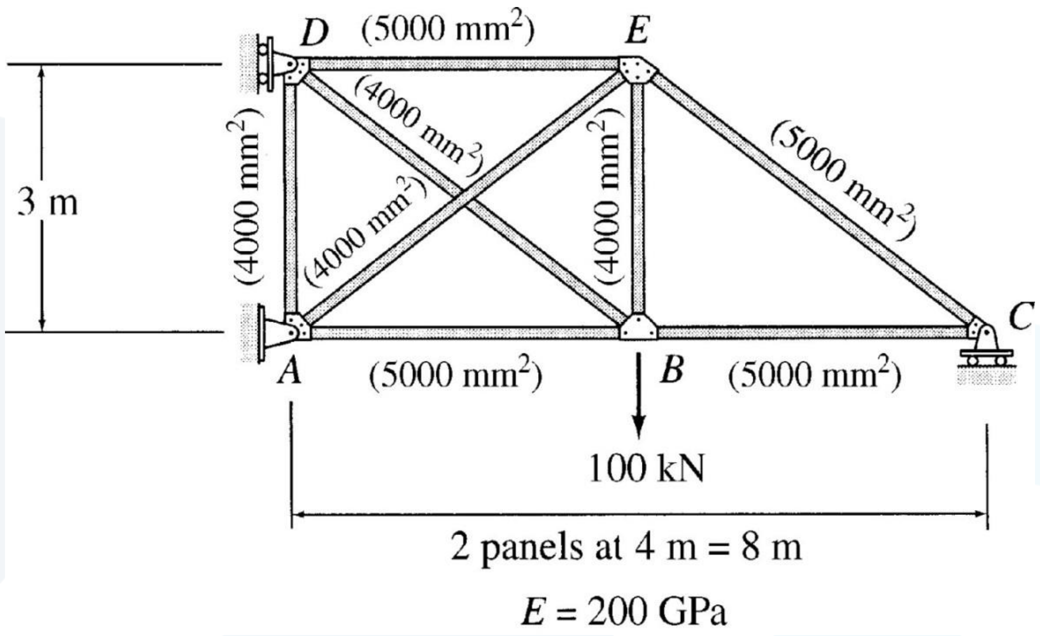
Solution:

The truss is indeterminate to the second degree, once externally and once internally.

C_y is taken as redundant reaction

N_{AE} is taken as redundant member

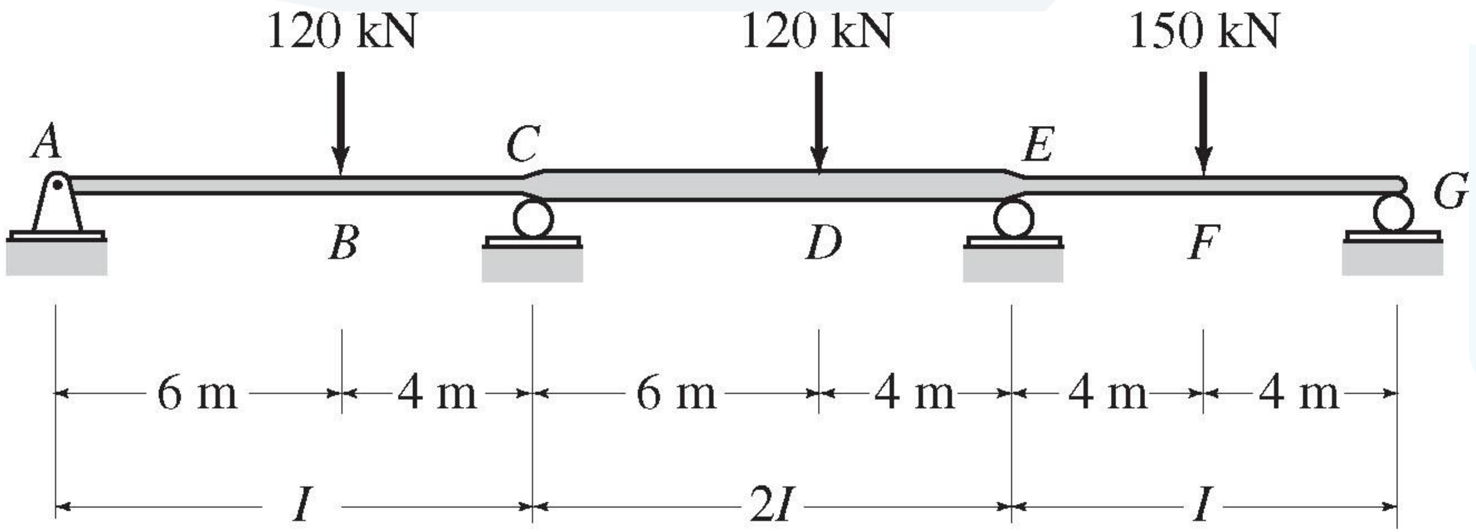
Analysis of S_0, S_1 & S_2 to get N_0, N_1 & N_2. Then use the next table to determine the coefficients of the two compatibility equations



Member	L	EA	N_0	N_1	N_2	N_0N_1L/EA	N_0N_2L/EA	N_1N_1L/EA	N_1N_2L/EA	N_2N_2L/EA

Homework

Pr-01: Using force method, determine the reactions and draw the shear and bending moment diagrams for the beam shown in the following figure.



$E = 200$ Gpa
$I = 500\,(10^6)$ mm^4

Homework

Pr-02: Using force method, determine the reactions and draw the shear and bending moment diagrams for the frame shown in the following figure.