Lecture Outline

- Condition for the Equilibrium of a Particle.
- Concurrent Coplanar Systems.
- The Free-Body Diagram.
- Examples

1. Condition for the Equilibrium of a Particle

- **A Particle is at *equilibrium* if:**

- At rest, or
- Moving at constant velocity.

} Acceleration must vanish: $\boldsymbol{a} = 0$

- **Newton's second law of motion:**

$\sum \boldsymbol{F} = m\boldsymbol{a}$, where $\sum \boldsymbol{F}$ is the vector sum of all the forces acting on the particle

- Condition for the Equilibrium of a Particle: $\sum \boldsymbol{F} = \mathbf{0}$

2. Concurrent Coplanar Systems

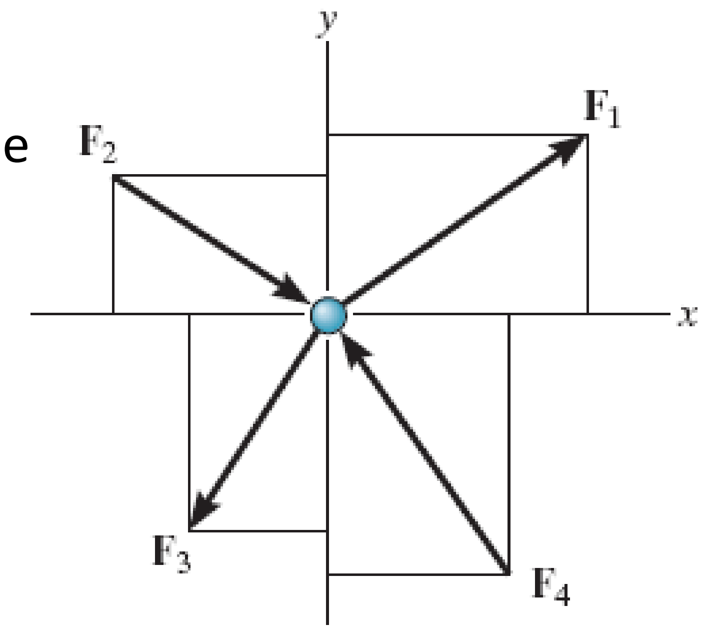
A particle is subjected to coplanar forces in the *x*-*y* plane

The vector equilibrium condition is:

$$\sum \boldsymbol{F} = 0$$

Resolve into *x* and *y* components for equilibrium

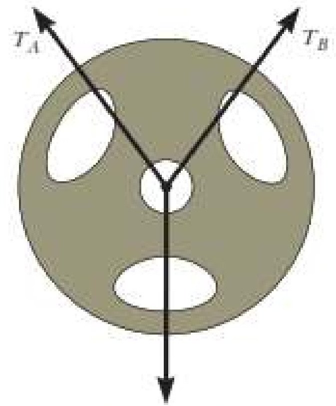
$$\sum F_x = 0 \quad \& \quad \sum F_y = 0$$



Scalar equations of equilibrium require that the algebraic sum of the *x* & *y* components to equal to zero.

3. The Free-Body Diagram

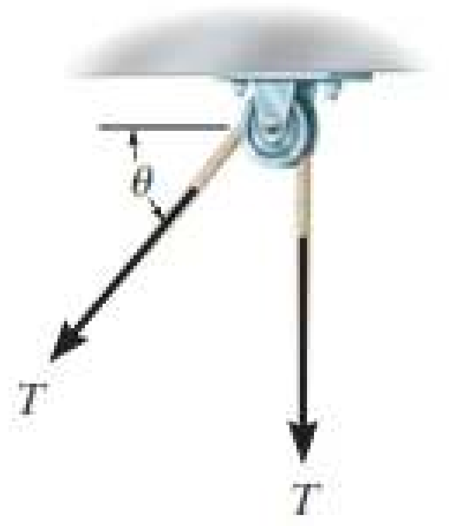
- Best representation of all the forces ($\sum \boldsymbol{F}$) which acts on a body (particle).
- A sketch showing the particle "free" from the surroundings with all the forces acting on it.
- Consider two common connections in this subject:
  - Cables with or without Pulley.
  - Spring or rubber cord.



3. The Free-Body Diagram … common connections

Cables and Pulley

- Cables (or cords) are assumed of negligible weight & cannot stretch.
- Tension always acts in the direction of the cable.
- Tension force must have a constant magnitude for equilibrium
- For any angle θ, the cable is subjected to a constant tension *T*

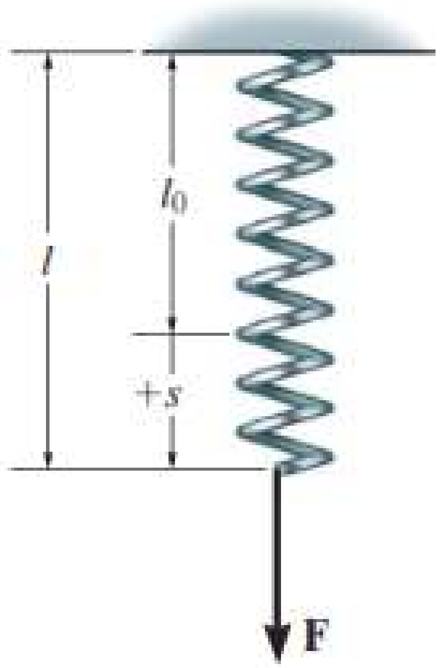


Cable is in tension

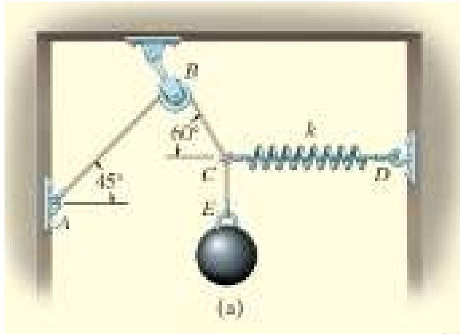
3. The Free-Body Diagram … common connections

Spring or Rubber Cord.

- Linear elastic spring: change in length is directly proportional to the force acting on it.
- Spring constant or *stiffness* k : defines the elasticity of the spring.
- Magnitude of force when spring is elongated or compressed, is: $F = k s$



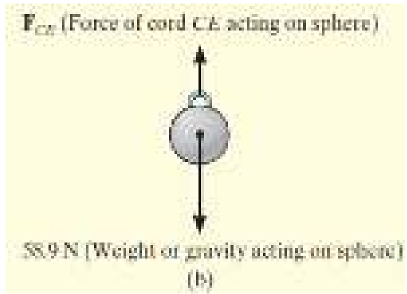
Example 1. A sphere has a mass of 6kg and is supported. Draw a free-body diagram of the sphere, cord CE and the knot at C.



FBD of Sphere. Fig. b.

Two forces acting, weight and the force on cord CE.

Weight of [6kg]=6×(9.81m/s^2)=58.9N

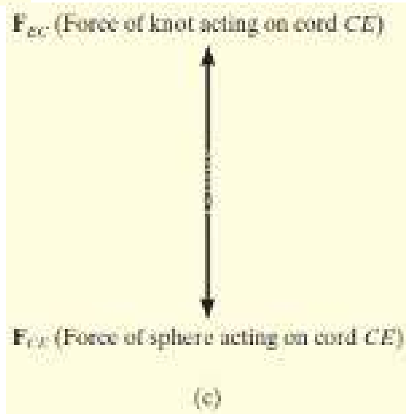


FBD of Cord CE. Fig. c.

Two forces acting: sphere and knot. Newton's 3rd Law:

$\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull the cord in tension

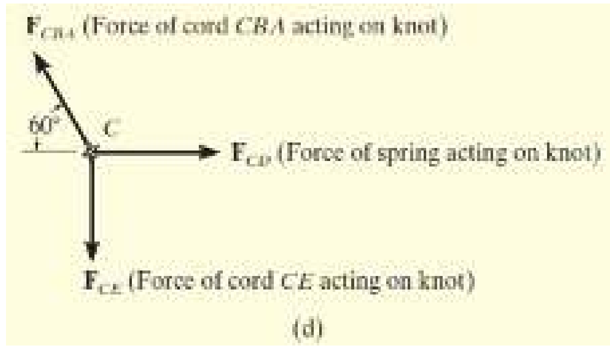
For equilibrium, $\mathbf{F}_{CE} = \mathbf{F}_{EC}$



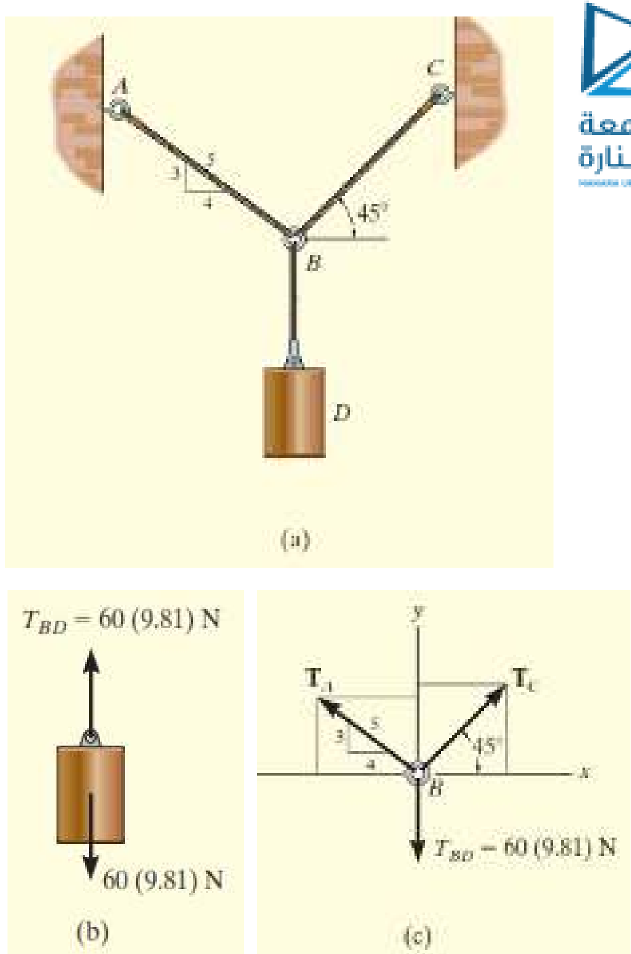
FBD at Knot. Fig. d.

3 forces acting: cord CBA, cord CE and spring CD.

Important to know that the weight of the sphere does not act directly on the knot but subjected to by the cord CE.



Example 2 . Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig.a.



Solution

Free-Body Diagrams.

The tension in cable BD, acting on the cylinder in addition to its weight, is shown in the free-body diagram (Fig.b.). Due to equilibrium, this tension is T_{BD}=60(9.81)N= 589 N.

The forces in cables BA, BC can be determined by investigating the equilibrium of ring B. Its free-body diagram is shown in Fig.c.

The magnitudes of T_A and T_C are unknown, but their directions are known. From the Eq. Eqs.

Equilibrium Equations (Eq. Eqs.).

$$\sum F_x = 0: -T_A(4/5)+T_C\cos45° = 0$$

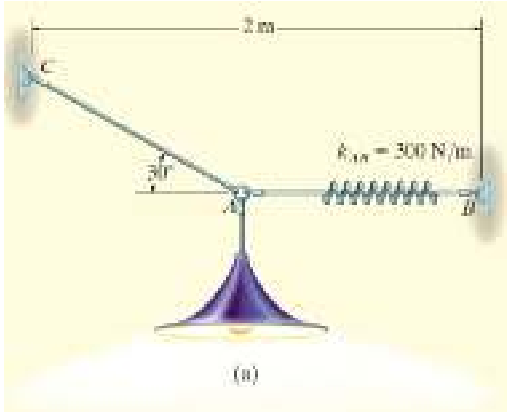
$$\sum F_y = 0: T_A(3/5)+T_C\sin45° - 589 = 0$$

Solving to get T_A =420N & T_C =476N

Example 3.

1. Determine the two forces T_{AB} & T_{AC} that the spring AB & the cord AC apply to the ring *A*, if the spring *AB* stays horizontal and the mass of the suspended lamp is 8 kg.

2. Determine the length of the cord *AC* if the undeformed length of the spring *AB* is l^0_{AB} = 0.4 m, and the its stiffness is k = 300N/m.

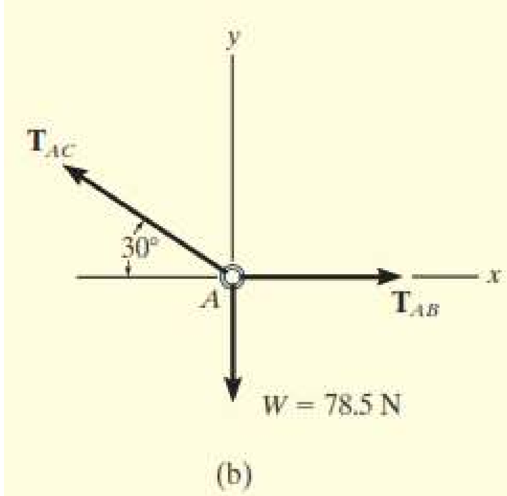


Solution 1.

FBD of Ring A

Three forces acting, force by cable AC, force in spring AB and weight of the lamp:

$W = gm = (9.81)(8) \approx 78.5\text{N}$



Eq. Eqs.

$$\sum F_x = 0;\ T_{AB} - T_{AC}\cos 30^\circ = 0$$
$$\sum F_y = 0;\ T_{AC}\sin 30^\circ - 78.5N = 0$$

Solving to get $T_{AC} = 157\text{N}$ and $T_{AB} = 136\text{N}$

Solution 2.

If force on cable AB is known, stretch of the spring is found by:

$$T_{AB} = k_{AB} s_{AB} \Rightarrow 136 = 300 s_{AB} \Rightarrow s_{AB} = 0.453\text{m}$$

For stretched length, $l_{AB} = l^\circ_{AB} + s_{AB} \Rightarrow l_{AB} = 0.4\text{m} + 0.453\text{m} = 0.853\text{m}$ For horizontal distance BC,

$$2\text{m} = l_{AC}\cos 30^\circ + 0.853\text{m} \Rightarrow l_{AC} = 1.32\text{m}$$