

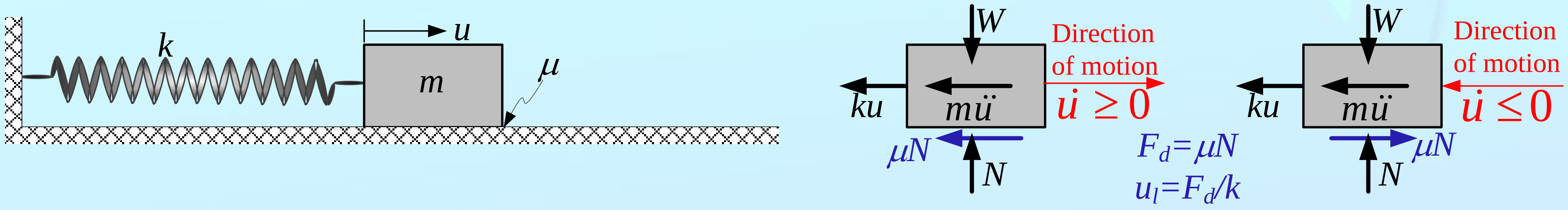



Bracing members
Pall Friction Damper


Coulomb Damping:

Viscous damping is convenient mathematically but not realistic and is not the only type of dissipative mechanism present in modern structural systems.

One such mechanism is sliding friction known as Coulomb Damping. Coulomb damping is a force used to model dry friction that takes place between two surfaces in contact and moving relative to each other.



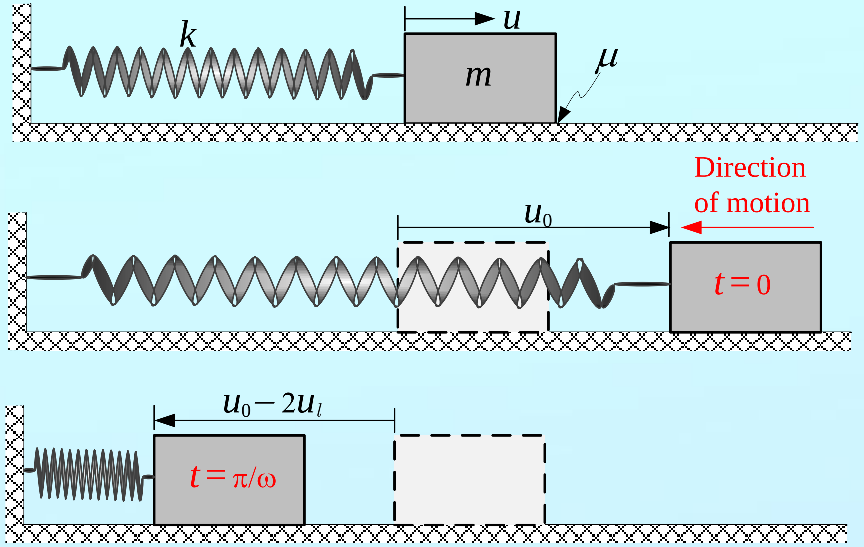
Any initial displacement greater than the locking one, $u_l = F_d/k$, can put the system in free vibration regime.

The Eq. of motion depends on its direction: $m\ddot{u} + ku = -F_d$ or, $m\ddot{u} + ku = +F_d$

Both forms can be written as: $\ddot{u} + \omega^2 u = -(\text{sign}(\dot{u}))(F_d / m)$, where $\omega^2 = k/m$

The complete solution of this ODE, is : $u(t) = A\cos(\omega t) + B\sin(\omega t) - (\text{sign}(\dot{u}))u_l$

The constants *A* and *B*, are determined according to the initial conditions (ICs). One possible scenario is to displace the mass to the right by $u_0 > u_l$, and let free to vibrate.



$$u(t)=A\cos(\omega t)+B\sin(\omega t)-(\text{sign}(\dot{u}))u_l$$

First half cycle starts at $t=0$: $u(0)=u_0>u_l$, or : $ku_0>F_d$.

So motion is to left and velocity at $t=0$, is null. Solution (displacement & velocity) has the form:

$$u(t)=A\cos(\omega t)+B\sin(\omega t)+u_l$$

$$\dot{u}(t)=-A\omega\sin(\omega t)+B\omega\cos(\omega t)$$

with $\dot{u}(0)=0,\ B=0,$ & with $u(0)=u_0,\ A=u_0-u_l$

So the displacement and velocity functions are:

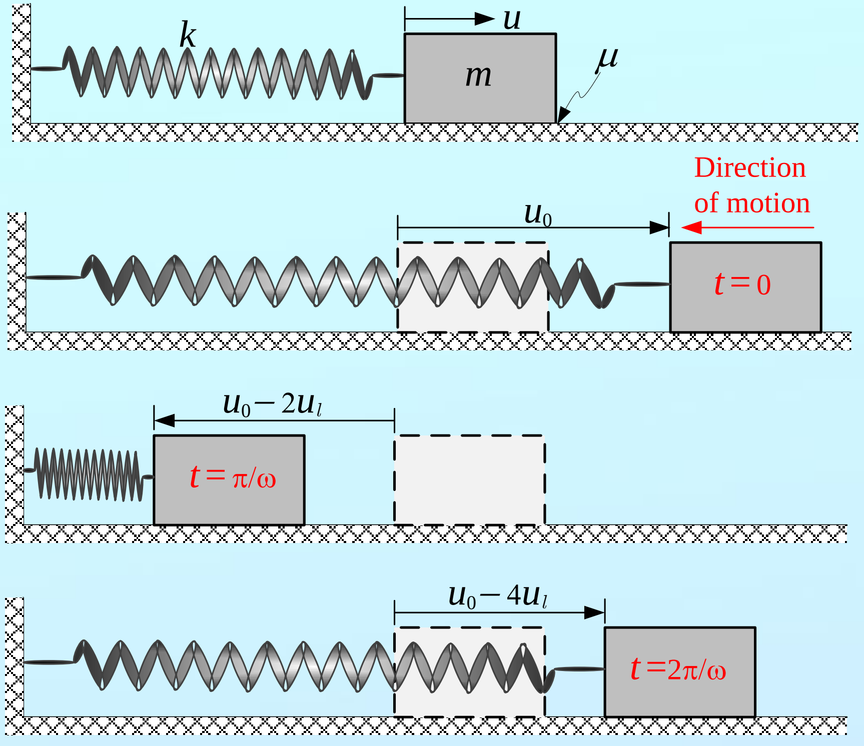
$$u(t)=(u_0-u_l)\cos(\omega t)+u_l,\ \&\ \ \dot{u}(t)=-\omega(u_0-u_l)\sin(\omega t)$$

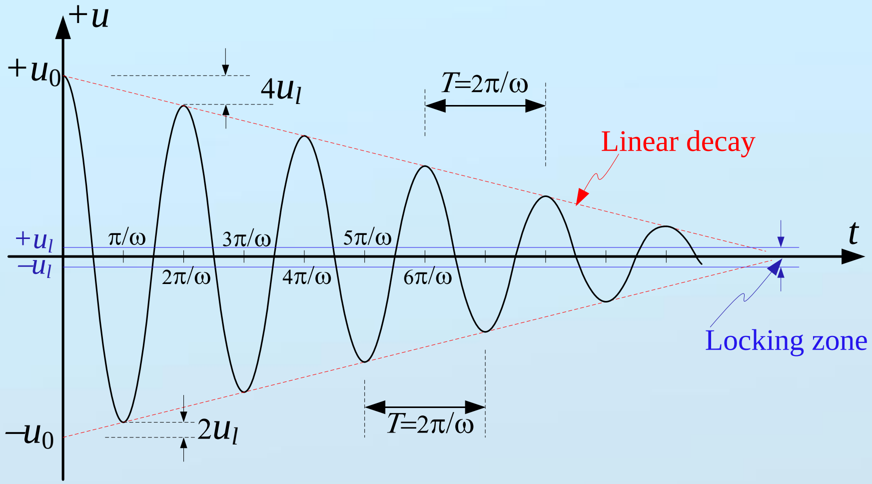
The motion continues to the left until:

$$\dot{u}(t)=0\Rightarrow\sin(\omega t)=0,\quad \text{when}\quad t=\pi/\omega$$

Where the displacement reaches the value:

$$u(\pi/\omega)=(u_0-u_l)\cos(\pi)+u_l=-u_0+2u_l$$





$$u(t) = A\cos(\omega t) + B\sin(\omega t) - (\text{sign}(\dot{u}))u_l$$

If $|u(\pi/\omega)| > u_l$, the motion restarts to right with:

$$u(t) = A\cos(\omega t) + B\sin(\omega t) - u_l$$

$$\dot{u}(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

with $\dot{u}(\pi/\omega) = 0,\ B = 0,$

& with $u(\pi/\omega) = -u_0 + 2u_l,\ A = u_0 - 3u_l$

So the displacement and velocity functions are:

$$u(t) = (u_0 - 3u_l)\cos(\omega t) - u_l,$$

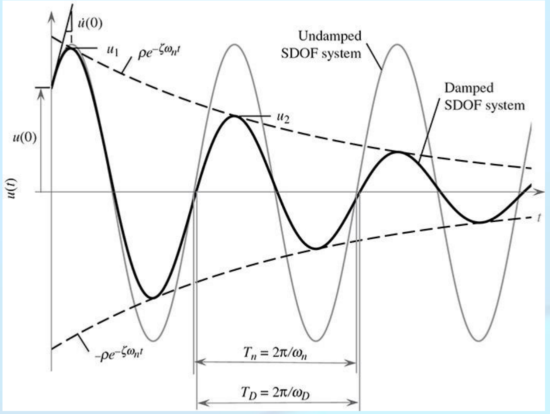
$$\&\ \dot{u}(t) = -\omega(u_0 - 3u_l)\sin(\omega t).$$

The motion continues to the right until:

$$\dot{u}(t) = 0 \Rightarrow \sin(\omega t) = 0, \quad \text{when} \quad t = 2\pi/\omega$$

Where the displacement reaches the value:

$$u(2\pi/\omega) = (u_0 - 3u_l)\cos(2\pi) - u_l = u_0 - 4u_l$$



Final Result:

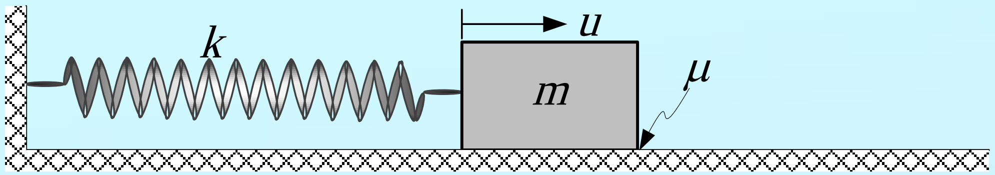
For every half-cycle of motion the amplitude loss is $2u_l = 2F_d/k$, and for a complete cycle the loss is $4u_l$.

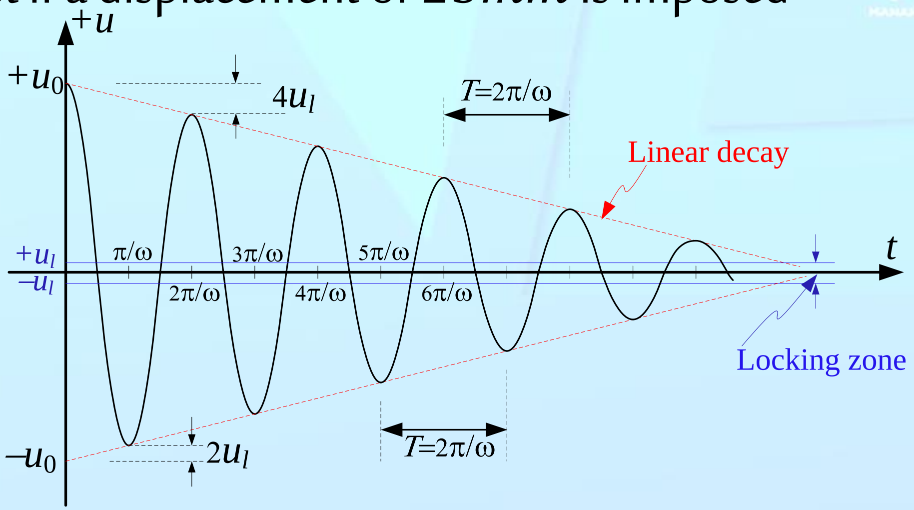
So for n cycle ($2n$ half-cycles) this loss is $4nu_l$.

When the amplitude becomes less than u_l, the motion stops and the mass is trapped in the locking zone.

EX. 1. A block of mass $m = 10kg$ is restrained by a spring of stiffness $k = 5000\ N/m$ and rests on a rough surface with coefficient of friction $\mu = 0.10$.

Calculate the number of half-cycles required for the mass to come to rest if a displacement of $25mm$ is imposed on the block and released with zero velocity.





SOLUTION:

1. The lock displacement u_l is:

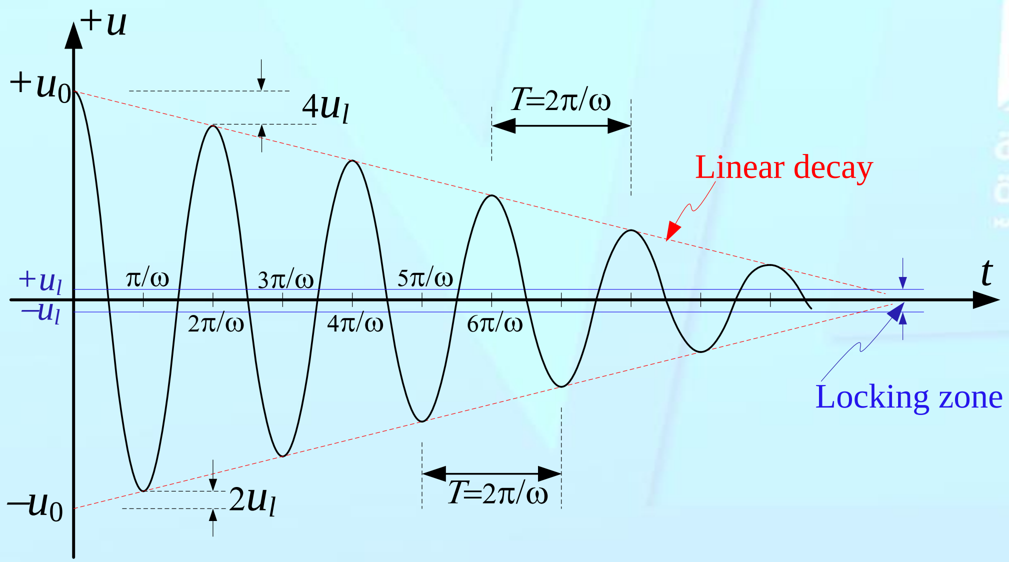
$$u_l = \frac{F_d}{k} = \frac{\mu g m}{k} = \frac{0.1 \times 9.81 \times 10}{5000} = 1.96 \times 10^{-3}\ \text{m}$$

2. The number of half-period of motion n, is given by

$$-u_l \le u_0 - 2nu_l \le u_l$$

EX. 2. A block of mass $m = 200kg$ is restrained by a spring of stiffness $k = 6000\ N/m$ and rests on a rough surface with coefficient of friction $\mu = 0.15$.

Calculate the number of half-cycles required for the mass to come to rest if a displacement of $20mm$ is imposed on the block and released with zero velocity.



Ex. 3. For the system shown in figure, $m = 500$ kg, $k = 400$ kN/m , $\mu = 0.15$ and the initial conditions are: $u_0 = 16$ cm, $\dot{u}_0 = 0$. Determine the amplitude after 8 half-cycles and the number of half-cycles of motion completed before the mass comes to rest