Lecture 2

Number Systems

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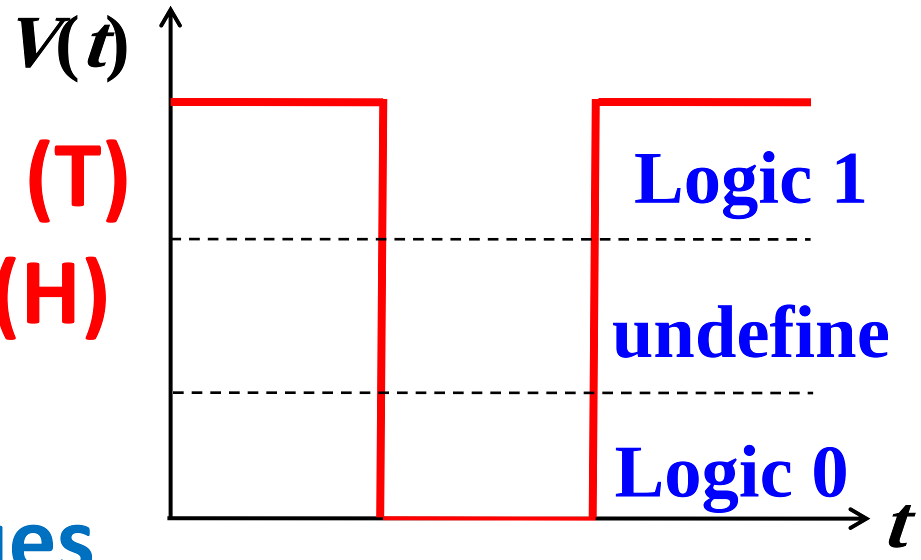
Outline

1. Digital Systems
2. Binary Numbers
3. Number-base Conversions
4. Octal and Hexadecimal Numbers
5. Complements
6. Signed Binary Numbers
7. Binary Codes
8. Binary Storage and Registers
9. Binary Logic

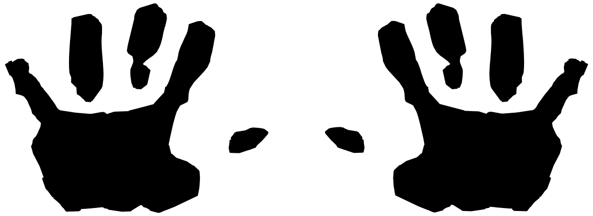
BINARY DIGITAL SIGNAL

- An information variable represented by physical quantity.
- For digital systems, the variable takes on discrete values.
  - ➢ Two level, or binary values are the most prevalent values.
- Binary values are represented abstractly by:
  - ➢ Digits 0 and 1
  - ➢ Words (symbols) False (F) and True (T)
  - ➢ Words (symbols) Low (L) and High (H)
  - ➢ And words On and Off
- Binary values are represented by values or ranges of values of physical quantities.



Binary digital signal

DECIMAL NUMBER SYSTEM



- **Base** (also called radix) = **10**
  - 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
- **Digit Position**
  - Integer & fraction
- **Digit Weight**
  - Weight = $(Base)^{Position}$
- **Magnitude**
  - Sum of "*Digit* x *Weight*"
- **Formal Notation**

Position	2	1	0	-1	-2
Digit	5	1	2	7	4
Weight	*100*	*10*	*1*	*0.1*	*0.01*
Magnitude	500	10	2	0.7	0.04

$$d_2*B^2+d_1*B^1+d_0*B^0+d_{-1}*B^{-1}+d_{-2}*B^{-2}$$

$$(512.74)_{10}$$

OCTAL NUMBER SYSTEM

- Base = 8
  - 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }
- Weights
  - Weight = $(Base)^{Position}$
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

$(512.74)_8$

64	8	1		1/8	1/64
5	1	2		7	4
2	1	0		-1	-2

$5 * 8^2 + 1 * 8^1 + 2 * 8^0 + 7 * 8^{-1} + 4 * 8^{-2}$

$= (330.9375)_{10}$

BINARY NUMBER SYSTEM

- **Base = 2**
  - **2 digits** { 0, 1 }, called *b*inary dig*its* or "*bits*"
- Weights
  - ➢ Weight = $(Base)^{Position}$
- Magnitude
  - ➢ Sum of "*Bit* x *Weight*"
- Formal Notation
- Groups of bits

4 bits = *Nibble* 1 0 1 1

8 bits = *Byte* 1 1 0 0 0 1 0 1

$(101.01)_2$

4	2	1	1/2	1/4
1	0	1	0	1
2	1	0	-1	-2

$1 *2^{2}+0 *2^{1}+1 *2^{0}+0 *2^{-1}+1 *2^{-2}$

$=(5.25)_{10}$

HEXADECIMAL NUMBER SYSTEM

- **Base = 16**
  - **16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }**
- Weights
  - ➢ Weight = (*Base*)^{Position}
- Magnitude
  - ➢ Sum of "*Digit* x *Weight*"
- Formal Notation

$(1E5.7A)_{16}$

256 16 1 1/16 1/256

1 E 5 7 A

2 1 0 -1 -2

$1 * 16^{2} + 14 * 16^{1} + 5 * 16^{0} + 7 * 16^{-1} + 10 * 16^{-2}$

$= (485.4765625)_{10}$

The Power of 2

n	2^n
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
4	$2^4=16$
5	$2^5=32$
6	$2^6=64$
7	$2^7=128$

n	2^n	
8	$2^8=256$	
9	$2^9=512$	
10	$2^{10}=$**1024**	**Kilo**
11	$2^{11}=2048$	
12	$2^{12}=4096$	
20	$2^{20}=$**1M**	**Mega**
30	$2^{30}=$**1G**	**Giga**
40	$2^{40}=$**1T**	**Tera**

Addition

- Decimal Addition

	1	1	
		5	5
+		5	5
	1	1	0

← Carry

= *Ten* ≥ *Base*

➔ **Subtract a Base**

Binary Addition

- Column Addition

	1	1	1	1	1	1		
		1	1	1	1	0	1	= 61
+			1	0	1	1	1	= 23
	1	0	1	0	1	0	0	= 84

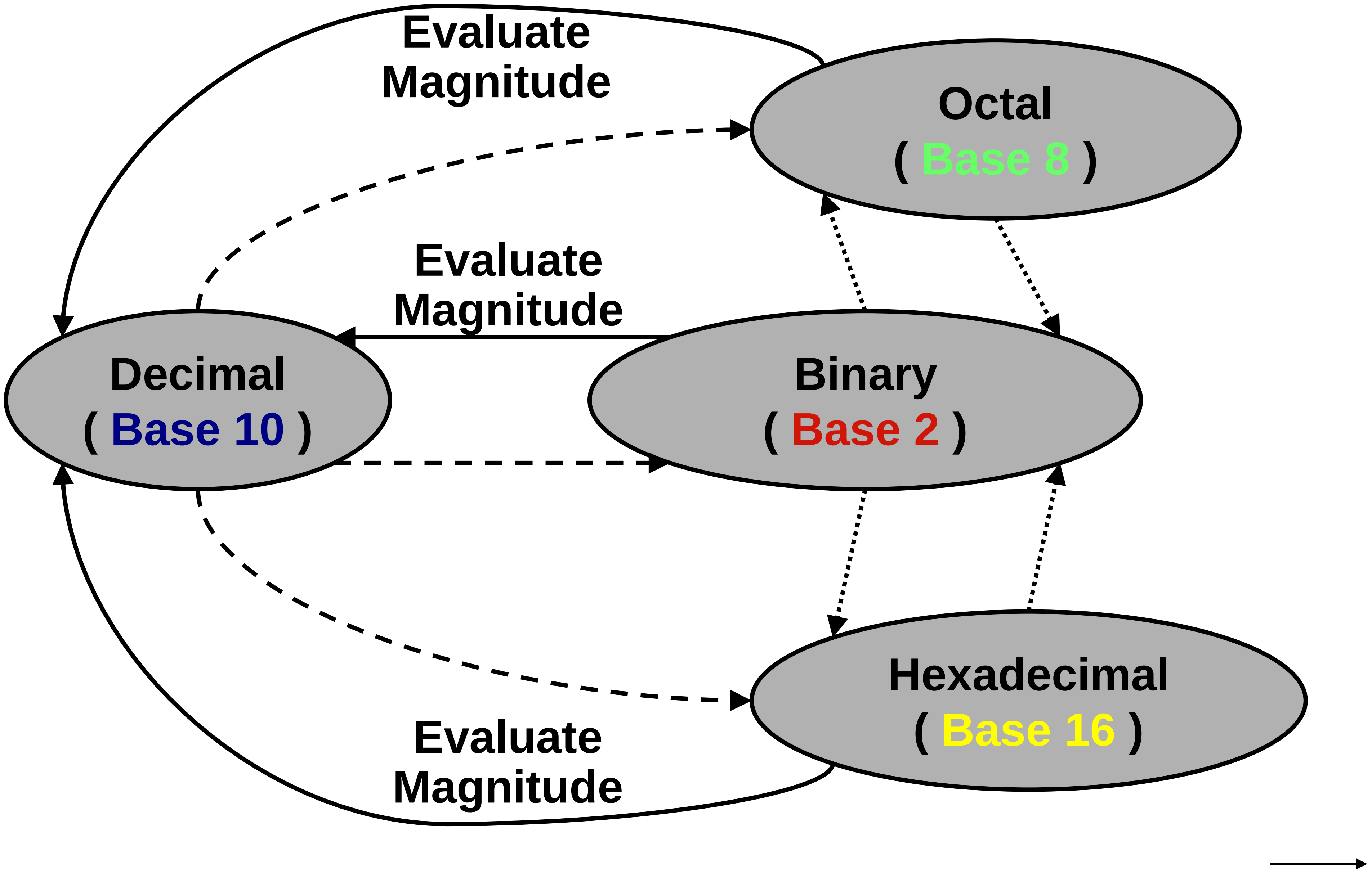
$\geq (2)_{10}$

Binary Subtraction

- Borrow a “Base” when needed

		1			2			$= (10)_2$
	0	~~2~~	2	0	0	2		
	~~1~~	0	0	~~1~~	~~1~~	0	1	77
−			1	0	1	1	1	- 23
	0	1	1	0	1	1	0	= 54

Number Base Conversions



Binary Multiplication

- Bit by bit

				1	0	1	1	1
x					1	0	1	0
				0	0	0	0	0
			1	0	1	1	1	
		0	0	0	0	0		
	1	0	1	1	1			
	1	1	1	0	0	1	1	0

Decimal (*Fraction*) to Binary Conversion

- Multiply the number by the 'Base' (=2)
- Take the integer (either 0 or 1) as a coefficient
- Take the resultant fraction and repeat the division

Example: $(0.625)_{10}$

	Integer	Fraction	Coefficient
0.625 * 2 =	1	. 25	$a_{-1} = 1$
0.25 * 2 =	0	. 5	$a_{-2} = 0$
0.5 * 2 =	1	. 0	$a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1}\,a_{-2}\,a_{-3})_2 = (0.101)_2$

MSB LSB

Decimal (Integer) to Binary Conversion

- Divide the number by the 'Base' (=2)
- Take the remainder (either 0 or 1) as a coefficient
- Take the quotient and repeat the division

Example: $(13)_{10}$

	Quotient	Remainder	Coefficient
13 / 2 =	6	1	$a_0 = 1$
6 / 2 =	3	0	$a_1 = 0$
3 / 2 =	1	1	$a_2 = 1$
1 / 2 =	0	1	$a_3 = 1$

Answer: $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

MSB LSB

Decimal to Octal Conversion

Example: $(175)_{10}$

	Quotient	Remainder	Coefficient
175 / 8 =	21	7	$a_0 = 7$
21 / 8 =	2	5	$a_1 = 5$
2 / 8 =	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_2\, a_1\, a_0)_8 = (257)_8$

Example: $(0.3125)_{10}$

	Integer		Fraction	Coefficient
0.3125 * 8 =	2	.	5	$a_{-1} = 2$
0.5 * 8 =	4	.	0	$a_{-2} = 4$

Answer: $(0.3125)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_8 = (0.24)_8$

Binary – Octal Conversion

- $8 = 2^3$
- Each group of 3 bits represents an octal digit

Example:

Assume Zeros

$$(\;10110.01\;)_2$$

$$(\;2\quad 6\;.\;2\;)_8$$

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

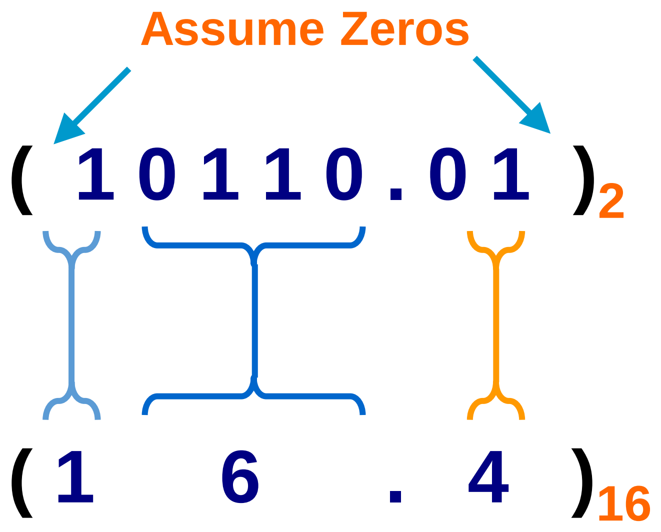
Works both ways (*Binary* to *Octal* & *Octal* to *Binary*)

Binary – Hexadecimal Conversion

- $16 = 2^4$
- Each group of 4 bits represents a hexadecimal digit

Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

Example:

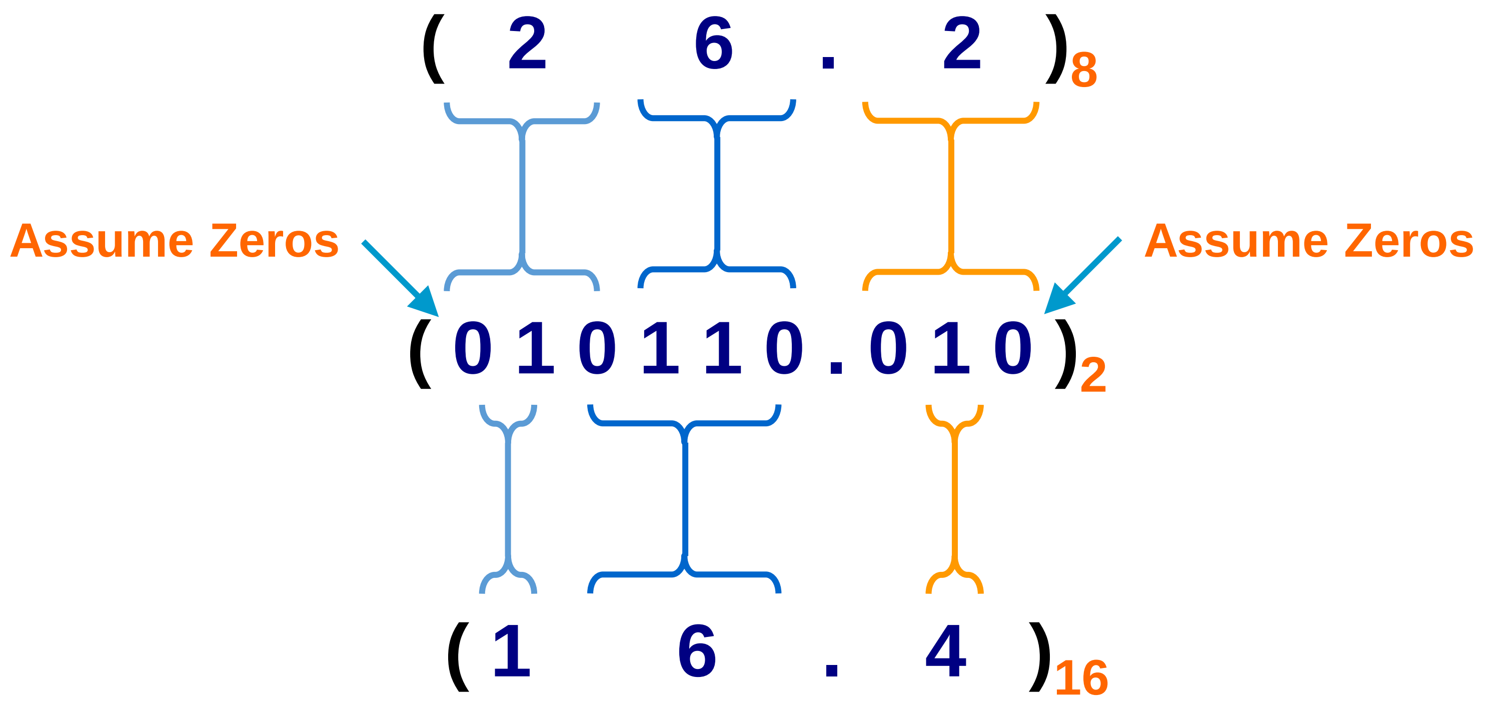


Works **both** **ways (*Binary* to *Hex* & *Hex* to *Binary*)**

Octal – Hexadecimal Conversion

- Convert to Binary as an intermediate step

Example:



Works both ways (*Octal* to *Hex* & *Hex* to *Octal*)

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A

Decimal	Binary	Octal	Hex
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F