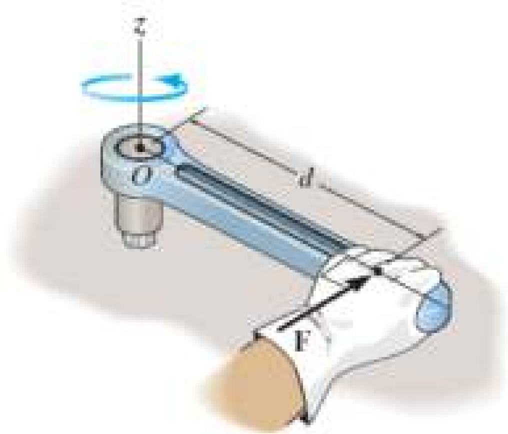
Moment of a Force – Scalar Formulation

عزم القوة – الصياغة السلمية (غير الشعاعية)

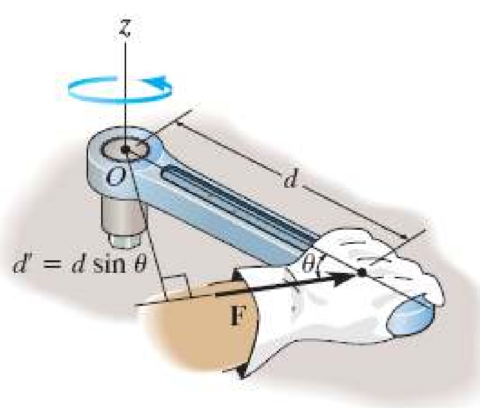
Moment of a force about a point or axis: "*Moment sometime is called Torque*"

A measure of its tendency to cause a body to rotate about the point or axis

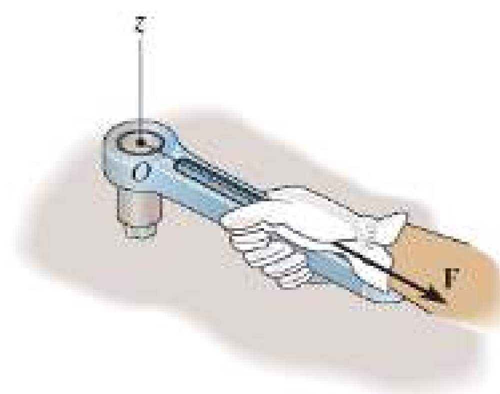
عزم القوة : مقياس لنزعة القوة على تدوير جسم ما حول نقطة ثابتة أو حول محور



(a)



(b)



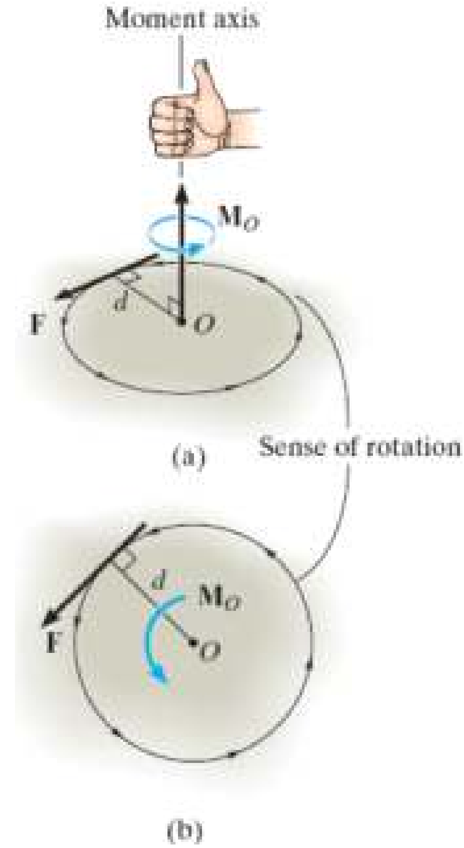
(c)

Moment of a Force – Scalar Formulation

عزم القوة – الصياغة السلمية (غير الشعاعية)

Magnitude: $M_O = Fd$ (Nm)

where d "perpendicular distance" from O to its line of action of force

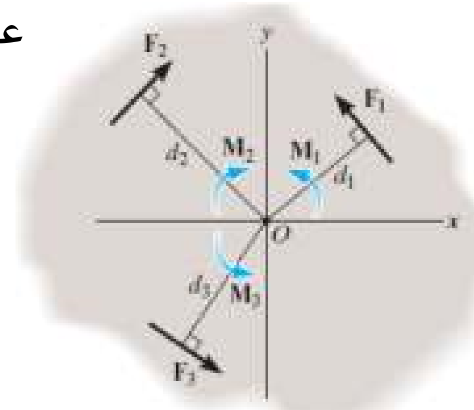


Direction:

Direction using "right hand rule"

Moment of a Force – Scalar Formulation

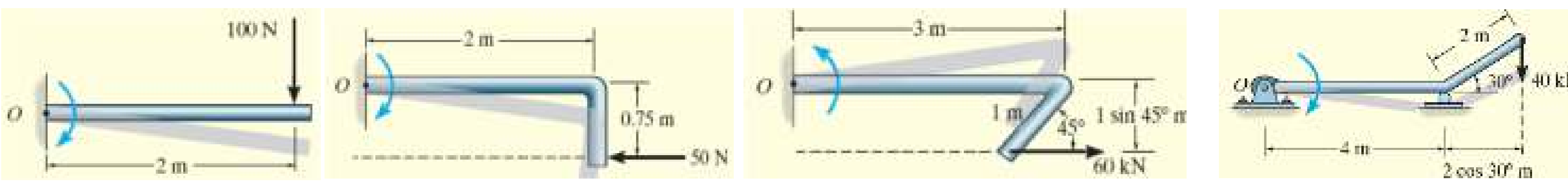
عزم القوة – الصياغة السلمية (غير الشعاعية)



Resultant Moment:

$\mathbf{M}_{Ro}$ = moments of all the forces $= \sum \boldsymbol{Fd}$

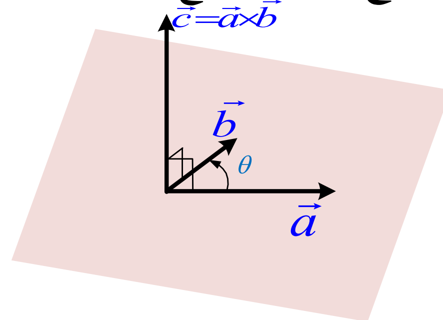
Example 1. For each case, determine the moment of the force about point *O*.

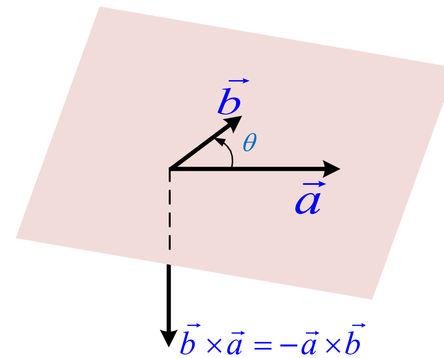


Solutions

M_o=–(100)(2)=–200 Nm M_o=–(50)(0.75)=–37.5 Nm M_o=+(60)(1sin45°)=42.4 kNm M_o=–(40)(4+2cos30°)=–229 Nm

الجداء الشعاعي لشعاعين ناتجه شعاع متعامد مع مستويهما





$$|\vec{c}|=|\vec{a}||\vec{b}|\sin\theta$$

تبديلي عكسا، توزيعي مع جمع الأشعة. معدوم لشعاعين متوازيين. وتحليلياً لدينا

$$\vec{a}\times\vec{b}=(a_x\vec{i}+a_y\vec{j}+a_z\vec{k})\times(b_x\vec{i}+b_y\vec{j}+b_z\vec{k})$$

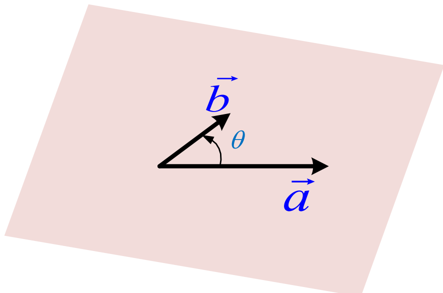
$$=a_xb_x(\vec{i}\times\vec{i})+a_xb_y(\vec{i}\times\vec{j})+a_xb_z(\vec{i}\times\vec{k})$$

$$+a_yb_x(\vec{j}\times\vec{i})+a_yb_y(\vec{j}\times\vec{j})+a_yb_z(\vec{j}\times\vec{k})$$

$$+a_zb_x(\vec{k}\times\vec{i})+a_zb_y(\vec{k}\times\vec{j})+a_zb_z(\vec{k}\times\vec{k})$$

$$=\vec{i}(a_yb_z-a_zb_y)+\vec{j}(a_zb_x-a_xb_z)+\vec{k}(a_xb_y-a_yb_x)$$

الجداء السلمي (النقطي) لشعاعين



ناتجه عدد حقيقي معرف بالعلاقة

$$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$$

تبديلي، توزيعي مع جمع الأشعة. معدوم لمتعامدين.

في جملة احداثيات ديكارتية متعامدة نظامية

$$\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$$

$$\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$$

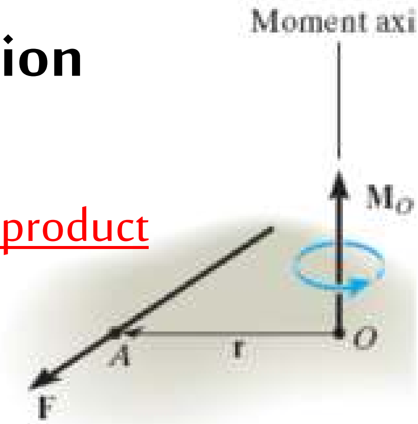
يعطى الجداء والزاوية بين الشعاعين بالعلاقتين

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z$$

$$\cos\theta=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a_x^2+a_y^2+a_z^2}\sqrt{b_x^2+b_y^2+b_z^2}}$$

Moment of a Force – Vector Formulation

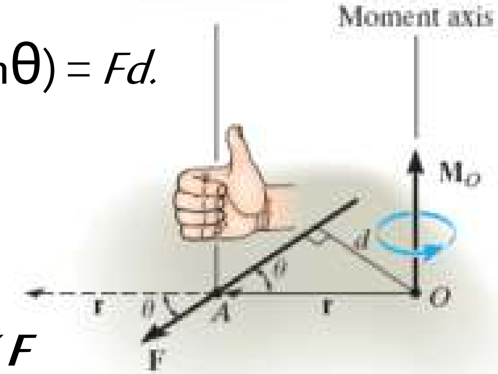
عزم القوة – الصياغة الشعاعية



- Moment of force $\boldsymbol{F}$ about point O can be expressed using cross (vector) product

$$\mathbf{M}_O = \boldsymbol{r} \times \boldsymbol{F} \qquad \textit{Hand writing}: \vec{M}_o = \vec{r} \times \vec{F}$$

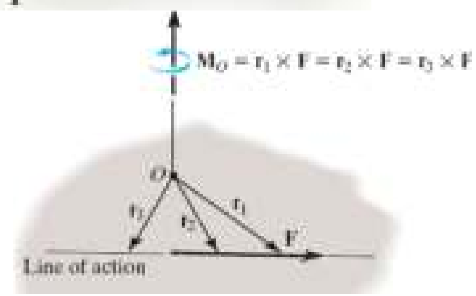
- Magnitude: For magnitude of cross product: $M_O = rF\sin\theta.$
- Treat $\boldsymbol{r}$ as a sliding vector. Since $d = r\sin\theta$, $\Rightarrow M_O = rF\sin\theta = F(r\sin\theta) = Fd.$
- Direction and sense of $\mathbf{M}_O$ are determined by right-hand rule.



Principle of Transmissibility:

For force $\boldsymbol{F}$ applied at any point A, moment created about O is: $\mathbf{M}_O = \boldsymbol{r}_A \times \boldsymbol{F}$

The force $\boldsymbol{F}$ acting on rigid body can slide and the cross product properties allow to write: $\boldsymbol{M}_O = \boldsymbol{r}_1 \times \boldsymbol{F} = \boldsymbol{r}_2 \times \boldsymbol{F} = \boldsymbol{r}_3 \times \boldsymbol{F}$



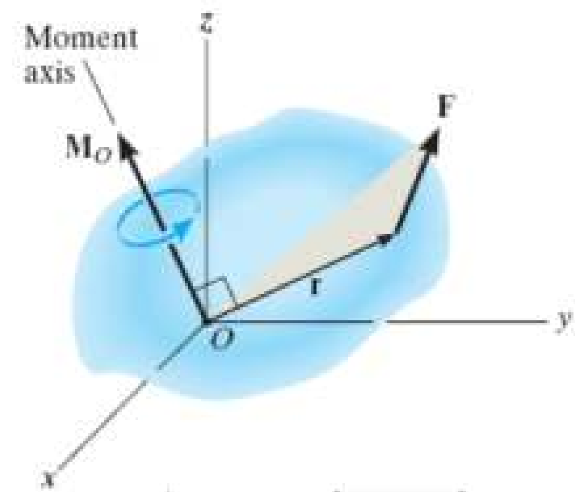
$$\boldsymbol{r}_1 = \boldsymbol{r}_2 + \lambda \boldsymbol{u}_F,\ \boldsymbol{r}_1 \times \boldsymbol{F} = (\boldsymbol{r}_2 + \lambda\, \boldsymbol{u}_F) \times \boldsymbol{F} = \boldsymbol{r}_2 \times \boldsymbol{F} + \lambda\, \boldsymbol{u}_F \times \boldsymbol{F} = \boldsymbol{r}_2 \times \boldsymbol{F} + \boldsymbol{0} = \boldsymbol{r}_2 \times \boldsymbol{F}$$

Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

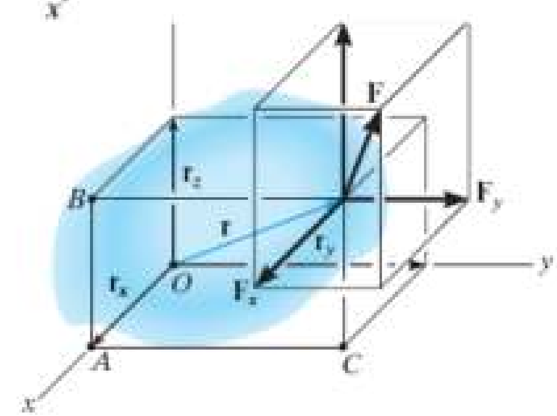
Cartesian Vector Formulation: For force expressed in Cartesian form,

$\vec{r} = r_x\vec{i} + r_y\vec{j} + r_z\vec{k}$ and $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$, the vector moment is:



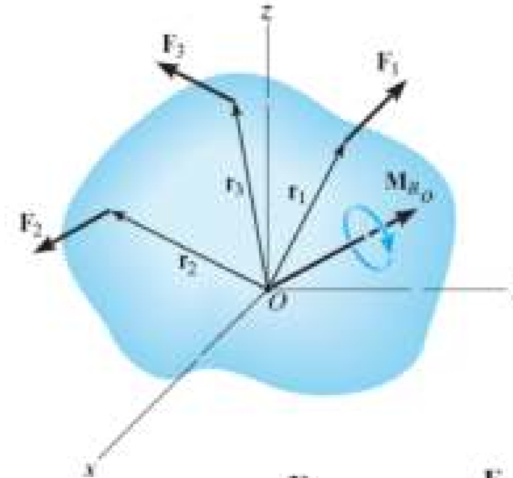
With the determinant expended,

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F} = \mathbf{i}\,(r_yF_z - r_zF_y) + \mathbf{j}\,(r_zF_x - r_xF_z) + \mathbf{k}\,(r_xF_y - r_yF_x)$$



Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

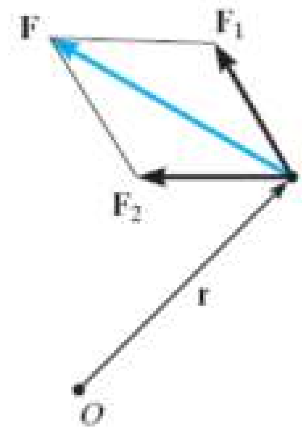


Resultant Moment of a System of Forces

Resultant moment of forces about point O can be determined by vector addition

$$M_{R_O} = \sum (\boldsymbol{r} \times \boldsymbol{F})$$

Varignon's Principle



"Moment of a force about a point is equal to the sum of the moments of the forces' components about the point"

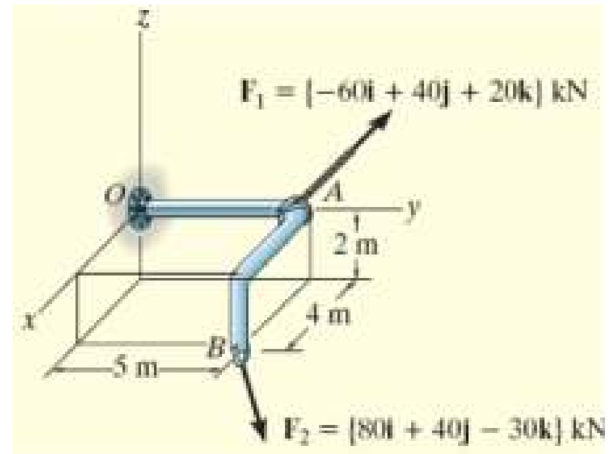
Since $\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2$, then $\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F} = \boldsymbol{r} \times (\boldsymbol{F}_1 + \boldsymbol{F}_2) = \boldsymbol{r} \times \boldsymbol{F}_1 + \boldsymbol{r} \times \boldsymbol{F}_2$

Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

Example 2:

Two forces act on the pipe system. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.



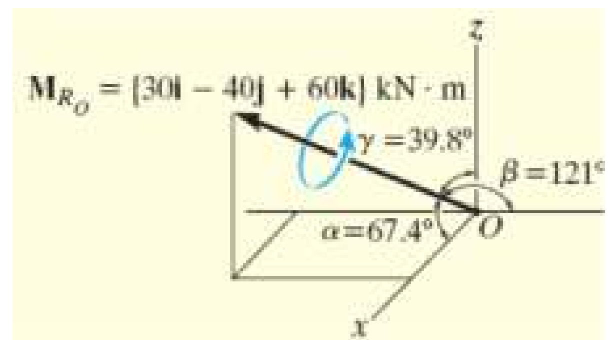
Solution:

Position vectors are directed from O to each force as shown.

$$\vec{r}_A = \{5\vec{j}\}\ m \quad and \quad \vec{r}_B = \{4\vec{i} + 5\vec{j} - 2\vec{k}\}\ m$$

The resultant moment about O is:

$$\vec{M}_O = \sum(\vec{r} \times \vec{F}) = \vec{r}_A \times \vec{F}_1 + \vec{r}_B \times \vec{F}_2 = \{30\vec{i} - 40\vec{j} + 60\vec{k}\}\ kN \cdot m$$

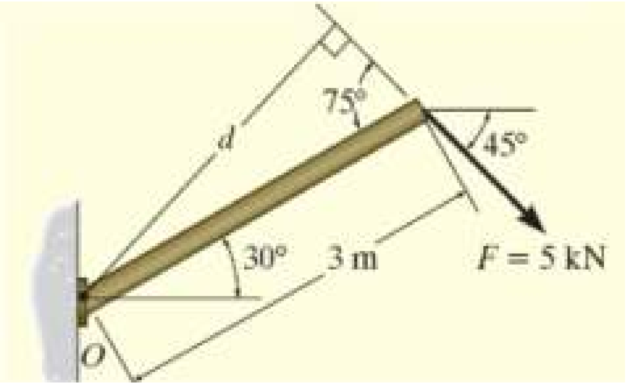


Moment of a Force – Vector Formulation

عزم القوة – الصياغة الشعاعية

Example 3:

Determine the moment of the force about point *O*.



Solution:

The moment arm *d* can be found from trigonometry,

$d = (3)\sin 75° = 2.898\text{ m}$ Thus, $M_O = Fd = -(5)(2.898) = -14.5\text{ kN}\cdot\text{m}$

Without using the arm *d*, the moment can be found from the components by

$$M_O = |d_x F_y| + |d_y F_x| = -(3\cos 30^\circ)(5\sin 45^\circ) - (3\sin 30^\circ)(5\cos 45^\circ) = -14.5\ \text{kN}\cdot\text{m}$$

$$= -15(\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ) = -15\sin 75^\circ$$