Structural Mechanics (1)

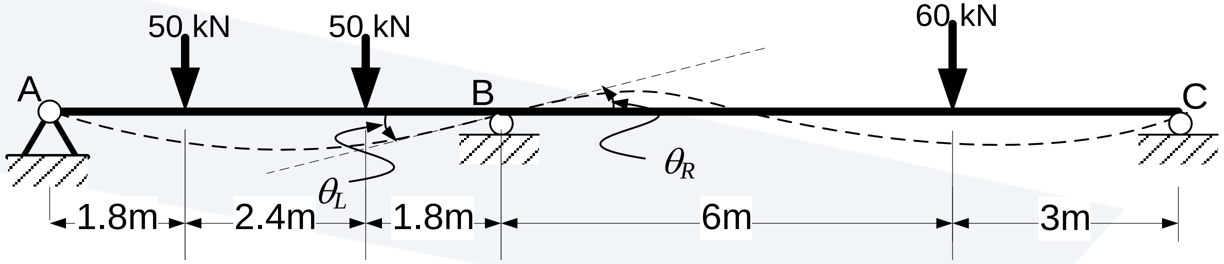
Week No-09

Analysis of Indeterminate Structures - Force Method

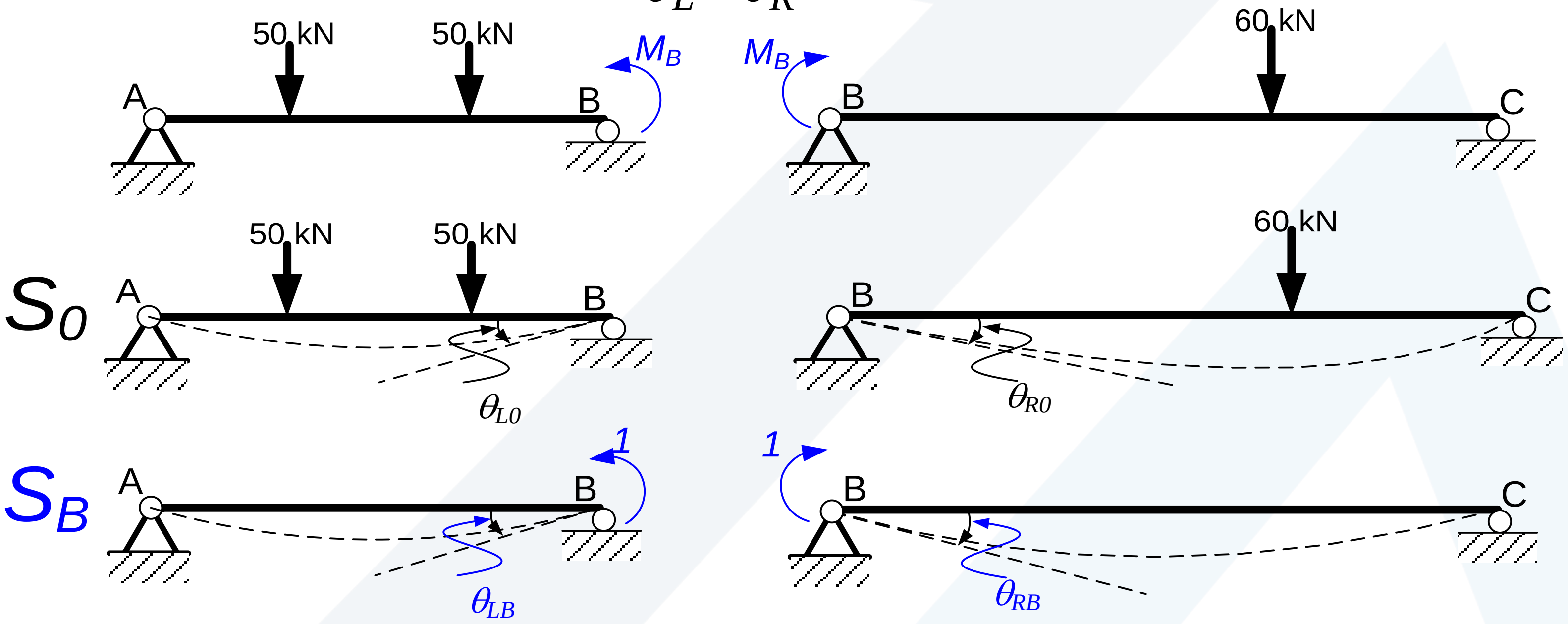
- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- Structures with multiple Degrees of Indeterminacy
- Support Settlements
- Three-Moment Equation for Continuous Beams

The Three-Moment Equation - Force Method for Continuous Beams
Introduction: Support Moments as Redundant

The analysis of structure with one degree of indeterminacy can also be done by choosing an internal force or moment as the redundant, provided that the released structure is determinate and stable



$$\theta_L = \theta_R$$



$$\theta_L = \theta_{L0} + M_B\theta_{LB} \qquad \theta_R = \theta_{R0} + M_B\theta_{RB}$$

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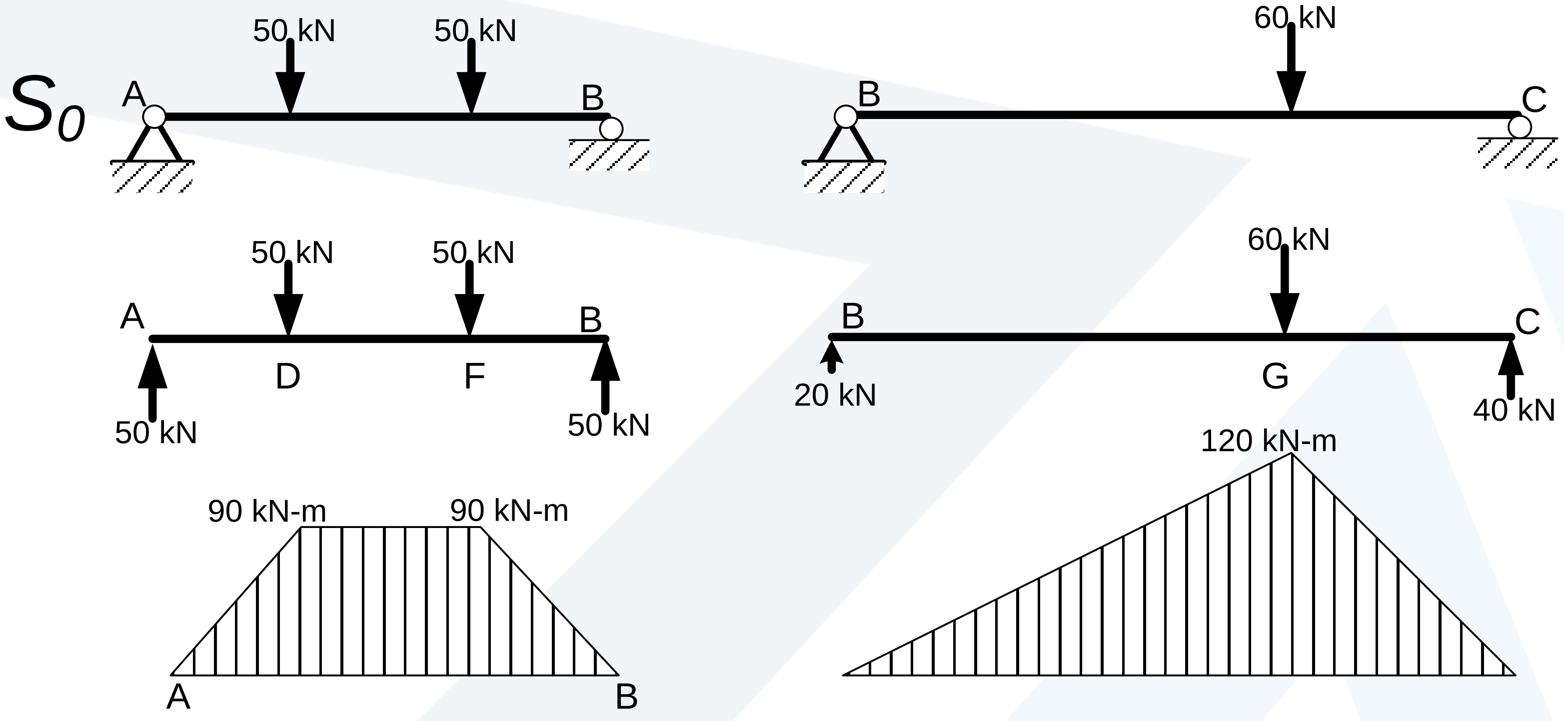
Analysis of S_0 Structure



M_{L0}

$\text{In}AD: M_{L0} = 50x$

$\text{In}DF: M_{L0} = 90$

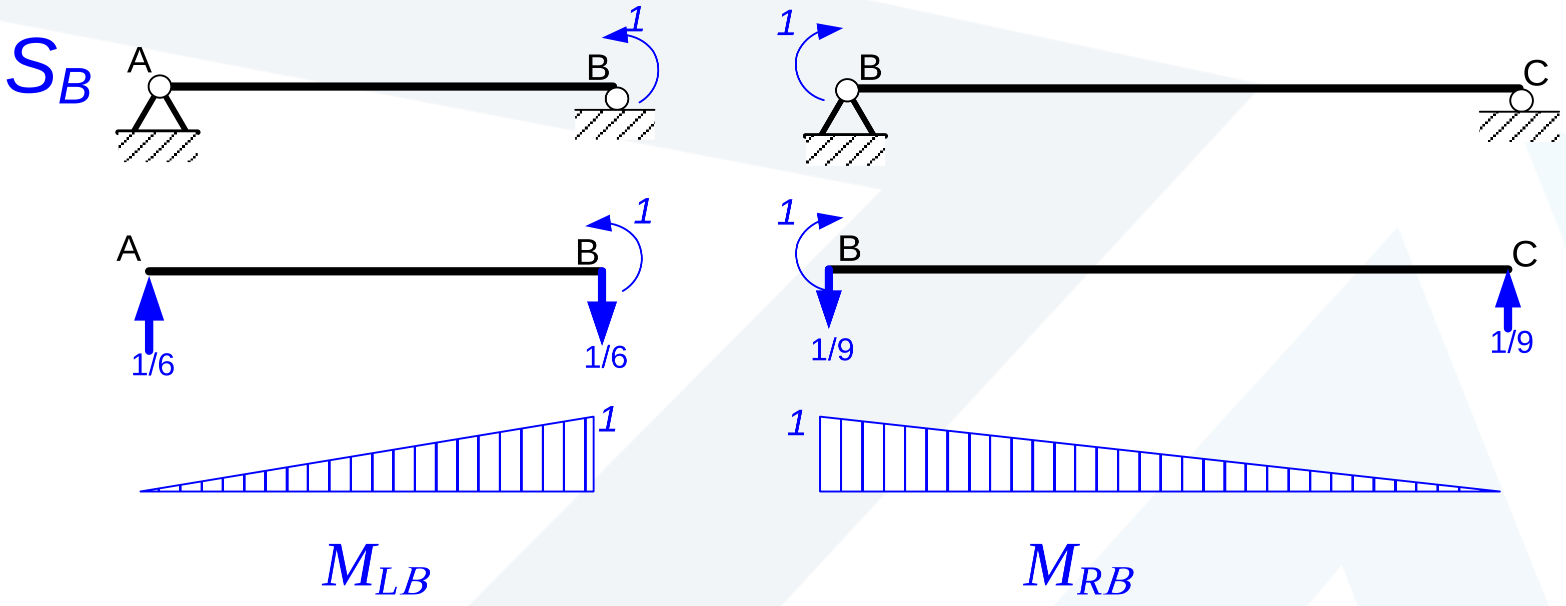
$\text{In}FB: M_{L0} = 50(6 - x)$

M_{R0}

$\text{In}BG: M_{R0} = 20x$

$\text{In}GC: M_{R0} = 40(9 - x)$

Analysis of S_B Structure



In AB: $M_{LB} = x/6$ In BC: $M_{RB} = (9-x)/9$

The Three-Moment Equation - Force Method for Continuous Beams
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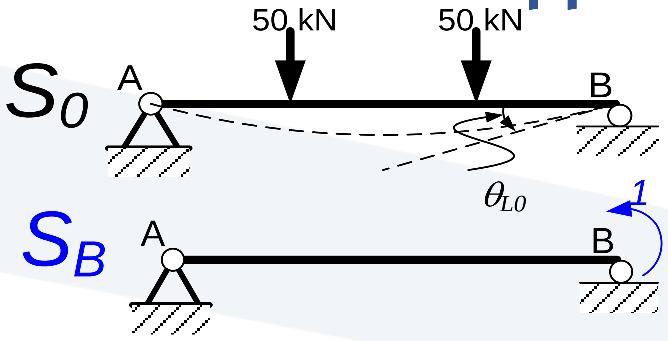
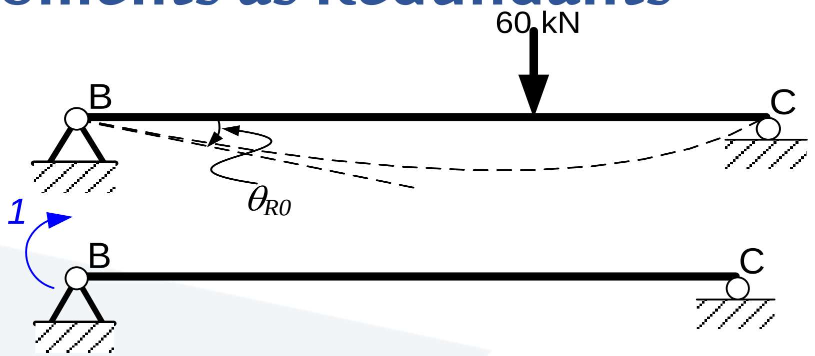




$$\theta_L = \theta_{L0} + M_B \theta_{LB}$$

$$EI\theta_{L0} = \int_0^6 M_{L0} M_{LB}\, dx$$

$$= \int_0^{1.8} (50x)(x/6)\, dx$$

$$+ \int_{1.8}^{4.2} (90)(x/6)\, dx$$

$$+ \int_{4.2}^{6} 50(6-x)(x/6)\, dx$$

$$EI\theta_{L0} = 189 \quad +$$

$$\theta_R = \theta_{R0} + M_B \theta_{RB}$$

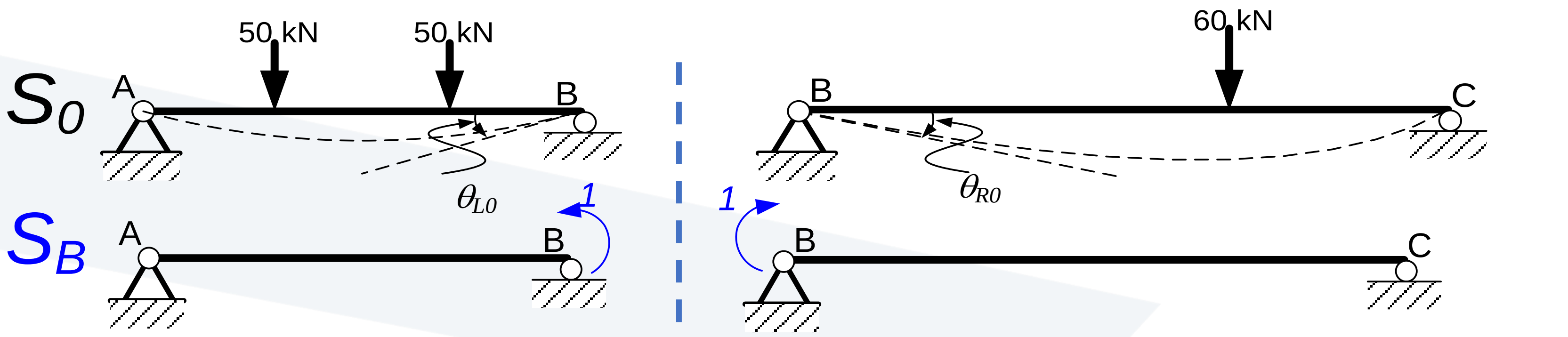
$$EI\theta_{R0} = \int_0^9 M_{R0} M_{RB}\, dx$$

$$= \int_0^{6} (20x)[(9-x)/9]\, dx$$

$$+ \int_{6}^{9} [40(9-x)][(9-x)/9]\, dx$$

$$EI\theta_{R0} = 240 \quad +$$

Introduction: Support Moments as Redundants



$$\theta_L = \theta_{L0} + M_B \theta_{LB} \qquad \theta_R = \theta_{R0} + M_B \theta_{RB}$$

$$EI\theta_{LB} = \int_0^6 M_{LB} M_{LB} dx \qquad EI\theta_{RB} = \int_0^9 M_{RB} M_{RB} dx$$

$$= \int_0^6 (x/6)(x/6) dx \qquad = \int_0^9 [(9-x)/9][(9-x)/9] dx$$

$$EI\theta_{LB} = 2 \quad + \qquad EI\theta_{L0} = 189 \quad + \qquad EI\theta_{RB} = 3 \quad + \qquad EI\theta_{R0} = 240 \quad +$$

$$\theta_L = \theta_{L0} + M_B \theta_{LB} \equiv \theta_R = -(\theta_{R0} + M_B \theta_{RB})$$

$$(\theta_{L0} + \theta_{R0}) + M_B(\theta_{LB} + \theta_{RB}) = 0$$

$$M_B(\theta_{LB} + \theta_{RB}) = -(\theta_{L0} + \theta_{R0}) \Rightarrow \quad M_B = -429/5 = -85.5 \text{ kN.m}$$

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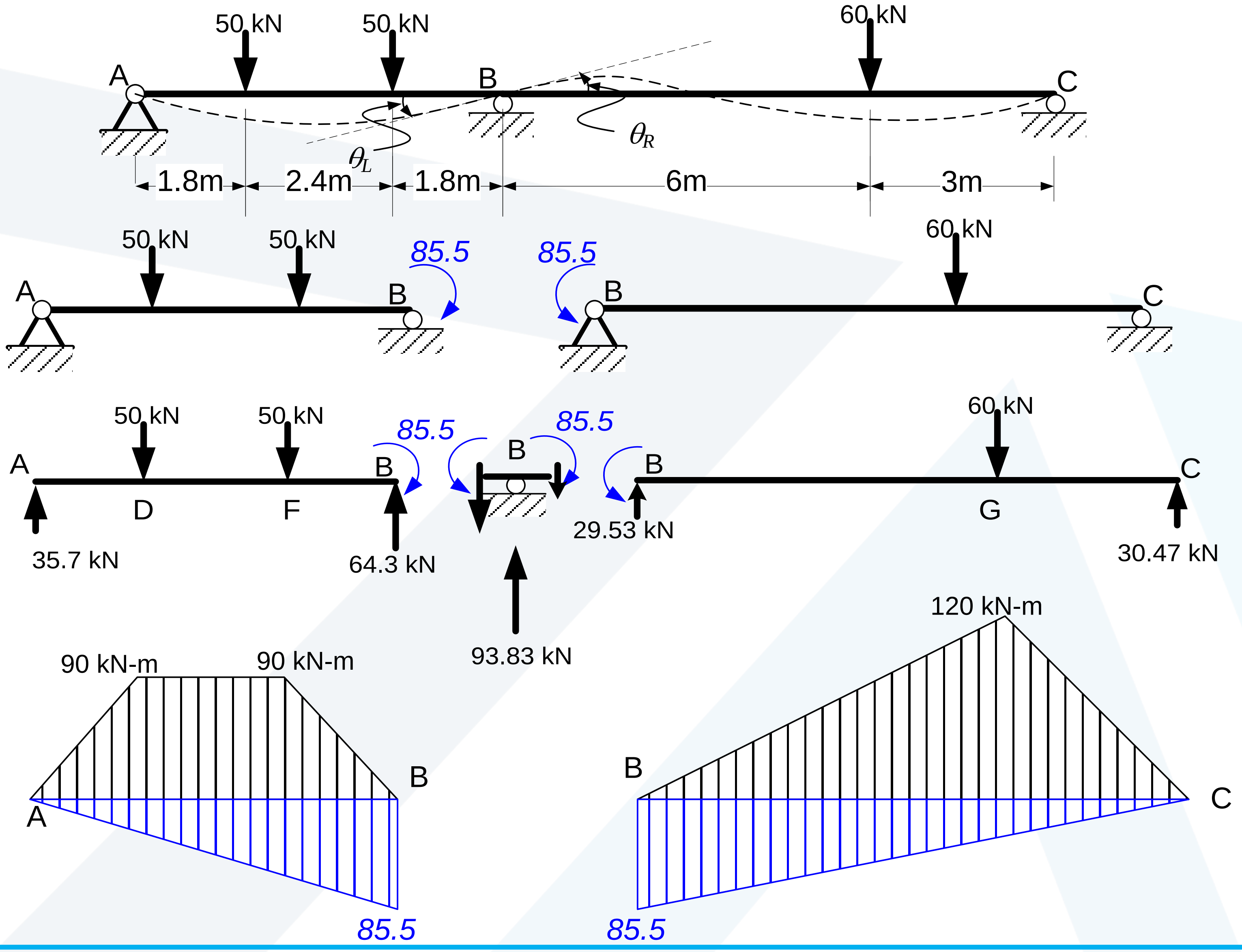

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