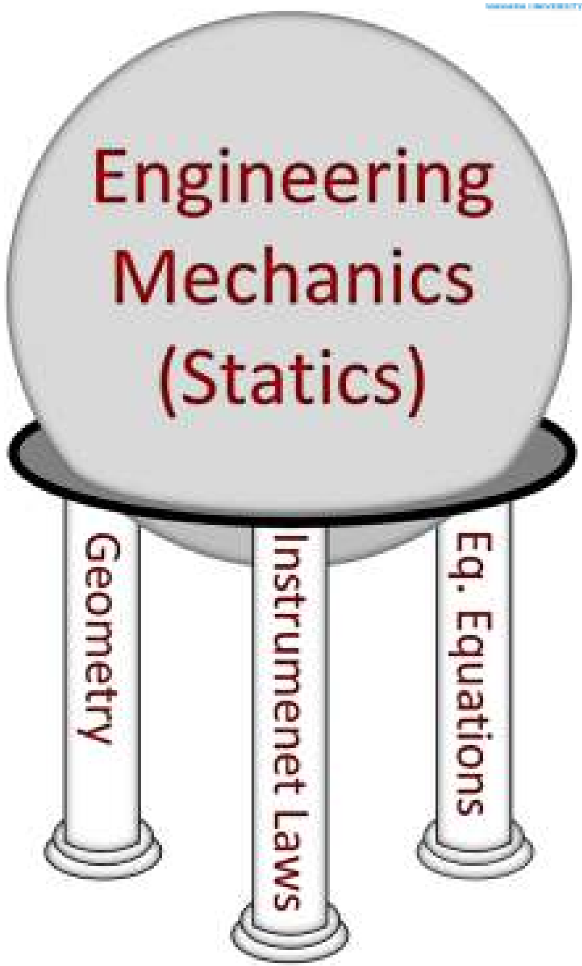
Lecture Outline

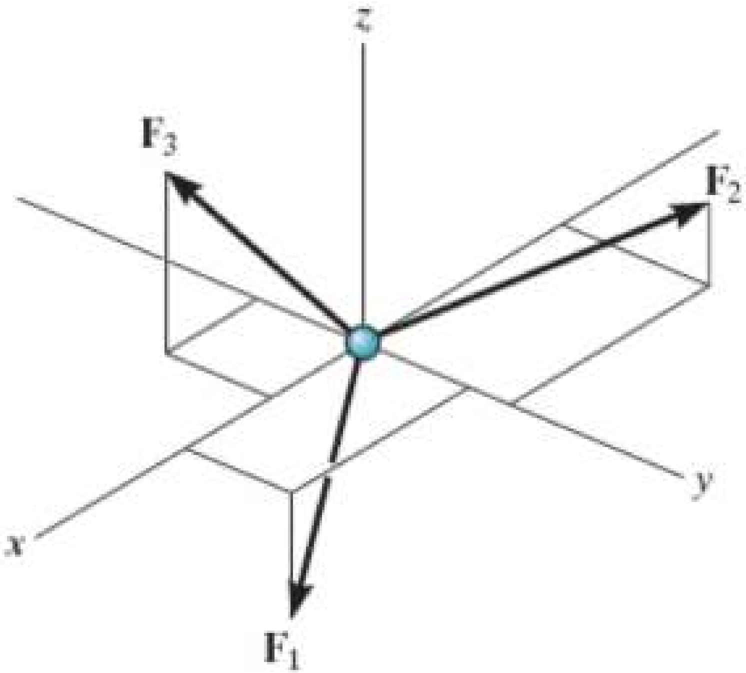
- Concurrent 3-Dimensional Force Systems.
- Expressing 3D Force as Cartesian Vector.
- Position Vectors and Relative Position Vectors.
- Examples [(F.B.D), (Eq. Eqs.), (Geometry)]



Last week Outline

- Condition for the Equilibrium of a Particle. $\sum \boldsymbol{F} = 0$
- Concurrent Coplanar Systems. $\sum F_x = 0$ & $\sum F_y = 0$
- The Free-Body Diagram.
- Examples & 3 Exercises.

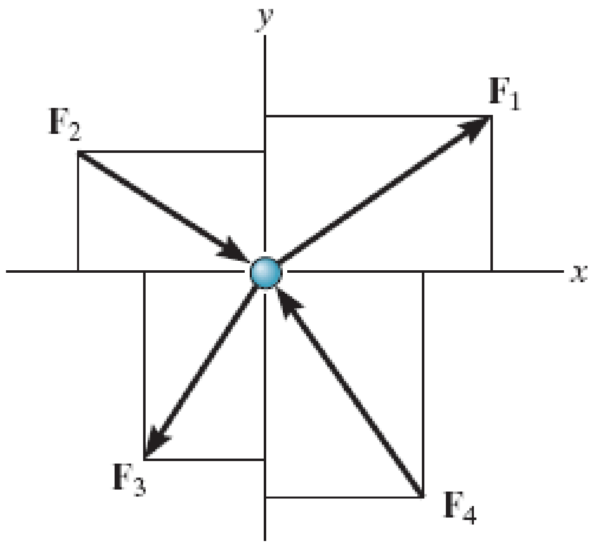
1. Concurrent *Three-Dimensional* Force Systems.



The vector equilibrium condition is: $\sum \boldsymbol{F}=0$

Resolve into *x*, *y* and *z* components for equilibrium

$$\sum \boldsymbol{F}=0 \Leftrightarrow \left[\sum F_x = 0 \;,\; \sum F_y = 0 \;\&\; \sum F_z = 0\right]$$



Concurrent Coplanar Systems

$$\sum \boldsymbol{F}=0 \Leftrightarrow \sum F_x = 0 \;\&\; \sum F_y = 0$$

Expressing 3D Force as Cartesian Vector

Direction of the Force Vector ***F*** is defined by the coordinate direction angles α, β & γ Measured between the tail of ***F*** and the positive *x*, *y* and *z* axes. $0° \leq \alpha, \beta \ \& \ \gamma \leq 180°$.

From the figure, $F = F_x\mathrm{i} + F_y\mathrm{j} + F_Z\mathrm{k}$,

The *magnitude* of ***F*** is: $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

And its *direction cosines* are: $\cos\alpha = \dfrac{F_x}{F}, \ \cos\beta = \dfrac{F_y}{F}, \ \cos\gamma = \dfrac{F_z}{F}.$

The components in terms of *direction cosines & magnitude,* are:

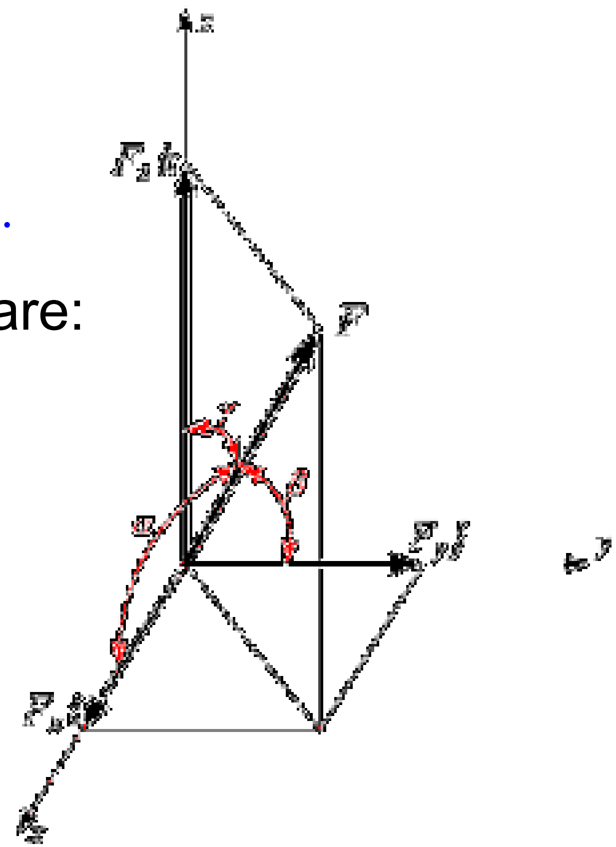
$F_x = F\cos\alpha, \qquad F_y = F\cos\beta, \qquad F_z = F\cos\gamma.$

then, $F = F(\cos\alpha\,\mathrm{i} + \cos\beta\,\mathrm{j} + \cos\gamma\,\mathrm{k}) = F\,u_F$

Where u_F is a unity vector given by:

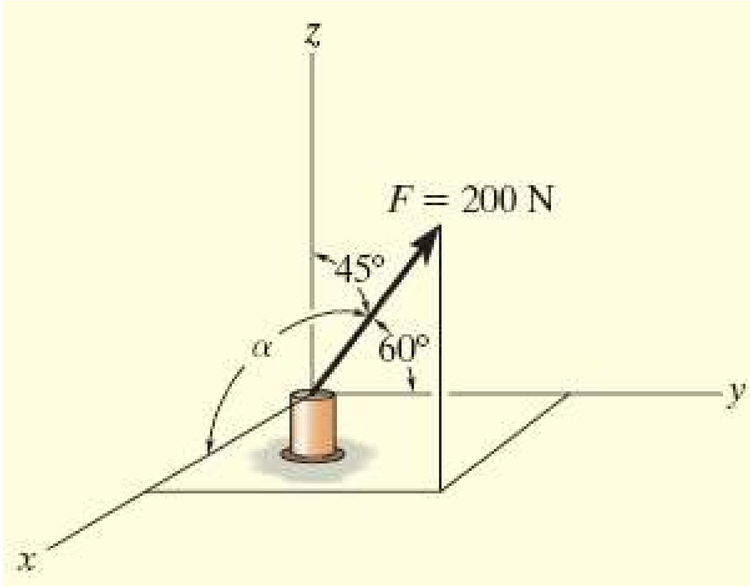
$u_F = \cos\alpha\,\mathrm{i} + \cos\beta\,\mathrm{j} + \cos\gamma\,\mathrm{k}.$

then, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



Example

Express the shown force $\boldsymbol{F}$ as Cartesian vector.



Solution:

Since two angles are specified, the third angle is found by: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, so

$$\cos^2\alpha + \cos^2 60° + \cos^2 45° = 1$$

then, $\cos\alpha = (1 - \cos^2 60° + \cos^2 45°)^{1/2} = \pm 0.5$

Two possibilities exit , namely: $(1)\ \alpha = \cos^{-1}(0.5) = 60°$, $(2)\ \alpha = \cos^{-1}(-0.5) = 120°$,

But by visual inspection, $\alpha = 60°$ since F_x is in the $+x$ direction

Given F = 200N,

then $\boldsymbol{F}$ = $F\cos\alpha$ **i** + $F\cos\beta$ **j** + $F\cos\gamma$ **k**= 200[N](cos60º**i** + cos60º**j** + cos45º**k)**

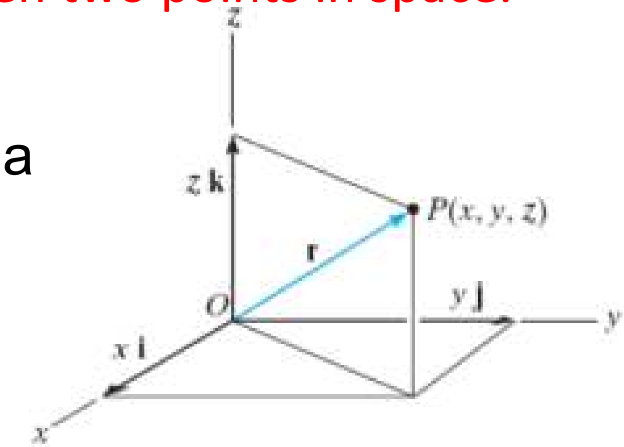
= {100 [N] **i** + 100 [N] **j** + 141 [N] **k**}

Position Vector and Relative Position Vector

We introduce the position vector and the relative position vector because they are used in cartesian formulation of any force vector directed between two points in space.

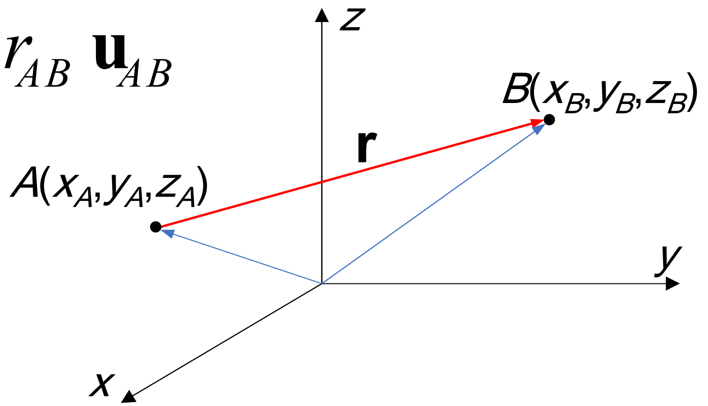
Position Vector

Position vector **r** is defined as a fixed vector which locates a point in space relative to the coordinates origin



$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

Relative Position Vector from *A* to *B*



$$\mathbf{r} = \mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k} = r_{AB}\,\mathbf{u}_{AB}$$

$$r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

$$\mathbf{u}_{AB} = \left(\frac{x_B - x_A}{r_{AB}}\right)\mathbf{i} + \left(\frac{y_B - y_A}{r_{AB}}\right)\mathbf{j} + \left(\frac{z_B - z_A}{r_{AB}}\right)\mathbf{k}$$

Examples (3-Dimensional Force Systems)

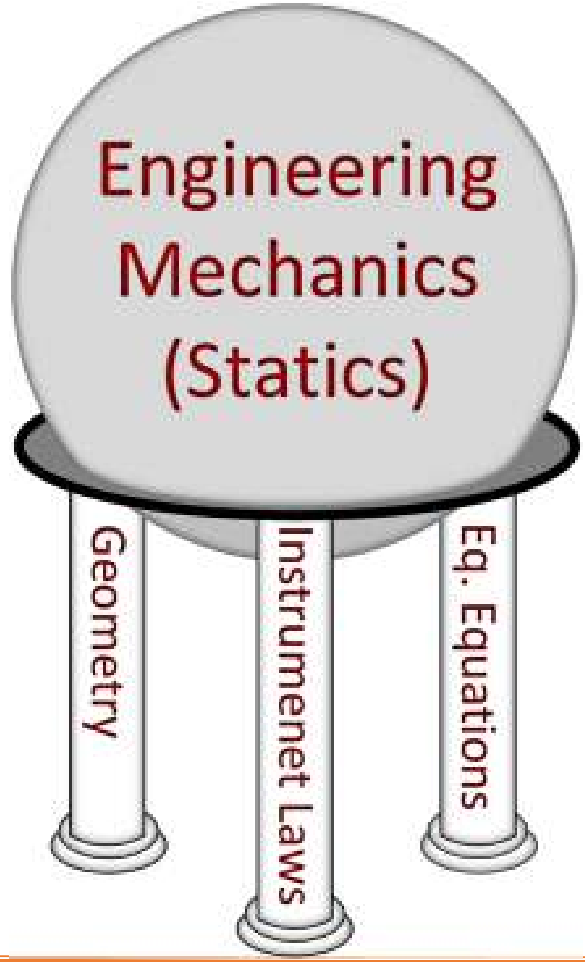
Procedure for Analysis

- **Free-body Diagram**

- Establish the z, y, z axes
- Label all known and unknown force

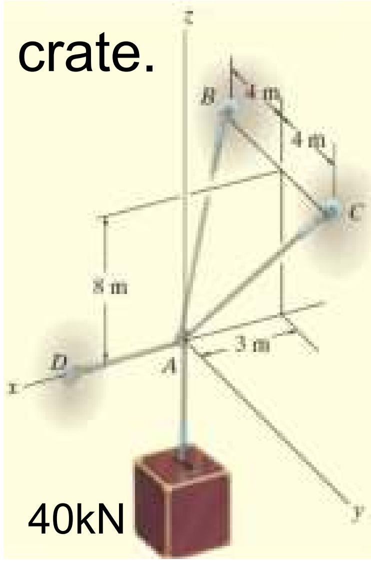
- **Equations of Equilibrium**

- Apply $\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$
- Negative results indicate that the sense of the force is opposite to that shown in the FBD.



Example 1.

Determine the force developed in each cable used to support the crate.



Solution:

$$r_{AB} = \sqrt{(-3-0)^2 + (-4-0)^2 + (8-0)^2} = 9.434\text{m}$$

$$\mathbf{u}_{AB} = \frac{-3}{9.434}\mathbf{i} + \frac{-4}{9.434}\mathbf{j} + \frac{+8}{9.434}\mathbf{k} = -0.318\mathbf{i} - 0.424\mathbf{j} + 0.848\mathbf{k}$$

$$r_{AC} = \sqrt{(-3-0)^2 + (4-0)^2 + (8-0)^2} = 9.434\text{m}$$

$$\mathbf{u}_{AC} = \frac{-3}{9.434}\mathbf{i} + \frac{+4}{9.434}\mathbf{j} + \frac{+8}{9.434}\mathbf{k} = -0.318\mathbf{i} + 0.424\mathbf{j} + 0.848\mathbf{k}$$

$$\mathbf{u}_{AD} = \mathbf{i}$$

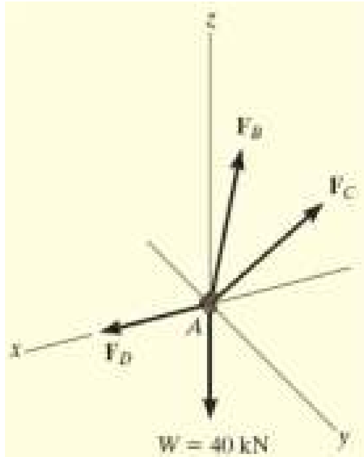
Expressing the four forces in Cartesian vectors,

$$\boldsymbol{F}_B = F_B\,\boldsymbol{u}_{AB} = -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$

$$\boldsymbol{F}_C = F_C\,\boldsymbol{u}_{AC} = -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$

$$\boldsymbol{F}_D = F_D\mathbf{i}$$

$$\boldsymbol{W} = -40\mathbf{k}$$



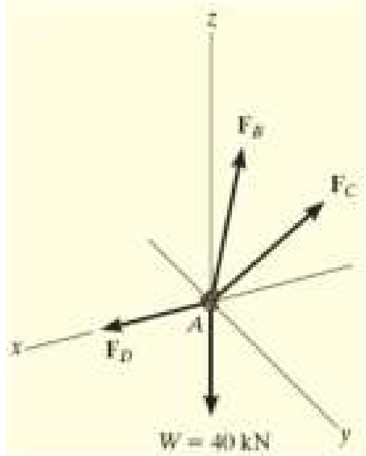
FBD at Point A

$\boldsymbol{F}_B = F_B\,\boldsymbol{u}_{AB} = -0.318F_B\,\mathbf{i} - 0.424F_B\,\mathbf{j} + 0.848F_B\,\mathbf{k}$

$\boldsymbol{F}_C = F_C\,\boldsymbol{u}_{AC} = -0.318F_C\,\mathbf{i} + 0.424F_C\,\mathbf{j} + 0.848F_C\,\mathbf{k}$

$\boldsymbol{F}_D = F_D\,\mathbf{i}$

$\boldsymbol{W} = -40\mathbf{k}$



For equilibrium,

$\sum \boldsymbol{F} = 0;\ \boldsymbol{F}_B + \boldsymbol{F}_C + \boldsymbol{F}_D + \boldsymbol{W} = 0$

Eq. Eqs.:

$\sum F_x = 0: \quad -0.318F_B - 0.318F_C + F_D = 0$

$\sum F_y = 0: \quad -0.424F_B + 0.424F_C = 0$

$\sum F_z = 0; \quad 0.848F_B + 0.848F_C = 40$

$\mathbf{u}_{AB} = -0.318\mathbf{i} - 0.424\mathbf{j} + 0.848\mathbf{k}$

$\mathbf{u}_{AC} = -0.318\mathbf{i} + 0.424\mathbf{j} + 0.848\mathbf{k}$

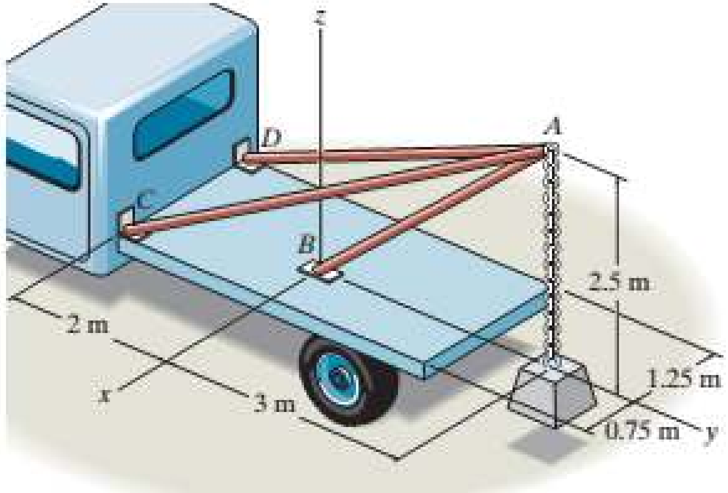
$\mathbf{u}_{AD} = \mathbf{i}$

$\mathbf{u}_W = -\mathbf{k}$

Solving, $F_B = F_C = 23.6\text{kN}\ \ F_D = 15.0\text{kN}$

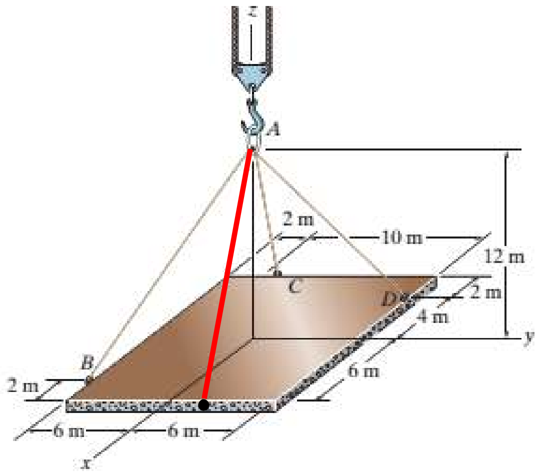
Example 2.

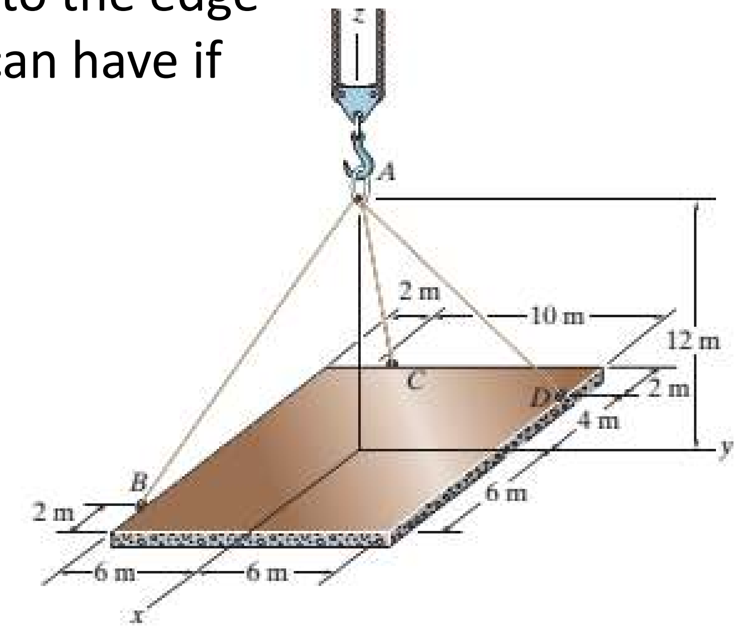
Determine the force acting along the axis of each of the three struts needed to support the 500-kg block



Example 3.

The ends of the three cables are attached to a ring at A and to the edge of the uniform plate. Determine the largest mass the plate can have if each cable can support a maximum tension of 15 kN.





Example 4.

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of k = 300 N/ m.