Center of Gravity, Center of Mass, Centroids مركز الثقل، مركز الكتلة، المركز الجيومتري

1. Center of Forces.
2. Center of Gravity and Center of Mass.
3. Centroids: Center of a Volume, Center of an Area and Center of a Line.
4. Examples and Exercises.

1. Center of Forces:

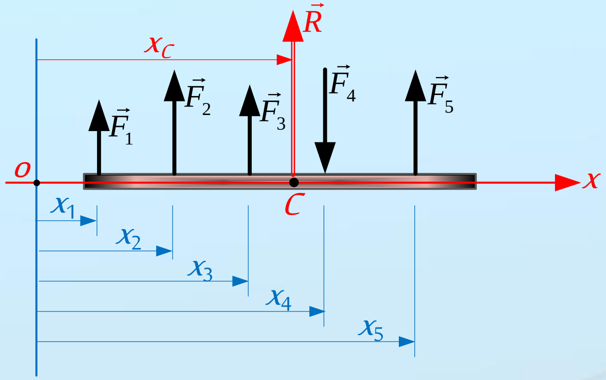
In Engineering Mechanics, it was shown that a system of forces that are not in equilibrium can be replaced by a single force, namely, the resultant R ($\vec{R}$), provided that the reduction does not lead to a couple.

رأينا في الميكانيك الهندسي أنه يمكن اختزال جملة من قوى غير متوازنة إلى قوة وحيدة تدعى المحصلة ونرمز لها $\vec{R}$، شريطة أن لا يؤول الاختزال إلى مزدوجة.

For parallel forces, the direction of the resultant R ($\vec{R}$) coincides with the direction of the forces.

1.1 Concentrated Forces:

$$\vec{R} = \sum \vec{F}_i \quad \Rightarrow R = \sum F_i = F_1 + F_2 + F_3 - F_4 + F_5$$



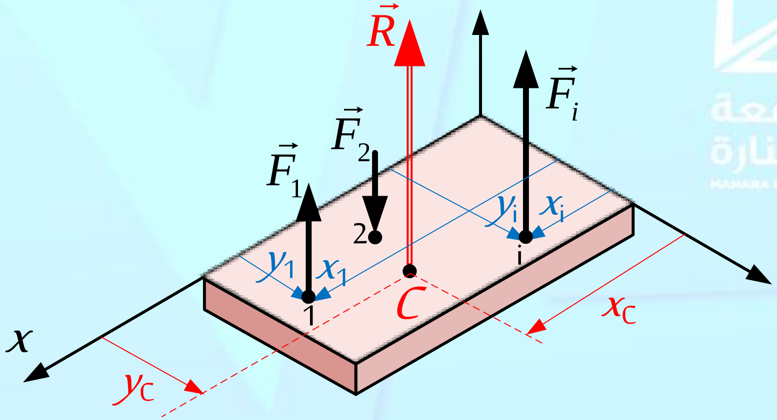
The action line of the resultant R ($\vec{R}$) can be found from

$$x_C R = \sum x_i F_i \quad \Rightarrow x_C R = x_1F_1 + x_2F_2 + x_3F_3 - x_4F_4 + x_5F_5 +$$

$$\Rightarrow x_C = \frac{x_1F_1 + x_2F_2 + x_3F_3 - x_4F_4 + x_5F_5}{R} \quad \Rightarrow \quad \boxed{x_C = \frac{\sum x_i F_i}{\sum F_i}}$$

$$R = \sum F_i$$

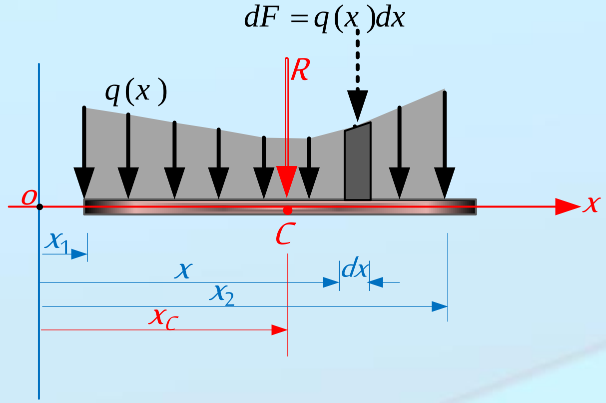
$$x_C = \frac{\sum x_i F_i}{\sum F_i} \qquad y_C = \frac{\sum y_i F_i}{\sum F_i}$$



1.2 Distributed Forces:

1.2.1 Line Forces:

Dim. :$[q] \equiv [F]/[L]$, S.I. Units: N/m, kN/m, . . .



$$R = \sum dF_i = \int_{x_1}^{x_2} q(x)dx$$

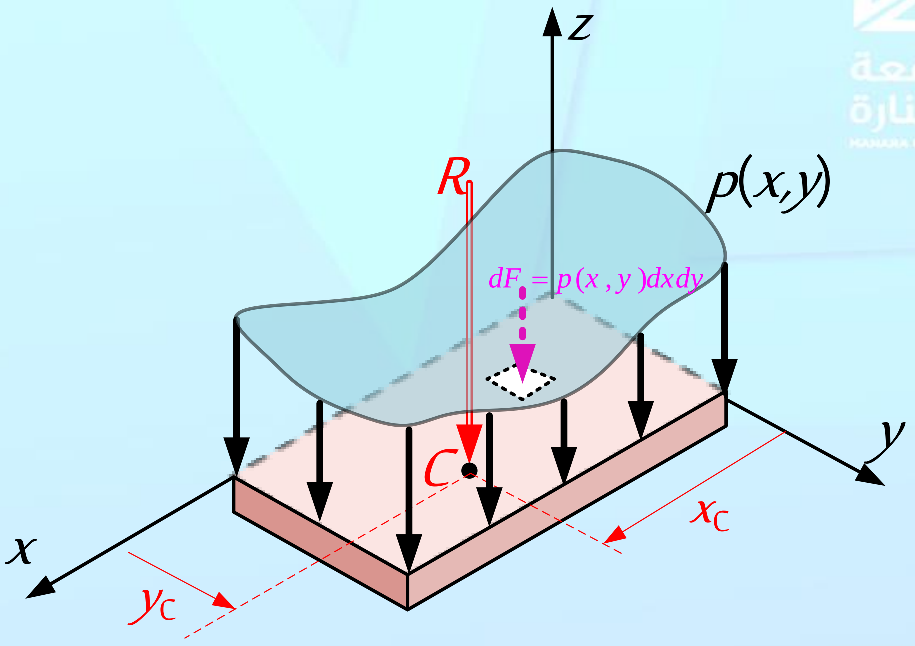
$$x_C = \frac{\sum x_i F_i}{\sum F_i} = \frac{\int_{x_1}^{x_2} xq(x)dx}{\int_{x_1}^{x_2} q(x)dx}$$

1.2.2 Area (Surface) Forces: Dim. :[p]≡[F]/[L^2], S.I. Units: N/m^2, kN/m^2, . . .

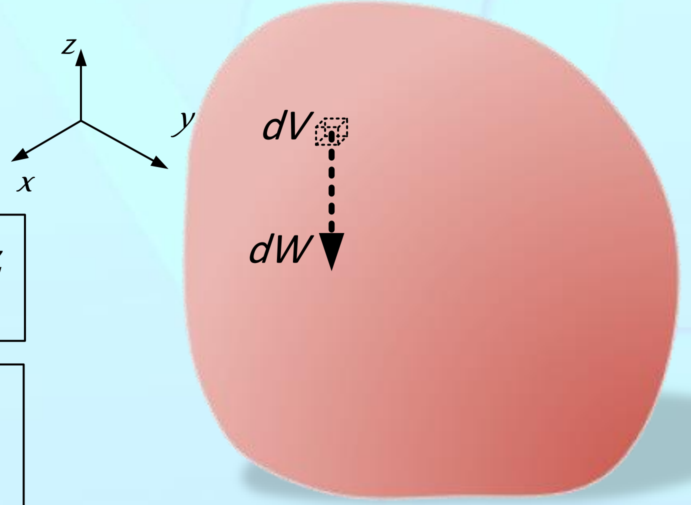
$$R = \sum dF = \int_A p(x, y)dxdy$$

$$x_C = \frac{\int_A xp(x, y)dxdy}{\int_A p(x, y)dxdy}$$

$$y_C = \frac{\int_A yp(x, y)dxdy}{\int_A p(x, y)dxdy}$$



2. Center of Gravity and Center of Mass.



Center of Gravity

$$W = \sum dW = \int_V \gamma(x,y,z)\,dx\,dy\,dz$$

$$x_C = \frac{\int_V x\,\gamma(x,y,z)\,dx\,dy\,dz}{\int_V \gamma(x,y,z)\,dx\,dy\,dz}$$

$$y_C = \frac{\int_V y\,\gamma(x,y,z)\,dx\,dy\,dz}{\int_V \gamma(x,y,z)\,dx\,dy\,dz}$$

$$z_C = \frac{\int_V z\,\gamma(x,y,z)\,dx\,dy\,dz}{\int_V \gamma(x,y,z)\,dx\,dy\,dz}$$

Center of Mass

$$M = \sum dM = \int_V \rho(x,y,z)\,dx\,dy\,dz$$

$$x_C = \frac{\int_V x\,\rho(x,y,z)\,dx\,dy\,dz}{\int_V \rho(x,y,z)\,dx\,dy\,dz}$$

$$y_C = \frac{\int_V y\,\rho(x,y,z)\,dx\,dy\,dz}{\int_V \rho(x,y,z)\,dx\,dy\,dz}$$

$$z_C = \frac{\int_V z\,\rho(x,y,z)\,dx\,dy\,dz}{\int_V \rho(x,y,z)\,dx\,dy\,dz}$$

$\gamma(x,y,z)$

Weight per Volume Unit

الوزن الحجمي

$\rho(x,y,z)$

Mass per Volume Unit

الكتلة الحجمية

On Earth عند سطح الأرض

$$\gamma(x,y,z) = g\,\rho(x,y,z)$$

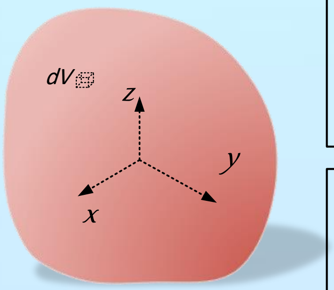
3. Centroids: Center of a Volume, Center of an Area and Center of a Line.

When $\gamma(x,y,z)$ and $\rho(x,y,z)$ are constant (homogeneous materials), the gravity and mass centers become geometric centers "Centroids". The previous equations take the forms

تكون الكتلة الحجمية والوزن الحجمي ثابتان في المواد المتجانسة وعندئذ يصبح مركز الثقل أو مركز الكتلة، مجرد مركز جيومتري. وتكون العلاقات السابقة:

For Volumes

$$V = \sum V_i = \sum \int_{V_i} dx\,dy\,dz$$



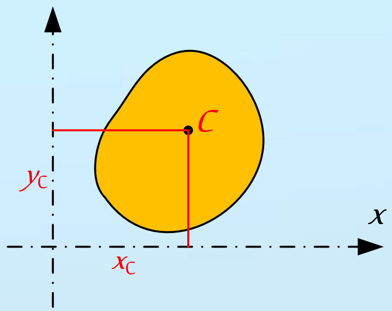
$$x_C = \frac{\int_V x\,dx\,dy\,dz}{V}$$

$$y_C = \frac{\int_V y\,dx\,dy\,dz}{V}$$

$$z_C = \frac{\int_V z\,dx\,dy\,dz}{V}$$

For Areas

$$A = \sum A_i = \sum \int_{A_i} dx\,dy$$



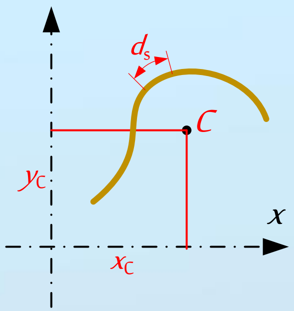
$$x_C = \frac{\int_A x\,dx\,dy}{A}$$

$$y_C = \frac{\int_A y\,dx\,dy}{A}$$

$$z_C = \frac{\int_A z\,dx\,dy}{A}$$

For Lines

$$s = \int_S ds = \int_S \sqrt{dx^2 + dy^2}$$



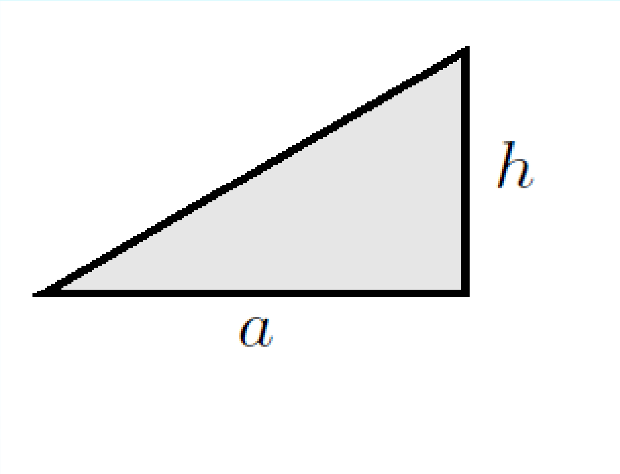
$$x_C = \frac{\int_S x\,ds}{S}$$

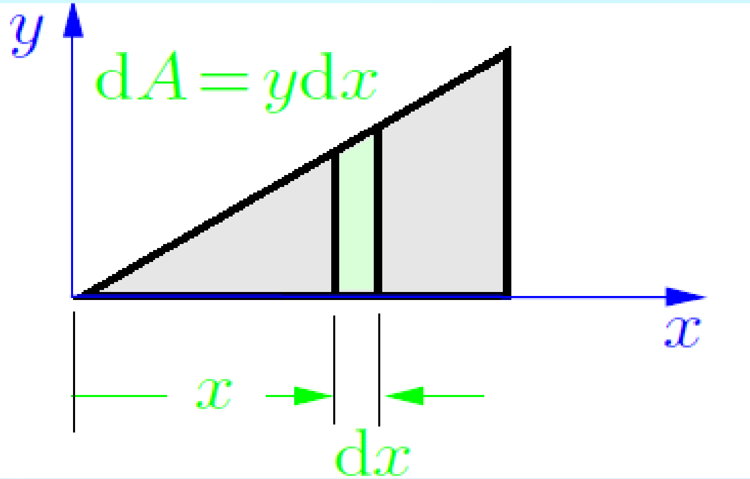
$$y_C = \frac{\int_S y\,ds}{S}$$

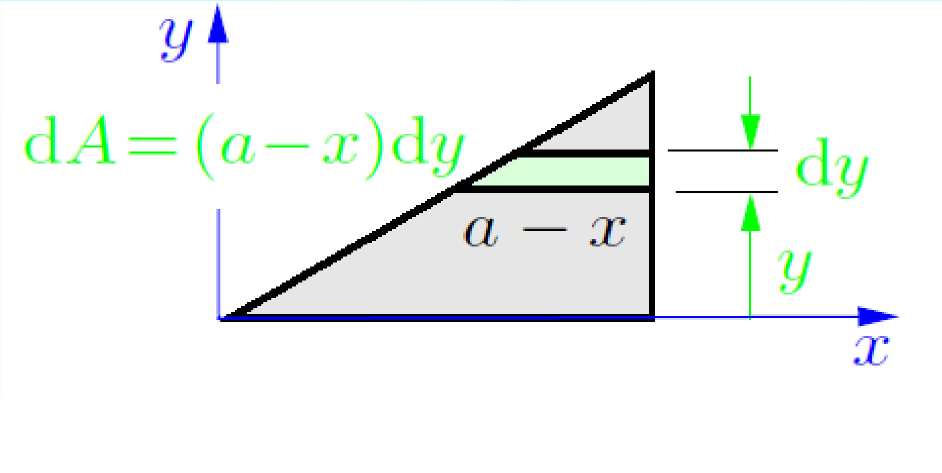
$$z_C = \frac{\int_S z\,ds}{S}$$

Example 1.

Locate the centroid of a rectangular triangle with baseline *a* and height *h*.







Example.

A wire is bent into the shape of a circular arc with an opening angle 2α