Basic Definitions

- Binary Operators
  - AND

    $z = x \bullet y = x\,y$ $z=1$ *if* $x=1$ *AND* $y=1$
  - OR

    $z = x + y$ $z=1$ *if* $x=1$ *OR* $y=1$
  - NOT

    $z = \overline{x} = x'$ $z=1$ *if* $x=0$
- Boolean Algebra
  - Binary Variables: only '0' and '1' values
  - Algebraic Manipulation

Boolean Algebra Postulates

- Commutative Law

$$x \bullet y = y \bullet x \qquad x + y = y + x$$

- Identity Element

$$x \bullet 1 = x \qquad x + 0 = x$$

- Complement

$$x \bullet x' = 0 \qquad x + x' = 1$$

Boolean Algebra Theorems

- Duality
  - The *dual* of a Boolean algebraic expression is obtained by interchanging the AND and the OR operators and replacing the 1's by 0's and the 0's by 1's.
  - $x \bullet (y + z) = (x \bullet y) + (x \bullet z)$
  - $x + (y \bullet z) = (x + y) \bullet (x + z)$

Applied to a valid equation produces a valid equation

- Theorem 1
  - $x \bullet x = x$ $\quad$ $x + x = x$
- Theorem 2
  - $x \bullet 0 = 0$ $\quad$ $x + 1 = 1$

Boolean Algebra Theorems

- Theorem 3: *Involution*
  - $(x')' = x$ $\qquad$ $\overline{(\overline{x})} = x$
- Theorem 4: *Associative & Distributive*
  - $(x \bullet y) \bullet z = x \bullet (y \bullet z)$ $\qquad$ $(x + y) + z = x + (y + z)$
  - $x \bullet (y + z) = (x \bullet y) + (x \bullet z)$

    $x + (y \bullet z) = (x + y) \bullet (x + z)$
- Theorem 5: *DeMorgan*
  - $(x \bullet y)' = x' + y'$ $\qquad$ $(x + y)' = x' \bullet y'$
  - $\overline{(x \bullet y)} = \overline{x} + \overline{y}$ $\qquad$ $\overline{(x + y)} = \overline{x} \bullet \overline{y}$
- Theorem 6: *Absorption*
  - $x \bullet (x + y) = x$ $\qquad$ $x + (x \bullet y) = x$

Operator Precedence

- Parentheses

  (. . .) • (. . .)

- NOT

  $x' + y$

- AND

  $x + x \cdot y$

- OR

$$x\left[y + z\,\overline{(w + x)}\right]$$

$$(w + x)$$

$$\overline{(w + x)}$$

$$z\,\overline{(w + x)}$$

$$y + z\,\overline{(w + x)}$$

$$x\left[y + z\,\overline{(w + x)}\right]$$

DeMorgan’s Theorem

$$\overline{a\,[b+c\,(d+\overline{e}\,)]}$$

$$\overline{a}+\overline{[b+c\,(d+\overline{e}\,)]}$$

$$\overline{a}+\overline{b}\,\overline{(c\,(d+\overline{e}\,))}$$

$$\overline{a}+\overline{b}\,(\overline{c}+\overline{(d+\overline{e}}\,))$$

$$\overline{a}+\overline{b}\,(\overline{c}+(\overline{d}\;\overline{\overline{e}}))$$

$$\overline{a}+\overline{b}\,(\overline{c}+\overline{d}\;e)$$

Boolean Functions

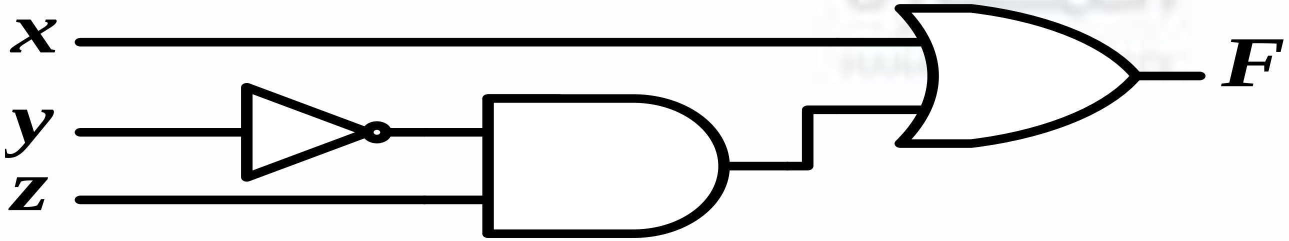
- Boolean Expression

  Example: $F = x + y'z$

- Truth Table

  All possible combinations of input variables

- Logic Circuit



x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Algebraic Manipulation

- *Literal*:

  A single variable within a term that may be complemented or not.

- Use Boolean Algebra to simplify Boolean functions to produce simpler circuits

  Example: Simplify to a minimum number of literals

  $F = x + x'y$ (3 *Literals*)

  $= x + (x'y)$ *Distributive law* (+ over •)

  $= (x + x')(x + y)$

  $= (1)(x + y) = x + y$ (2 *Literals*)

Complement of a Function

- DeMorgan's Theorm

$$F = A + B + C$$

$$\overline{F} = \overline{A + B + C}$$

$$\overline{F} = \overline{A} \bullet \overline{B} \bullet \overline{C}$$

- Duality & Literal Complement

$$F = A + B + C$$

$$F \quad A \bullet B \bullet C$$

$$\overline{F} = \overline{A} \bullet \overline{B} \bullet \overline{C}$$

Canonical Forms

- Minterm
  - Product (*AND* function)
  - Contains all variables
  - Evaluates to '1' for a specific combination

Example

$A = 0$
$B = 0$
$C = 0$

$$\overline{A} \quad \overline{B} \quad \overline{C}$$

$$\overline{(0)} \bullet \overline{(0)} \bullet \overline{(0)}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$1 \bullet 1 \bullet 1 = 1$$

	A B C	Minterm	
0	0 0 0	m_0	$\overline{A}\,\overline{B}\,\overline{C}$
1	0 0 1	m_1	$\overline{A}\,\overline{B}\,C$
2	0 1 0	m_2	$\overline{A}\,B\,\overline{C}$
3	0 1 1	m_3	$\overline{A}\,B\,C$
4	1 0 0	m_4	$A\,\overline{B}\,\overline{C}$
5	1 0 1	m_5	$A\,\overline{B}\,C$
6	1 1 0	m_6	$A\,B\,\overline{C}$
7	1 1 1	m_7	$A\,B\,C$

Canonical Forms

- Maxterm
  - Sum (*OR* function)
  - Contains all variables
  - Evaluates to '0' for a specific combination

Example

$A = 1$, $B = 1$, $C = 1$

$$\overline{A} \quad \overline{B} \quad \overline{C}$$

$$\overline{(1)} + \overline{(1)} + \overline{(1)}$$

$$0 + 0 + 0 = 0$$

	A B C	Maxterm	
0	0 0 0	M_0	$A+B+C$
1	0 0 1	M_1	$A+B+\overline{C}$
2	0 1 0	M_2	$A+\overline{B}+C$
3	0 1 1	M_3	$A+\overline{B}+\overline{C}$
4	1 0 0	M_4	$\overline{A}+B+C$
5	1 0 1	M_5	$\overline{A}+B+\overline{C}$
6	1 1 0	M_6	$\overline{A}+\overline{B}+C$
7	1 1 1	M_7	$\overline{A}+\overline{B}+\overline{C}$

Canonical Forms

- Truth Table *to* Boolean Function

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$$F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

Canonical Forms

- Sum of *Minterms*

$$F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$F = m_1 + m_4 + m_5 + m_7$$

$$F = \sum(1,4,5,7)$$

- Product of *Maxterms*

$$\bar{F} = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}$$

$$\bar{\bar{F}} = \overline{\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}}$$

$$F = \overline{\bar{A}\bar{B}\bar{C}} \cdot \overline{\bar{A}B\bar{C}} \cdot \overline{\bar{A}BC} \cdot \overline{AB\bar{C}}$$

$$F = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

$$F = M_0 \quad M_2 \quad M_3 \quad M_6$$

$$F = \prod(0,2,3,6)$$

	A B C	F	$\bar{F}$
0	0 0 0	0	1
1	0 0 1	1	0
2	0 1 0	0	1
3	0 1 1	0	1
4	1 0 0	1	0
5	1 0 1	1	0
6	1 1 0	0	1
7	1 1 1	1	0

Standard Forms

- Sum of Products (SOP)

$$F = \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

$$A\overline{B}(\overline{C} + C) = A\overline{B}(1) = A\overline{B}$$

$$AC(\overline{B} + B) = AC$$

$$\overline{B}C(\overline{A} + A) = \overline{B}C$$

$$F = \overline{B}C\cancel{(\overline{A} + A)} + A\overline{B}\cancel{(\overline{C} + C)} + AC\cancel{(\overline{B} + B)}$$

$$F = \overline{B}C + A\overline{B} + AC$$

Standard Forms

- Product of Sums (POS)

$$\overline{F} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + AB\overline{C}$$

- $\overline{A}B(\overline{C} + C)$
- $B\overline{C}(\overline{A} + A)$
- $\overline{A}\,\overline{C}(\overline{B} + B)$

$$\overline{F} = \overline{A}\,\overline{C}(\overline{B} + B) + \overline{A}B(\overline{C} + C) + B\overline{C}(\overline{A} + A)$$

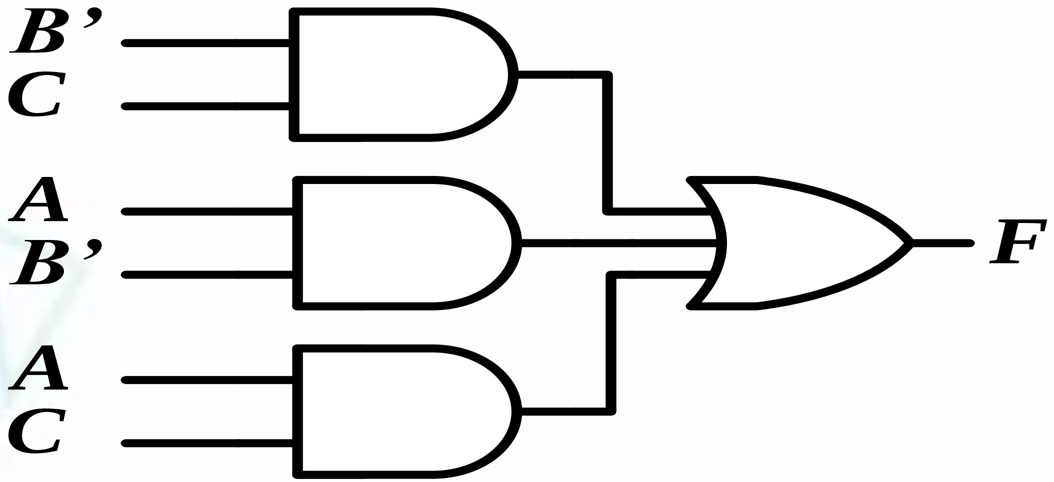
$$\overline{\overline{F}} = \overline{\overline{A}\,\overline{C} + \overline{A}B + B\overline{C}}$$

$$F = (A + C)(A + \overline{B})(\overline{B} + C)$$

Two-Level Implementations

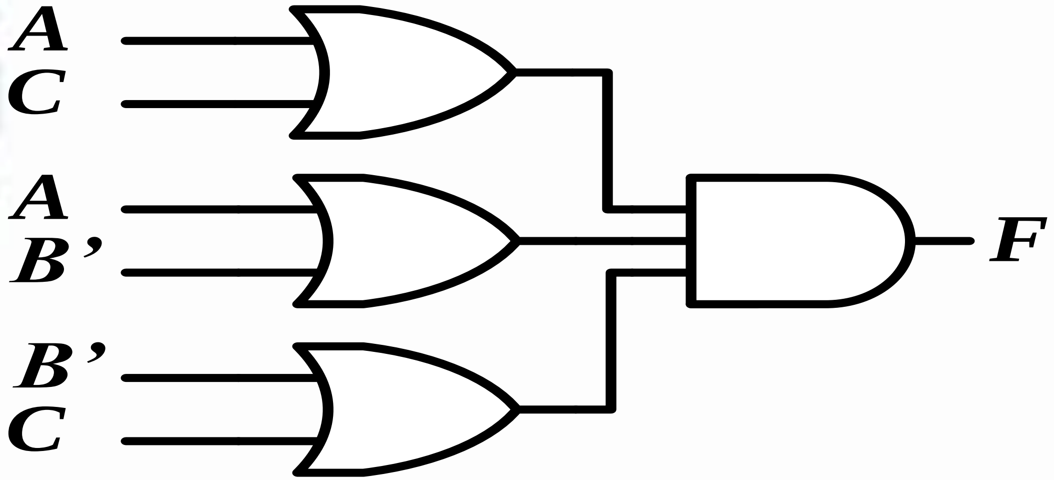
- Sum of Products (SOP)

$$F = \overline{B}C + A\overline{B} + AC$$



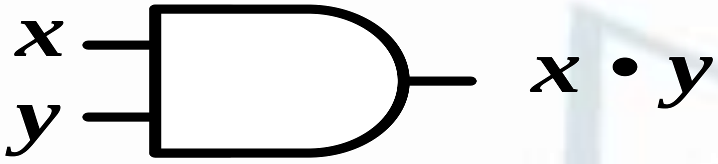
- Product of Sums (POS)

$$F = (A + C)(A + \overline{B})(\overline{B} + C)$$



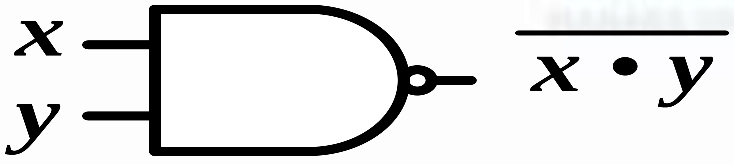
Logic Operators

- AND



x	y	AND
0	0	0
0	1	0
1	0	0
1	1	1

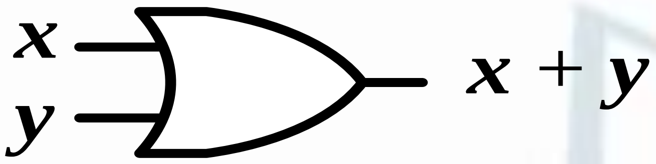
- NAND (Not AND)



x	y	NAND
0	0	1
0	1	1
1	0	1
1	1	0

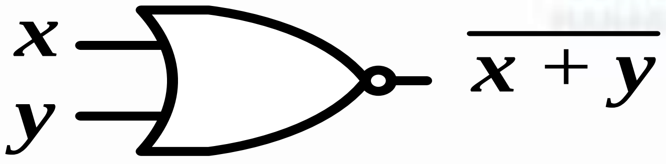
Logic Operators

- OR



x	y	OR
0	0	0
0	1	1
1	0	1
1	1	1

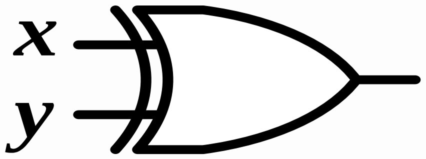
- NOR (Not OR)



x	y	NOR
0	0	1
0	1	0
1	0	0
1	1	0

Logic Operators

- XOR (Exclusive-OR)

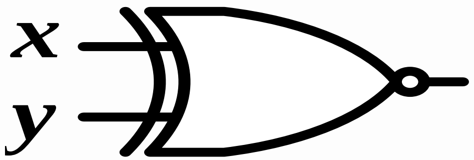


x □ y

$\overline{x}\,y + x\,\overline{y}$

x	y	XOR
0	0	0
0	1	1
1	0	1
1	1	0

- XNOR (Exclusive-NOR)
  (Equivalence)



$\overline{x \; □ \; y}$

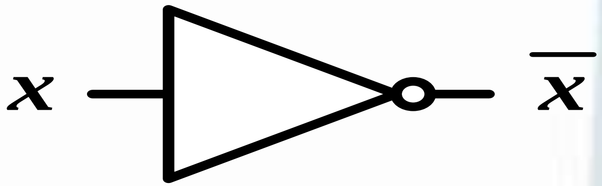
x ▯ y

$\overline{x}\,\overline{y} + x\,y$

x	y	$XNOR$
0	0	1
0	1	0
1	0	0
1	1	1

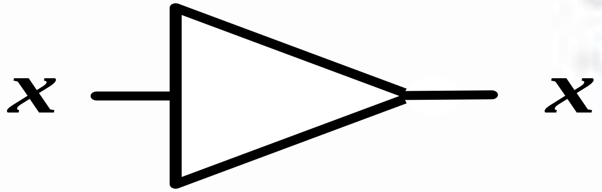
Logic Operators

- NOT (Inverter)



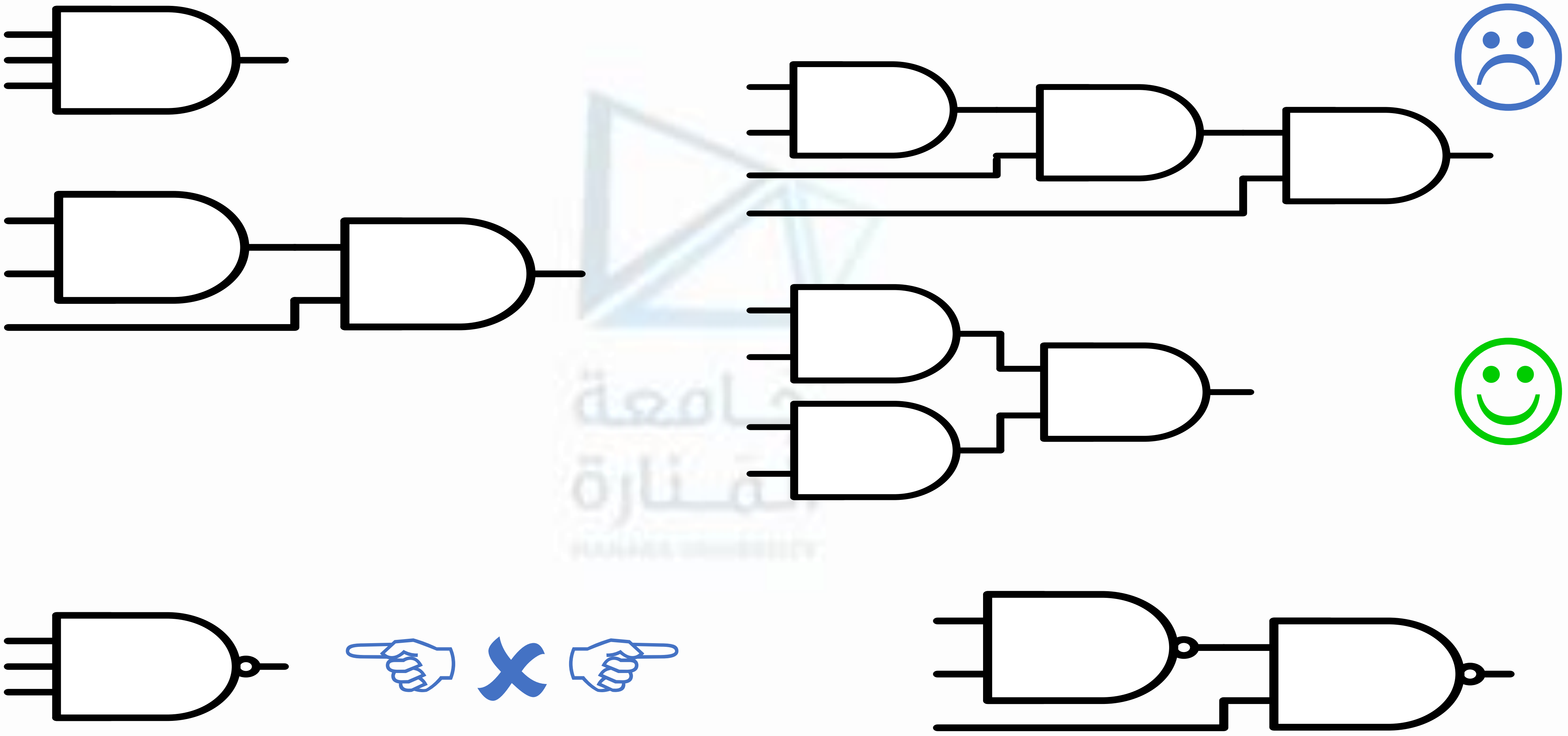
x	*NOT*
0	1
1	0

- Buffer



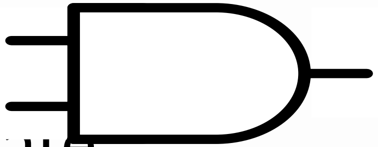
x	*Buffer*
0	0
1	1

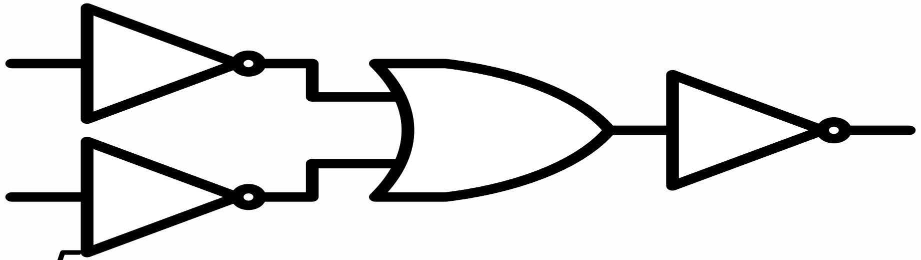
Multiple Input Gates



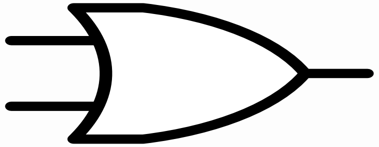
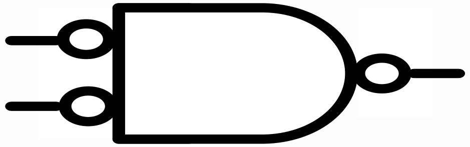
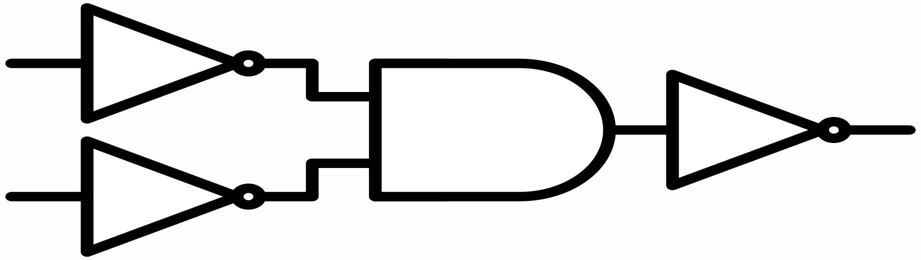
DeMorgan's Theorem on Gates

- AND Gate
  - $F = x \bullet y$ $\quad F = \overline{(x \bullet y)}$ $\quad F = \overline{\overline{x} + \overline{y}}$





- OR Gate
  - $F = x + y$ $\quad F = \overline{(x + y)}$ $\quad F = \overline{\overline{x} \bullet \overline{y}}$

➔ Change the "Shape" and "bubble" all lines

Homework

- Mano
  - Chapter 2
    - 2-4
    - 2-5
    - 2-6
    - 2-8
    - 2-9
    - 2-10
    - 2-12
    - 2-15
    - 2-18
    - 2-19

Homework

- Mano

2-4 **Reduce the following Boolean expressions to the indicated number of literals:**

(a) $A'C' + ABC + AC'$ to three literals

(b) $(x'y' + z)' + z + xy + wz$ to three literals

(c) $A'B(D' + C'D) + B(A + A'CD)$ to one literal

(d) $(A' + C)(A' + C')(A + B + C'D)$ to four literals

2-5 **Find the complement of $F = x + yz$; then show that $FF' = 0$ and $F + F' = 1$**

Homework

2-6 **Find the complement of the following expressions:**

(a) $xy' + x'y$ **(b)** $(AB' + C)D' + E$

(c) $(x + y' + z)(x' + z')(x + y)$

2-8 **List the truth table of the function:**

$$F = xy + xy' + y'z$$

2-9 **Logical operations can be performed on strings of bits by considering each pair of corresponding bits separately (this is called bitwise operation). Given two 8-bit strings**
A = 10101101 and B = 10001110, evaluate the 8-bit result after the following logical operations: (a) AND, (b) OR, (c) XOR, (d) NOT A, (e) NOT B.

Homework

2-10 **Draw the logic diagrams for the following Boolean expressions:**

(a) $Y = A'B' + B(A + C)$ (b) $Y = BC + AC'$

(c) $Y = A + CD$ (d) $Y = (A + B)(C' + D)$

2-12 **Simplify the Boolean function T_1 and T_2 to a minimum number of literals.**

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

Homework

2-15 **Given the Boolean function**

$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$

(a) Obtain the truth table of the function.

(b) Draw the logic diagram using the original Boolean expression.

(c) Simplify the function to a minimum number of literals using Boolean algebra.

(d) Obtain the truth table of the function from the simplified expression and show that it is the same as the one in part (a)

(e) Draw the logic diagram from the simplified expression and compare the total number of gates with the diagram of part (b).

Homework

2-18 **Convert the following to the other canonical form:**

(a) $F(x, y, z) = \sum (1, 3, 7)$

(b) $F(A, B, C, D) = \prod (0, 1, 2, 3, 4, 6, 12)$

2-19 **Convert the following expressions into sum of products and product of sums:**

(a) $(AB + C)(B + C'D)$

(b) $x' + x(x + y')(y + z')$