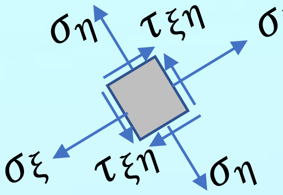
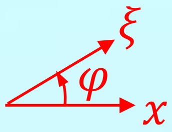
2 Stress

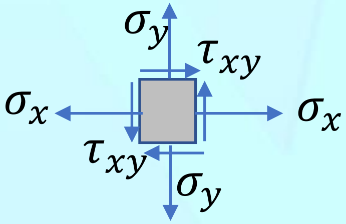
2 الإجهاد

2.2.3 Mohr's Circle

Transformation relations $\boldsymbol{\sigma} = \begin{bmatrix} \sigma_\xi & \tau_{\xi\eta} \\ \tau_{\xi\eta} & \sigma_\eta \end{bmatrix}$







$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$

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$$\sigma_\xi = \sigma_x \cos^2\varphi + \sigma_y \sin^2\varphi + 2\tau_{xy} \sin\varphi \cos\varphi$$
$$\sigma_\eta = \sigma_x \sin^2\varphi + \sigma_y \cos^2\varphi - 2\tau_{xy} \cos\varphi \sin\varphi$$
$$\tau_{\xi\eta} = -(\sigma_x - \sigma_y) \sin\varphi \cos\varphi + \tau_{xy} (\cos^2\varphi - \sin^2\varphi)$$

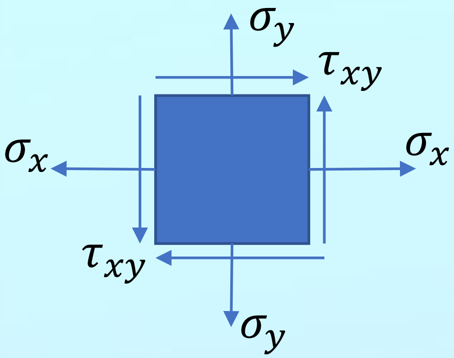
$$\sigma_\xi = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y) \cos 2\varphi + \tau_{xy} \sin 2\varphi\,,$$
$$\sigma_\eta = \tfrac{1}{2}(\sigma_x + \sigma_y) - \tfrac{1}{2}(\sigma_x - \sigma_y) \cos 2\varphi - \tau_{xy} \sin 2\varphi\,,$$
$$\tau_{\xi\eta} = -\tfrac{1}{2}(\sigma_x - \sigma_y) \sin 2\varphi + \tau_{xy} \cos 2\varphi\,.$$

$$\sigma_\xi - \frac{\sigma_x + \sigma_y}{2} = \tau_{max} \cos(2\varphi - 2\varphi^*)$$
$$\sigma_\eta - \frac{\sigma_x + \sigma_y}{2} = -\tau_{max} \cos(2\varphi - 2\varphi^*)$$
$$-\tau_{\xi\eta} = \tau_{max} \sin(2\varphi - 2\varphi^*)$$

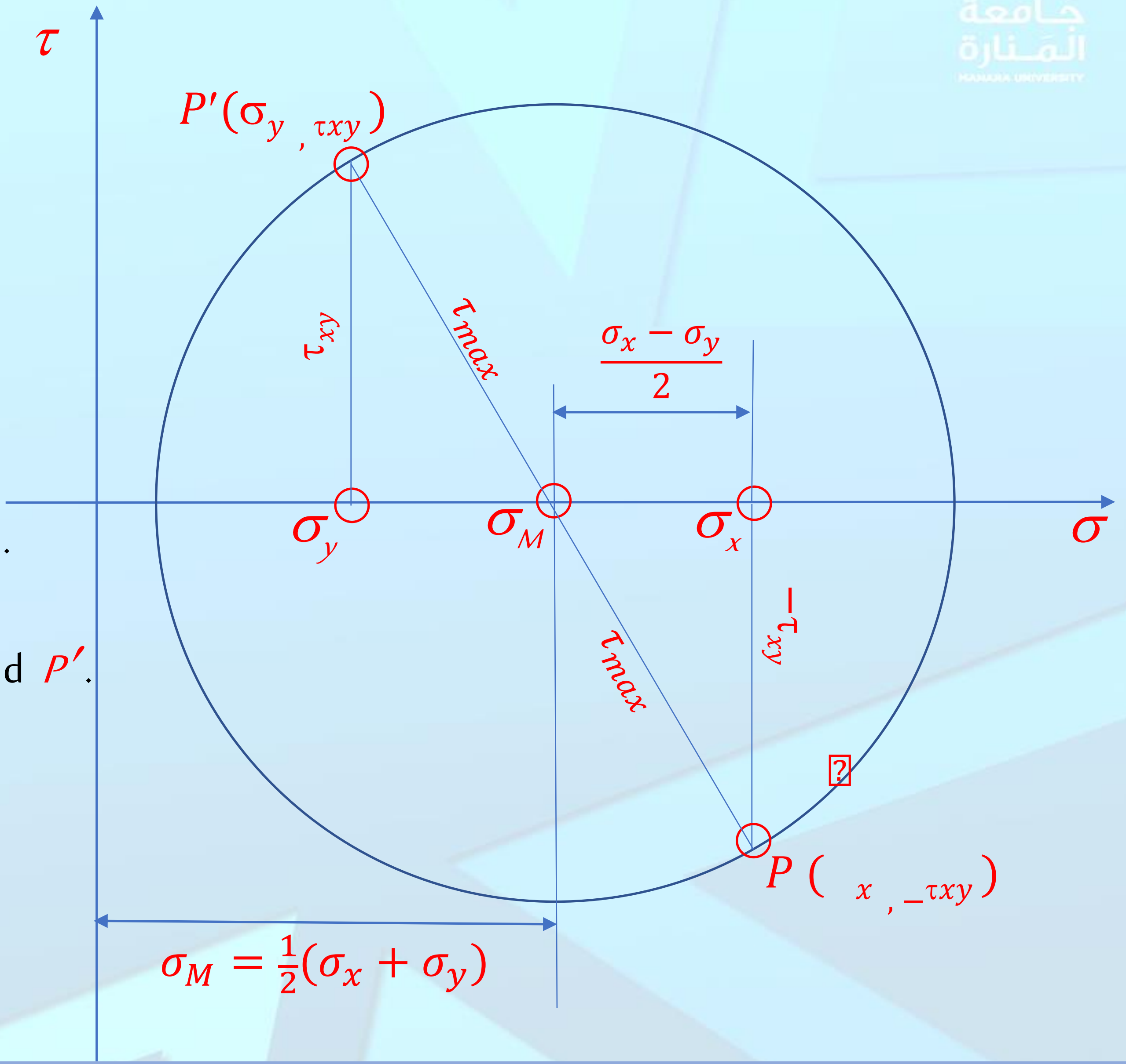
نركز اهتمامنا الآن على مركبتي الإجهاد المؤثرتين على الوجه ξ، أي على المركبتين σ_ξ & $\tau_{\xi\eta}$. تبين العبارتان الأولى والثالثة أن الثنائية $(\sigma_\xi, -\tau_{\xi\eta})$، تمثل وسيطياً دائرة في جملة إحداثيات محورها الأول يمثل الإجهاد الناظمي σ ، ومحورها الثاني هو الإجهاد المماسي τ ، ومركزها في النقطة $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$، ونصف قطرها τ_{max}، ومعادلتها الديكارتية:

$$\left[\sigma_\xi - \frac{\sigma_x + \sigma_y}{2}\right]^2 + (-\tau_{\xi\eta})^2 = \tau_{max}^2$$

$$(\sigma_\xi - \sigma_M)^2 + (-\tau_{\xi\eta})^2 = \tau_{max}^2 \qquad \sigma_M = \frac{\sigma_x + \sigma_y}{2} \qquad \tau_{max}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$



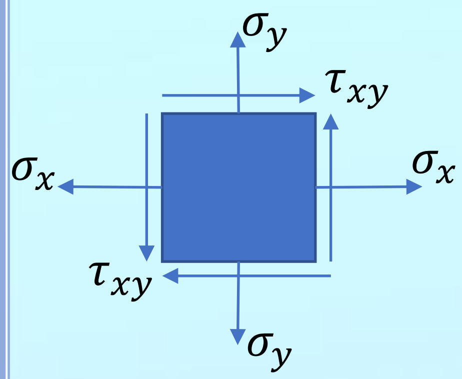
The state of stress at a point is known in the x, y system



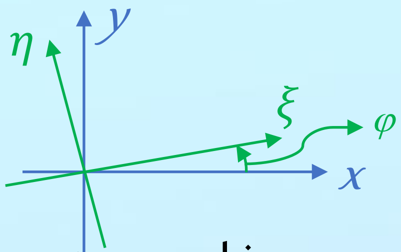
Drawing the Mohr's Circle

1. Draw two perpendicular axes σ & τ, with the same scale.
2. In the system σ, τ; locate points $P(\sigma_x, -\tau_{xy})$ & $P'(\sigma_y, \tau_{xy})$.
3. Draw line PP', to intersect the axis σ at the point $(0, \sigma_M)$.
4. Draw a circle with $(0, \sigma_M)$ as centre and passing through P and P'.
5. From the figure observe that: $\sigma_M = \frac{1}{2}(\sigma_x + \sigma_y)$.
6. And the radius is: $$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$(\sigma_\xi - \sigma_M)^2 + (-\tau_{\xi\eta})^2 = \tau_{max}^2 \qquad \sigma_M = \frac{\sigma_x + \sigma_y}{2} \qquad \tau_{max}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$



The state of stress at a point is known in the x, y system



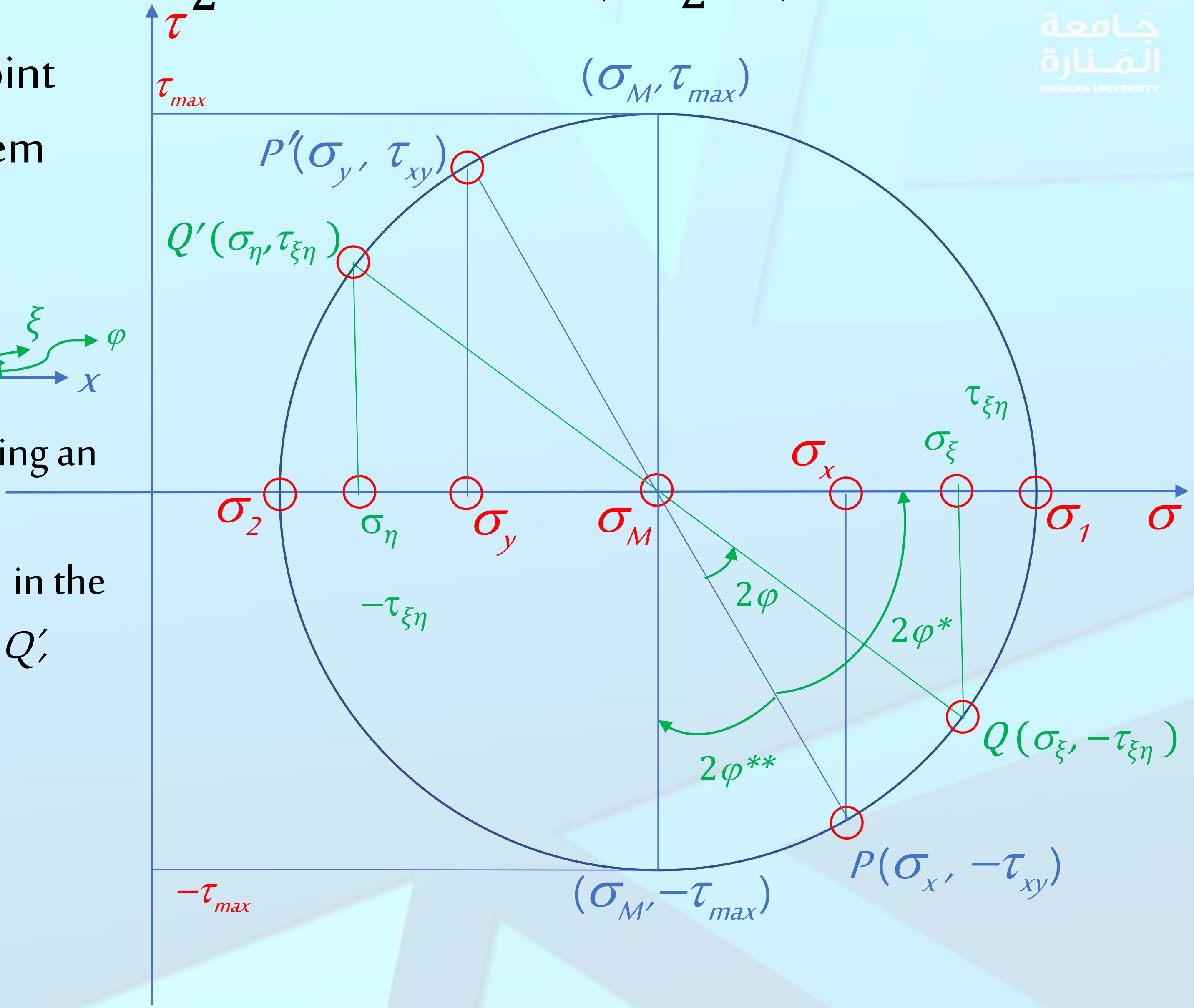
Using Mohr's Circle

1. To determine the stresses in the ξ, η system making an angle φ with x, y system.

At the circle rotate the diameter PP' by an angle 2φ in the opposite direction of φ, to QQ'. The two points Q & Q', represent the ξ, η system.

2. Principal stresses and principal directions.

3. Maximum shear stress and its directions.



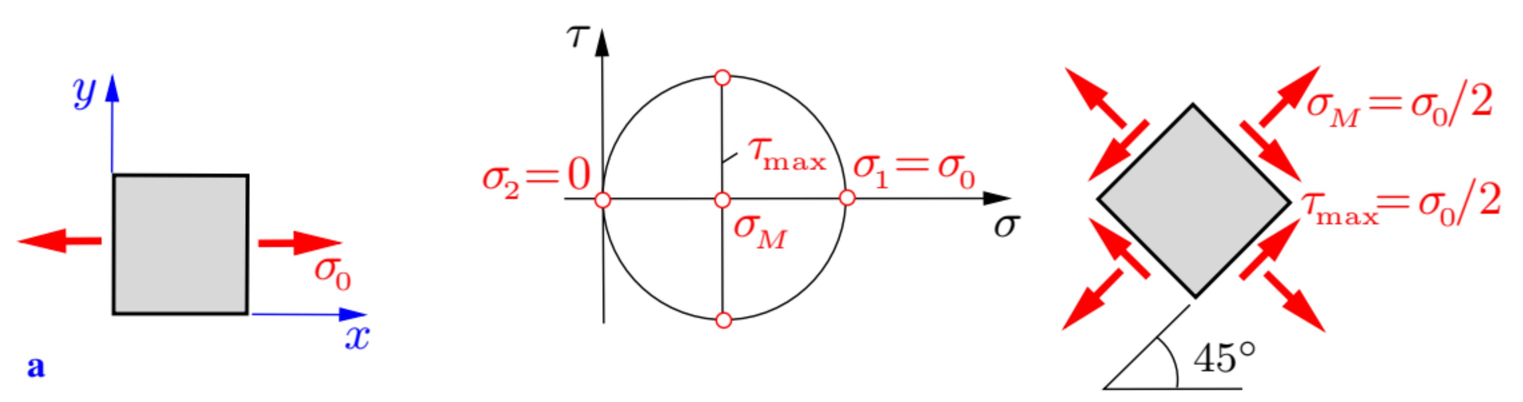
Three special cases

Uniaxial tension

$\sigma_x = \sigma_0 > 0,\ \sigma_y = 0,\ \tau_{xy} = 0$

$\tau_{max} = \sigma_0/2$

at $\varphi = 45°$

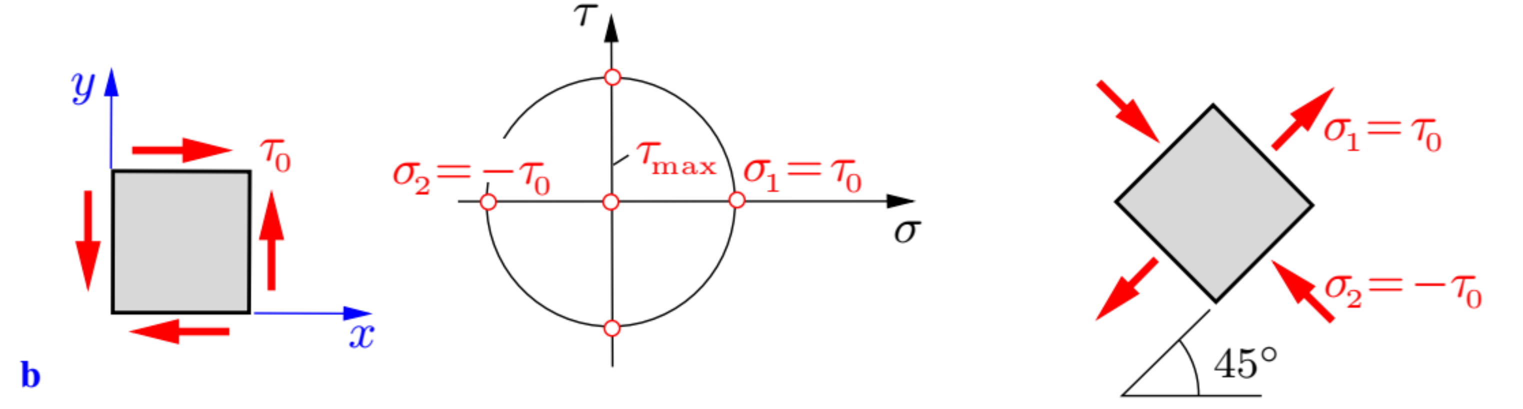


pure shear

$\sigma_x = 0,\ \sigma_y = 0\ \&\ \tau_{xy} = \tau_0$

$\sigma_1 = \tau_0\ \&\ \sigma_2 = -\tau_0$

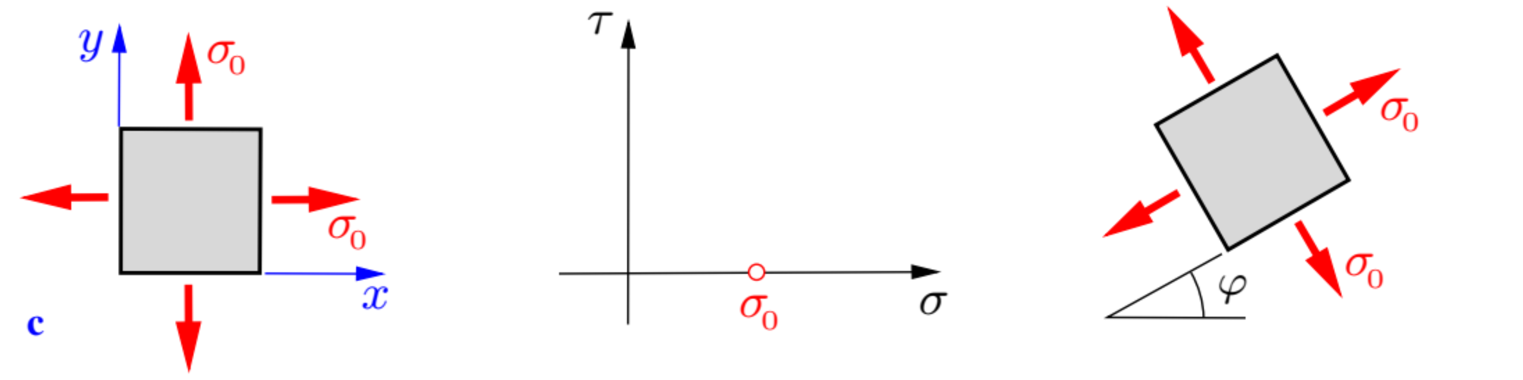
at $\varphi = 45°$



hydrostatic stress state

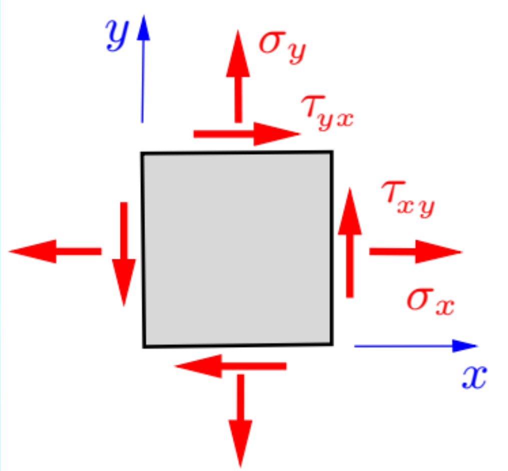
$\sigma_x = \sigma_y = \sigma_0\ \&\ \tau_{xy} = 0$

$\sigma_\xi = \sigma_\eta = \sigma_0$



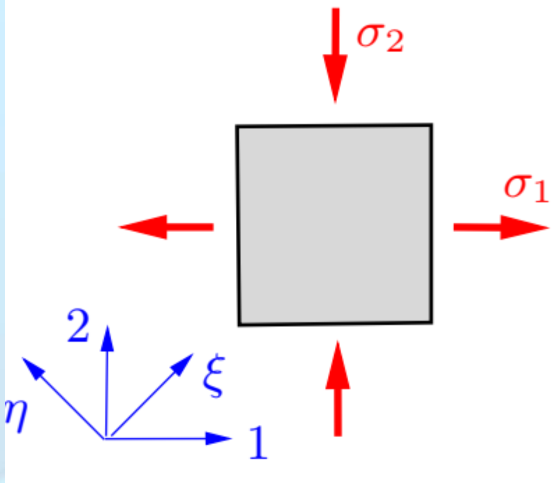
2.4 Supplementary Examples

Example 2.2 The stresses $\sigma_x = 20$ MPa, $\sigma_y = 30$ MPa and $\tau_{xy} = 10$ MPa in a metal sheet are known, see Fig.



Determine the principal stresses and their directions.

Example 2.3 A plane stress state is given by the principal stresses $\sigma_1 = 30$ MPa and $\sigma_2 = -10$ Mpa, seeFig.



a) Determine the stress components in a ξ, η-coordinate system which is inclined by 45° with respect to the principal axes.

b) Using Mohr's circle, determine the rotation angle α of an x, y-coordinate system where $\sigma_y = 0$ and $\tau_{xy} < 0$. Calculate σ_x and τ_{xy}.