Center of Gravity, Center of Mass, Centroids مركز الثقل، مركز الكتلة، المركز الجيومتري

1. Center of Forces.
2. Center of Gravity and Center of Mass.
3. Centroids: Center of a Volume, Center of an Area and Center of a Line.
4. Examples and Exercises.

4. Examples and Exercises. Centroids of plane areas and lines

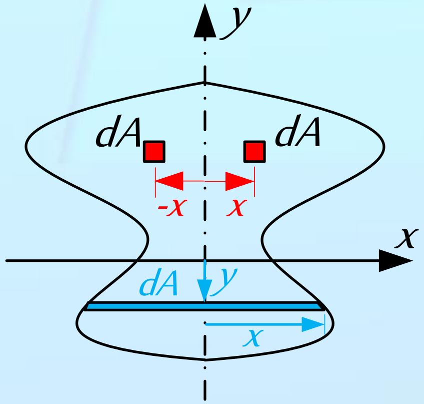
Three Remarks before starting

1. The term "centroid" is used when the material factors (γ, ρ) are omitted, i.e., when one is concerned with geometrical considerations only.

2. If the area has an axis of symmetry, the centroid of the area lies on this axis.

$$x_C = \frac{\int_A x\,dx\,dy}{A} = 0$$

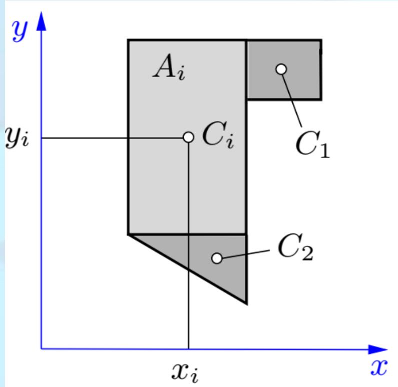
$$y_C = \frac{\int_A y\,dA}{A} = \frac{\int_A 2xy\,dy}{A}$$



3. For area composed of several parts of simple shape. The coordinates x_i, y_i of the centroids C_i and the areas A_i of the individual parts are assumed to be known.

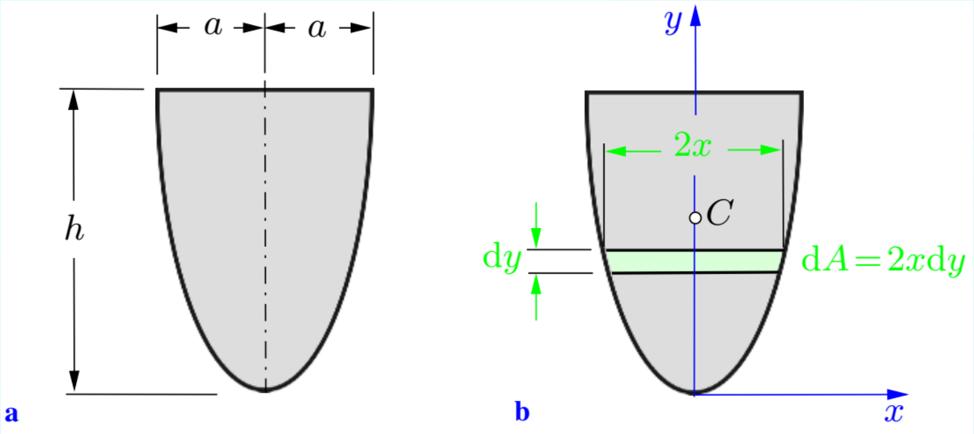
$$x_C = \frac{\sum_i x_i A_i}{\sum_i A_i}$$

$$y_C = \frac{\sum_i y_i A_i}{\sum_i A_i}$$



Example 1. Locate the centroid of the area that is bounded by a parabola, as in figure (a).

Solution



We use the coordinate system as shown in (b). Since the y-axis is an axis of symmetry, the centroid C lies on it: $x_c = 0$. To determine the coordinate y_c from:

$$y_C = \frac{\int_A y\,dA}{A} = \frac{\int_A 2xy\,dy}{A}$$

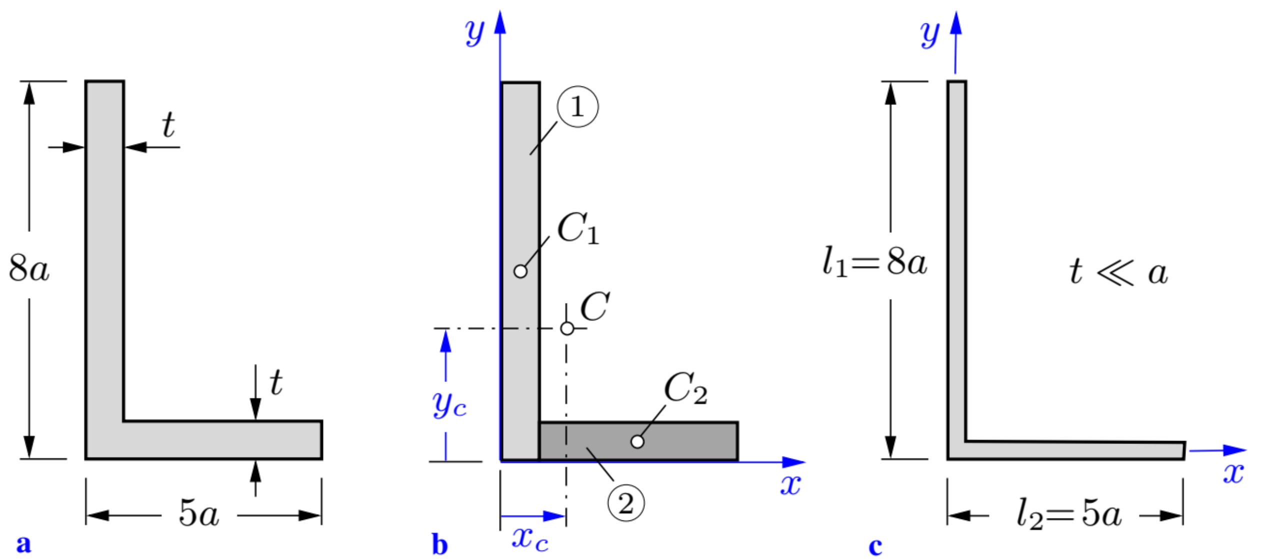
but $A = \int_A 2x\,dy = \int_0^h \ldots$ and $y = cx^2$, with $x = a$ when $y = h$, so $c = ?$ and $y = ?$

Example 2. Find the centroid of the L-shaped area in Fig. (a), then in Fig. (c).

Solution: We choose a coordinate system and consider the area to be composed of two rectangles (b):

$A_1 = 8at, \qquad A_2 = (5a - t)\,t$

The coordinates of their respective centroids are given by



$x_1 = t/2,\ y_1 = 4a\ ; \qquad x_2 = t + (5a - t)/2 = (5a + t)/2,\ y_2 = t/2$

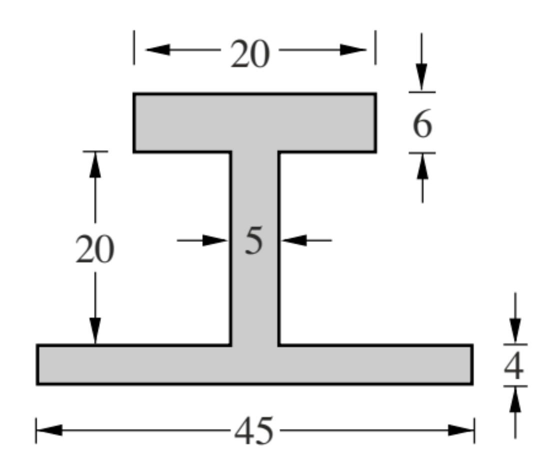
$$x_C = \frac{\sum_2 x_i A_i}{\sum_2 A_i} = \frac{(t/2)(8at) + [(5a+t)/2][(5a-t)t]}{(8at) + [(5a-t)t]} \qquad y_C = \frac{\sum_2 y_i A_i}{\sum_2 A_i} = \frac{(4a)(8at) + (t/2)[(5a-t)t]}{(8at) + [(5a-t)t]}$$

$$x_C = \frac{4at^2 + (25a^2 - t^2)(t/2)}{13at - t^2} = \frac{25a^2 + 8at - t^2}{26a - 2t} \qquad y_C = \frac{32a^2 t + (5a - t)(t^2/2)}{13at - t^2} = \frac{64a^2 + 5at - t^2}{26a - 2t}$$

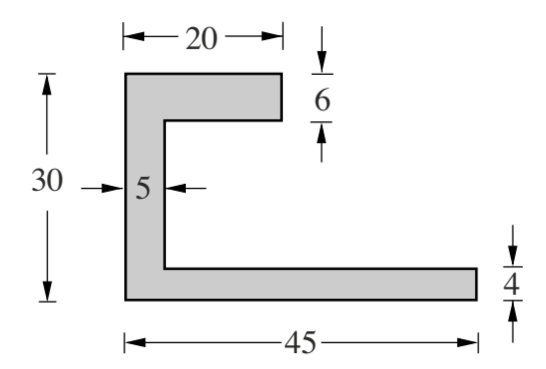
For $t << a$, as in Fig. (c), the areas become lines $x_C = \frac{25a}{26}$ and $y_C = \frac{32a}{13}$

Exercises.

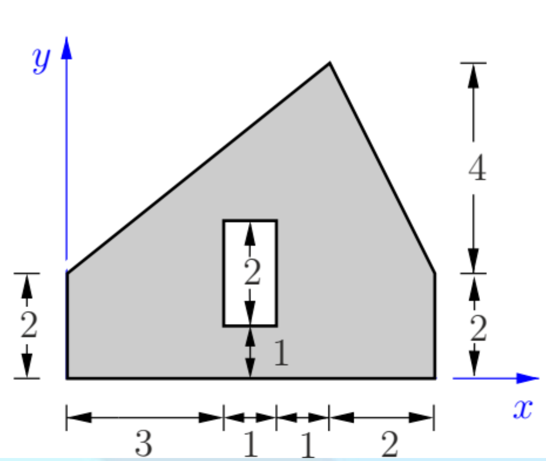
1. Locate the centroids of the depicted profile. The measurements are given in mm.



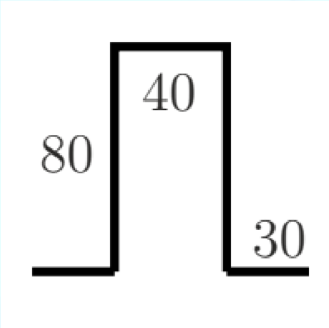
2. Locate the centroids of the depicted profile. The measurements are given in mm.



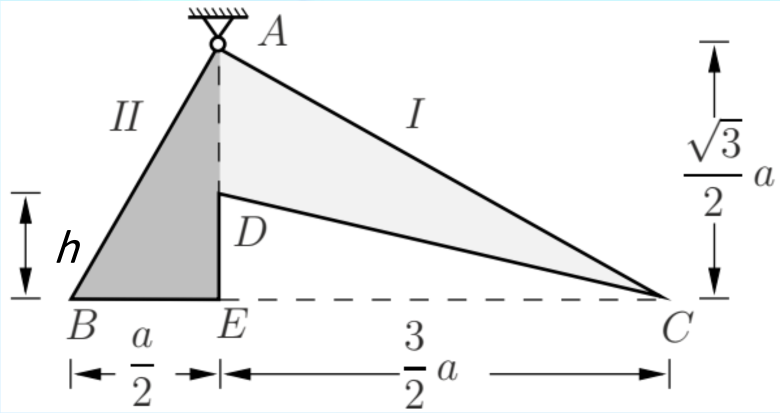
3. Locate the centroid of the depicted area with a rectangular cutout. The measurements are given in cm.



4. A wire with constant thickness is deformed into the depicted figure. The measurements are given in mm. Locate the centroid.



5. From the triangular-shaped metal sheet *ABC*, the triangle *CDE* has been cut out. The system is pin supported in *A*. Determine *h* such that *BC* adjusts horizontal.



6. A thin sheet with constant thickness and density, consisting of a square and two triangles, is bent to the depicted figure (measurements in cm). Locate the center of gravity