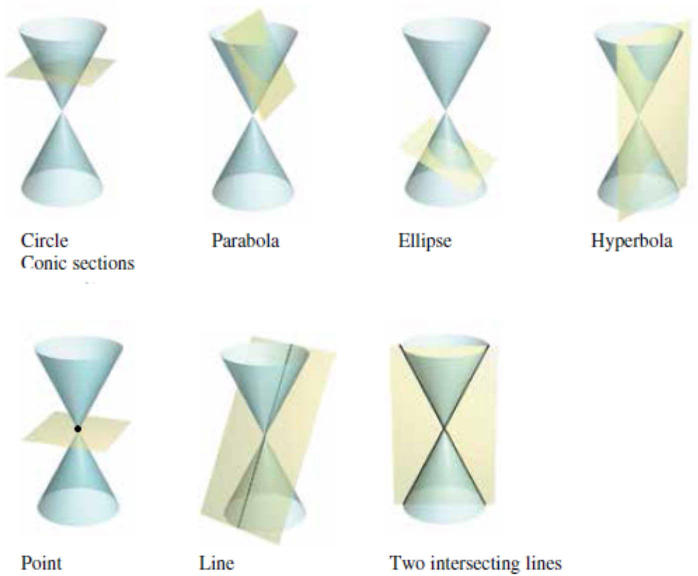
CALCULUS 2

Dr. Boushra ABBAS

Al-Manara University

2020-2021

Conics and Calculus:



Parabolas:

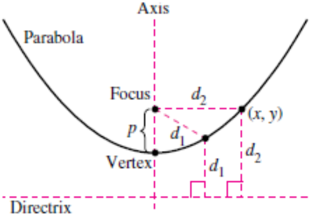
The general second-degree equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Standard equation of a circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

- A parabola is the set of all points (x, y) that are equidistant from a fixed line, the directrix, and a fixed point, the focus, not on the line.
- The midpoint between the focus and the directrix is the vertex,
- and the line passing through the focus and the vertex is the axis of the parabola.



Standard Equation of a Parabola

The **standard form** of the equation of a parabola with vertex (h, k) and directrix $y = k - p$ is

$$(x - h)^2 = 4p(y - k). \qquad \text{Vertical axis}$$

For directrix $x = h - p$, the equation is

$$(y - k)^2 = 4p(x - h). \qquad \text{Horizontal axis}$$

The focus lies on the axis p units (*directed distance*) from the vertex. The coordinates of the focus are as follows.

$(h, k + p)$ Vertical axis

$(h + p, k)$ Horizontal axis

Find the focus of the parabola

$$y = \frac{1}{2} - x - \frac{1}{2}x^2.$$

Solution To find the focus, convert to standard form by completing the square.

$$y = \frac{1}{2} - x - \frac{1}{2}x^2 \qquad \text{Write original equation.}$$

$$2y = 1 - 2x - x^2 \qquad \text{Multiply each side by 2.}$$

$$2y = 1 - (x^2 + 2x) \qquad \text{Group terms.}$$

$$2y = 2 - (x^2 + 2x + 1) \qquad \text{Add and subtract 1 on right side.}$$

$$x^2 + 2x + 1 = -2y + 2$$

$$(x + 1)^2 = -2(y - 1) \qquad \text{Write in standard form.}$$

Comparing this equation with

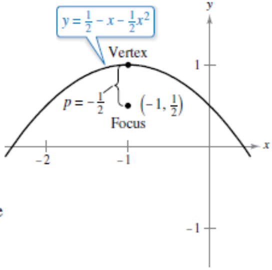
$$(x - h)^2 = 4p(y - k)$$

you can conclude that

$$h = -1, \quad k = 1, \quad \text{and} \quad p = -\frac{1}{2}.$$

Because p is negative, the parabola opens downward, . So, the focus of the parabola is p units from the vertex, or

$$(h, k + p) = \left(-1, \frac{1}{2}\right). \qquad \text{Focus}$$



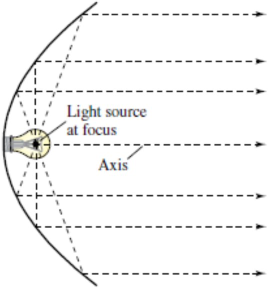
Parabola with a vertical axis, $p < 0$

Reflective Property of a Parabola:

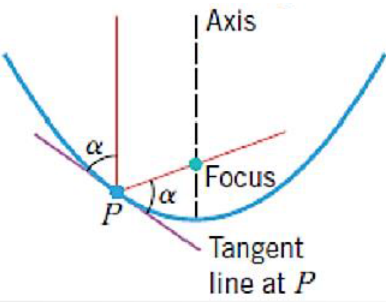
Reflective Property of a Parabola

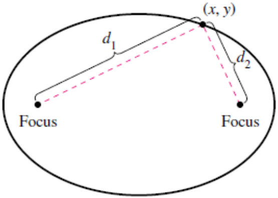
Let P be a point on a parabola. The tangent line to the parabola at point P makes equal angles with the following two lines.

1. The line passing through P and the focus
2. The line passing through P parallel to the axis of the parabola

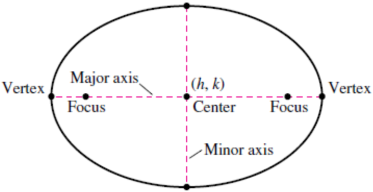


Parabolic reflector: light is reflected in parallel rays.





$d_1 + d_2$ is constant.



Standard Equation of an Ellipse

The **standard form** of the equation of an ellipse with center (h, k) and major and minor axes of lengths $2a$ and $2b$, respectively, where $a > b$, is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \text{Major axis is horizontal.}$$

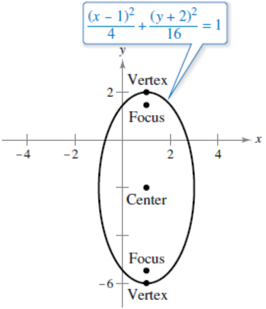
or

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1. \qquad \text{Major axis is vertical.}$$

The foci lie on the major axis, c units from the center, with

$$c^2 = a^2 - b^2.$$

Analyzing an Ellipse:



Ellipse with a vertical major axis.

Find the center, vertices, and foci of the ellipse

$$4x^2 + y^2 - 8x + 4y - 8 = 0.$$ General second-degree equatio

Solution Complete the square to write the original equation in standard form.

$$4x^2 + y^2 - 8x + 4y - 8 = 0$$ Write original equation.

$$4x^2 - 8x + y^2 + 4y = 8$$

$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4$$

$$4(x-1)^2 + (y+2)^2 = 16$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$ Write in standard form.

So, the major axis is vertical, where $h = 1$, $k = -2$, $a = 4$, $b = 2$, and

$$c = \sqrt{16-4} = 2\sqrt{3}.$$

So, you obtain the following.

Center: $(1, -2)$ (h, k)

Vertices: $(1, -6)$ and $(1, 2)$ $(h, k \pm a)$

Foci: $\left(1, -2 - 2\sqrt{3}\right)$ and $\left(1, -2 + 2\sqrt{3}\right)$ $(h, k \pm c)$

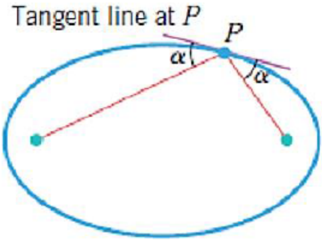
Reflective Property of an Ellipse

Let P be a point on an ellipse. The tangent line to the ellipse at point P makes equal angles with the lines through P and the foci.

Definition of Eccentricity of an Ellipse

The eccentricity e of an ellipse is given by the ratio

$$e = \frac{c}{a}.$$



Standard Equation of a Hyperbola

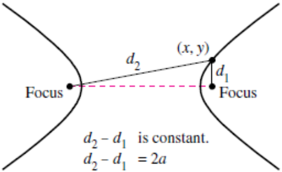
The **standard form** of the equation of a hyperbola with center at (h, k) is

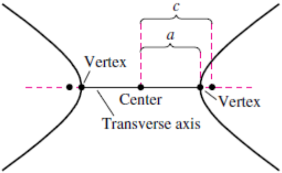
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \text{Transverse axis is horizontal.}$$

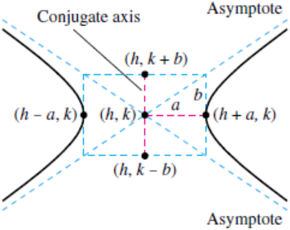
or

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1. \qquad \text{Transverse axis is vertical.}$$

The vertices are a units from the center, and the foci are c units from the center, where $c^2 = a^2 + b^2$.





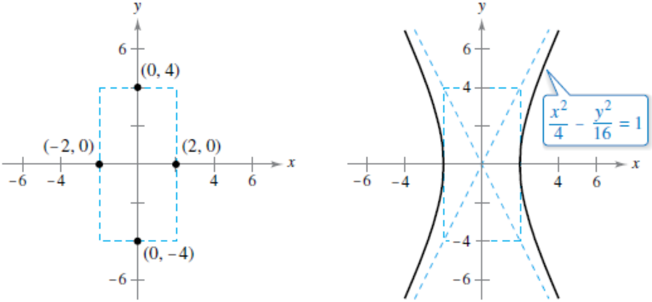


$4x^2 - y^2 = 16.$

Solution Begin by rewriting the equation in standard form.

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

The transverse axis is horizontal and the vertices occur at $(-2, 0)$ and $(2, 0)$. The ends of the conjugate axis occur at $(0, -4)$ and $(0, 4)$.



Examples

1. $x^2 + y^2 + 2x - 6y + 6 = 0$
2. $x^2 + y^2 - 10y - 11 = 0$
3. $x^2 - 2x - 8y + 9 = 0$
4. $x^2 + 6x + 12y - 3 = 0$
5. $y^2 - 4y - 6x - 8 = 0$
6. $y^2 + y + x = 0$
7. $x^2 + 4y^2 - 4x - 8y + 4 = 0$
8. $9x^2 + 4y^2 - 18x + 24y + 9 = 0$
9. $4x^2 - y^2 + 4y - 8 = 0$
10. $4x^2 - 5y^2 - 16x + 10y + 31 = 0$

Thank you for your attention