



Valid and Invalid Arguments

Epp Section 1.3



Argument حجة

An argument is a sequence of statements.

All statements but the final one are called **assumptions** or **hypothesis**.

The final statement is called the **conclusion**.

An argument is **valid** if:

whenever all the assumptions are true, then the conclusion is true.

If today is Wednesday, then yesterday is Tuesday.

Today is Wednesday.

∴ Yesterday is Tuesday.



Modus Ponens

طريقة التأكيد

If p then q.
p
∴ q

If typhoon, then class cancelled.
Typhoon.
∴ Class cancelled.

assumptions conclusion

p	q	$p \rightarrow q$	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Modus ponens is Latin meaning "method of affirming".

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Modus Tollens

طريقة الرفض

If p then q .
 $\sim q$
 $\therefore \sim p$

If typhoon, then class cancelled.
Class not cancelled.
 \therefore No typhoon.

assumptions conclusion

p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Modus tollens is Latin meaning "method of denying".

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Equivalence

A student is trying to prove that propositions P , Q , and R are all true. She proceeds as follows.

First, she proves three facts:

- P implies Q
- Q implies R
- R implies P .

Then she concludes,

`` Thus P , Q , and R are all true.``

Proposed argument:

$$(P \rightarrow Q), (Q \rightarrow R), (R \rightarrow P)$$

$$P \wedge Q \wedge R$$

assumption

Is it valid?

conclusion



Valid Argument?

$$(P \rightarrow Q), (Q \rightarrow R), (R \rightarrow P)$$

$$P \wedge Q \wedge R$$

Is it valid?

assumptions

conclusion

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

$P \rightarrow Q$	$Q \rightarrow R$	$R \rightarrow P$
T	T	T
T	F	T
F	T	T
F	T	T
T	T	F
T	F	T
T	T	F
T	T	T

$P \wedge Q \wedge R$	OK?
T	yes
F	yes
F	yes
F	yes
F	yes
F	yes
F	yes
F	yes
F	no

To prove an argument is not valid, we just need to find a counterexample.



Valid Arguments?

If p then q.
q
∴ p

	assumptions			conclusion
p	q	$p \rightarrow q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Assumptions are true, but not the conclusion.

If you are a fish, then you drink water.
You drink water.
You are a fish.



Valid Arguments?

assumptions

conclusion

p	q	$p \rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

If p then q.

$\sim p$

$\therefore \sim q$

If you are a fish, then you drink water.

You are not a fish.

You do not drink water.



Modus Ponens example

- Assume you are given the following two statements:

- “you are in this class”

 p

- “if you are in this class, you will get a grade”

 $\underline{p \rightarrow q}$ $\therefore q$

- Let p = “you are in this class”
- Let q = “you will get a grade”
- By Modus Ponens, you can conclude that you will get a grade



Modus Ponens

- Consider $(p \wedge (p \rightarrow q)) \rightarrow q$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

p
 $p \rightarrow q$
 $\therefore q$



Generalization & Specialization

- Generalization: If you know that p is true, then $p \vee q$ will ALWAYS be true

$$\frac{p}{\therefore p \vee q}$$

- Specialization: If $p \wedge q$ is true, then p will ALWAYS be true

$$\frac{p \wedge q}{\therefore p}$$



Example of proof

- We have the hypotheses:

- p** – “It is not sunny this afternoon and it is colder than yesterday”
- q** – “We will go swimming only if it is sunny”
- r** – “If we do not go swimming, then we will take a canoe trip”
- s** – “If we take a canoe trip, then we will be home by sunset”
- t** – “If we take a canoe trip, then we will be home by sunset”

- Does this imply that “we will be home by sunset”?

$$\neg p \wedge q$$

$$r \rightarrow p$$

$$\neg r \rightarrow s$$

$$\frac{s \rightarrow t}{t}$$

t



Example of proof

1. $\neg p \wedge q$

1st hypothesis

2. $\neg p$

Simplification using step 1

3. $r \rightarrow p$

2nd hypothesis

4. $\neg r$

Modus tollens using steps 2 & 3

5. $\neg r \rightarrow s$

3rd hypothesis

6. s

Modus ponens using steps 4 & 5

7. $s \rightarrow t$

4th hypothesis

8. t

Modus ponens using steps 6 & 7

$$\frac{p \wedge q}{\quad}$$

$\therefore p$

p

$$\frac{p \rightarrow q}{\quad}$$

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$\neg q$

$$\frac{p \rightarrow q}{\quad}$$

$\therefore \neg p$



So what did we show?

- We showed that:
 - $[(\neg p \wedge q) \wedge (r \rightarrow p) \wedge (\neg r \rightarrow s) \wedge (s \rightarrow t)] \rightarrow t$
 - That when the 4 hypotheses are true, then the implication is true
 - In other words, we showed the above is a tautology!
- To show this, enter the following into the truth table generator at
[http://sciris.shu.edu/~borowski/Truth/:](http://sciris.shu.edu/~borowski/Truth/)
 $((\sim P \wedge Q) \wedge (R \Rightarrow P) \wedge (\sim R \Rightarrow S) \wedge (S \Rightarrow T)) \Rightarrow T$



More rules of inference

- Conjunction: if p and q are true separately, then $p \wedge q$ is true

$$\frac{p}{q} \quad \frac{q}{\hline} \\ \therefore p \wedge q$$

- Elimination: If $p \vee q$ is true, and p is false, then q must be true

$$\frac{p \vee q}{\neg p} \quad \frac{\neg p}{\hline} \\ \therefore q$$

- Transitivity: If $p \rightarrow q$ is true, and $q \rightarrow r$ is true, then $p \rightarrow r$ must be true

$$\frac{p \rightarrow q}{q \rightarrow r} \quad \frac{q \rightarrow r}{\hline} \\ \therefore p \rightarrow r$$



Even more rules of inference

- Proof by division into cases:
if at least one of p or q is true, then r must be true

$$p \vee q$$

$$p \rightarrow r$$

$$\underline{q \rightarrow r}$$

$$\therefore r$$

- Contradiction rule: If $\neg p \rightarrow c$ is true, we can conclude p (via the contra-positive)

$$\underline{\neg p \rightarrow c}$$

$$\therefore p$$

- Resolution: If $p \vee q$ is true, and $\neg p \vee r$ is true, then $q \vee r$ must be true

$$p \vee q$$

$$\underline{\neg p \vee r}$$

$$\therefore q \vee r$$

– Not in the textbook Dr. Iyad Hatem



Example of proof

- Given the hypotheses:
 - “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on”
 - “If the sailing race is held, then the trophy will be awarded”
 - “The trophy was not awarded”
- Can you conclude: “It rained”?

$$(\neg r \vee \neg f) \rightarrow (s \wedge l)$$

$$s \rightarrow t$$

$$\neg t$$

$$r$$



Example of proof

- | | |
|--|--|
| 1. $\neg t$ | 3 rd hypothesis |
| 2. $s \rightarrow t$ | 2 nd hypothesis |
| 3. $\neg s$ | Modus tollens using steps 2 & 3 |
| 4. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$ | 1 st hypothesis |
| 5. $\neg(s \wedge l) \rightarrow \neg(\neg r \vee \neg f)$ | Contrapositive of step 4 |
| 6. $(\neg s \vee \neg l) \rightarrow (r \wedge f)$ | DeMorgan's law and double negation law |
| 7. $\neg s \vee \neg l$ | Addition from step 3 |
| 8. $r \wedge f$ | Modus ponens using steps 6 & 7 |
| 9. r | Simplification using step 8 |

$$\frac{p}{p \rightarrow q} \therefore q$$

$$\frac{p}{p \vee q} \therefore p \vee q$$

$$\frac{p \wedge q}{p} \therefore p$$

$$\frac{\neg q}{p \rightarrow q} \therefore \neg p$$



~~Modus Badus~~

Fallacy of affirming the conclusion

- Consider the following:

 q
 q
 $\underline{p \rightarrow q}$
 $\underline{\neg q \rightarrow \neg p}$
 $\therefore p$
 $\therefore p$

- Is this true?

p	q	$p \rightarrow q$	$q \wedge (p \rightarrow q)$	$(q \wedge (p \rightarrow q)) \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Not a valid rule!



Modus Badus example

- Assume you are given the following two statements:

- “you will get a grade”

 q

- “if you are in this class, you will get a grade”

 $p \rightarrow q$ $\therefore p$

- Let p = “you are in this class”
- Let q = “you will get a grade”
- You **CANNOT** conclude that you are in this class
 - You could be getting a grade for another class



~~Modus Badus~~

Fallacy of denying the hypothesis

- Consider the following:

$$\neg p$$

$$\underline{p \rightarrow q}$$

- Is this true?

$$\therefore \neg q$$

p	q	$p \rightarrow q$	$\neg p \wedge (p \rightarrow q)$	$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Not a valid rule!



Modus Badus example

- Assume you are given the following two statements:

- “you are not in this class”

$$\neg p$$

- “if you are in this class, you will get a grade”

$$\underline{p \rightarrow q}$$

$$\therefore \neg q$$

- Let p = “you are in this class”
- Let q = “you will get a grade”
- You CANNOT conclude that you will not get a grade
 - You could be getting a grade for another class