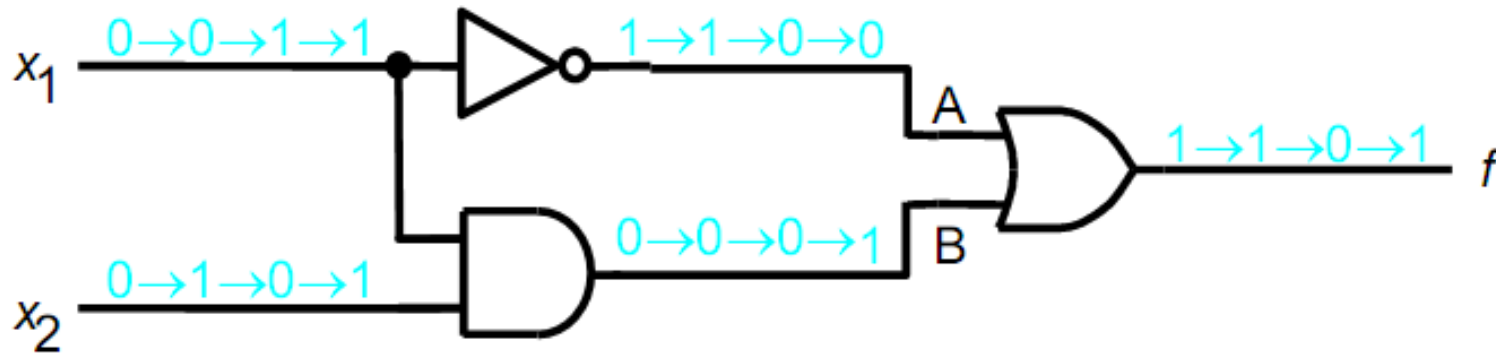


Lecture 4

Boolean algebra

Dr. BASSAM ATIEH

Timing diagram

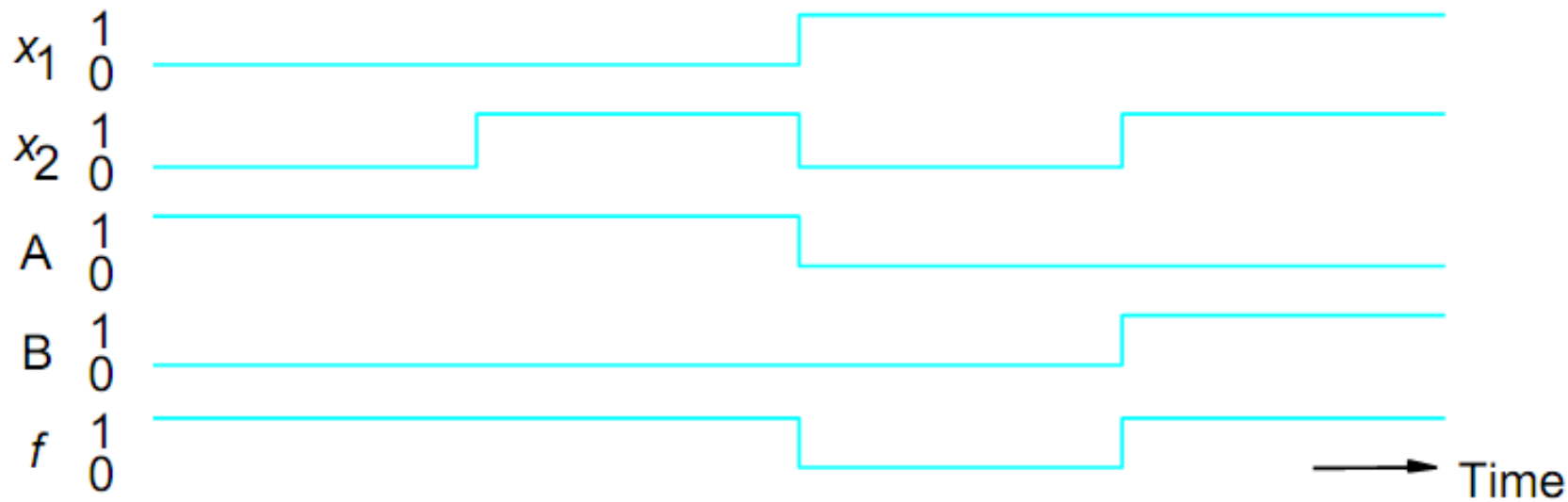


(a) Network that implements $f = x_1' + x_1 \cdot x_2$

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

A	B
1	0
1	0
0	0
0	1

(b) Truth table



(c) Timing diagram

Boolean Algebra

Axioms of Boolean Algebra

A1)

$$0 \cdot 0 = 0$$

A1')

$$1 + 1 = 1$$

A2)

$$1 \cdot 1 = 1$$

A2')

$$0 + 0 = 0$$

A3)

$$0 \cdot 1 = 1 \cdot 0 = 0$$

A3')

$$1 + 0 = 0 + 1 = 1$$

A4) if $x = 0$, then $x' = 1$

A4') if $x = 1$, then $x' = 0$

- **Single variable theorems**

$$T1) x \cdot 0 = 0$$

$$T1') x + 1 = 1$$

$$T2) x \cdot 1 = x$$

$$T2') x + 0 = x$$

$$T3) x \cdot x = x$$

$$T3') x + x = x$$

$$T4) x \cdot x' = 0$$

$$T4') x + x' = 1$$

$$T5) x'' = x$$

• Two and three variable theorems

$$T6) x \cdot y = y \cdot x$$

$$T6') x + y = y + x$$

$$T7) x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$$T7') x + (y + z) = (x + y) + z$$

$$T8) x \cdot (y + z) = x \cdot y + x \cdot z$$

$$T8') x + y \cdot z = (x + y) \cdot (x + z)$$

$$T9) x + x \cdot y = x$$

$$T9') x \cdot (x + y) = x$$

$$T10) x \cdot y + x \cdot y' = x$$

$$T10') (x + y) \cdot (x + y') = x$$

$$T11) (x \cdot y)' = x' + y'$$

$$T11') (x + y)' = x' \cdot y'$$

$$T12) x + x' \cdot y = x + y$$

$$T12') x \cdot (x' + y) = x \cdot y$$

$$T13) x \cdot y + y \cdot z + x' \cdot z = x \cdot y + x' \cdot z$$

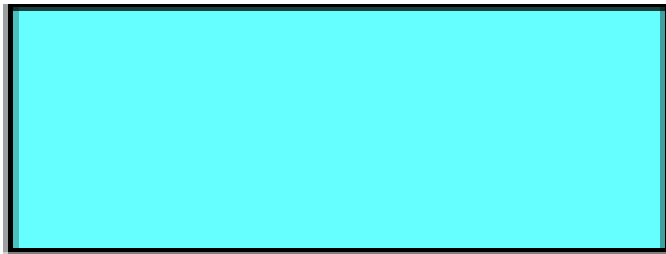
$$T13') (x + y) \cdot (y + z) \cdot (x' + z) = (x + y) \cdot (x' + z)$$

Example: Apply theorems of Boolean Algebra to prove that the left and right hand sides of the following logic equation are identical.

$$X_1 \cdot X_3' + X_2' \cdot X_3' + X_1 \cdot X_3 + X_2' \cdot X_3 = X_1' \cdot X_2' + X_1 \cdot X_2 + X_1 \cdot X_2'$$

• The Venn Diagram

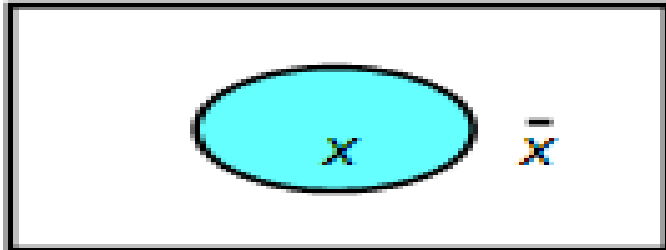
- Graphical illustration of various operations and relations in the algebra of sets
- A set s is a collection of elements that are said to be members of s
- In Venn diagram the elements of a set are represented by the area enclosed by a square, circle or ellipse
- In Boolean algebra there are only two elements in the universe, i.e. $\{0,1\}$. Then the area within a contour corresponding to a set s denotes that $s = 1$, while the area outside the contour denotes $s = 0$
- In a Venn diagram we shade the area where $s = 1$



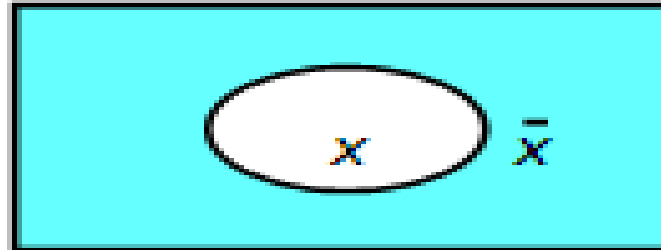
(a) Constant 1



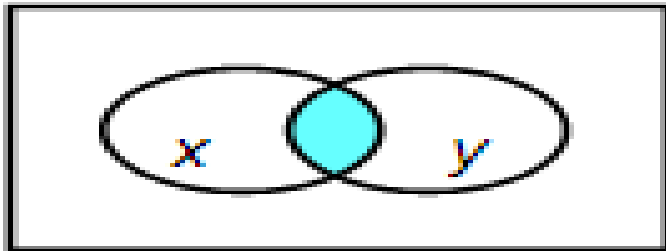
(b) Constant 0



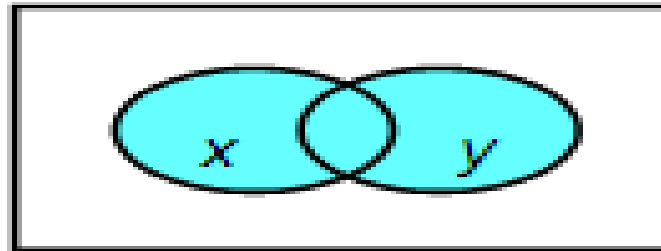
(c) Variable x



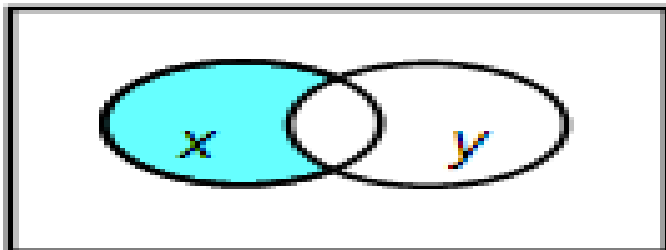
(d) \bar{x}



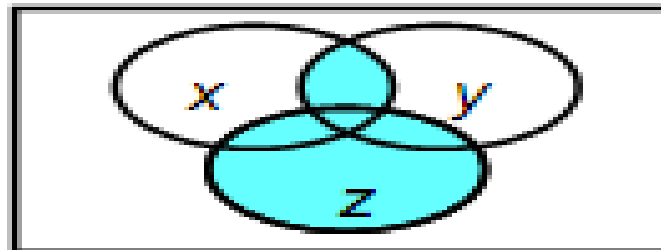
(e) $x \cdot y$



(f) $x + y$

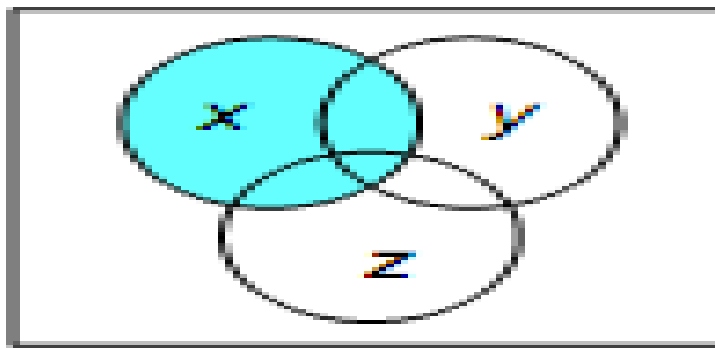


(g) $x \cdot \bar{y}$

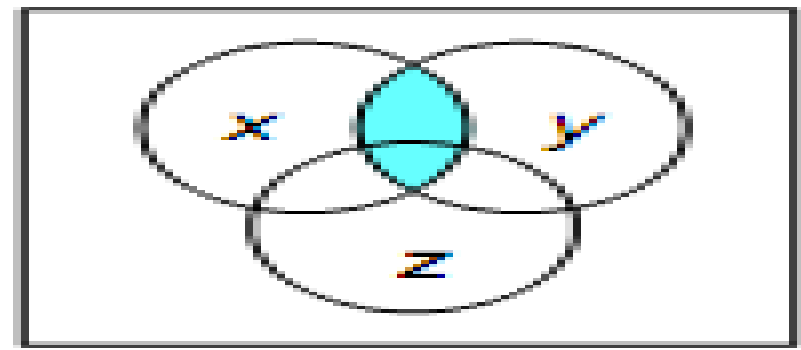


(h) $x \cdot y + z$

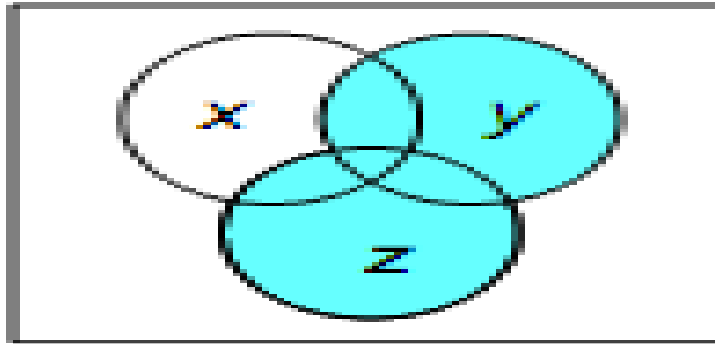
The Venn diagram representation.



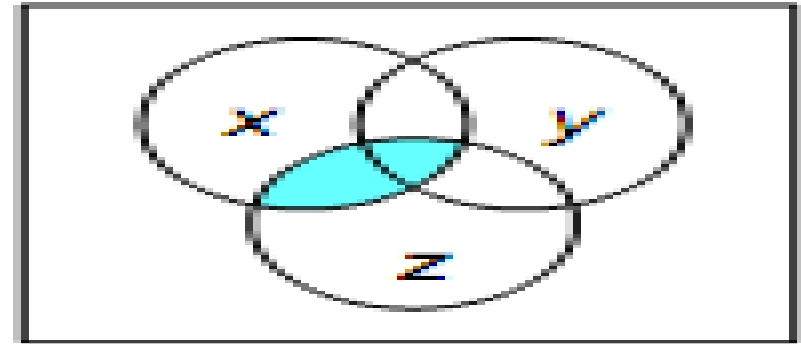
(a) x



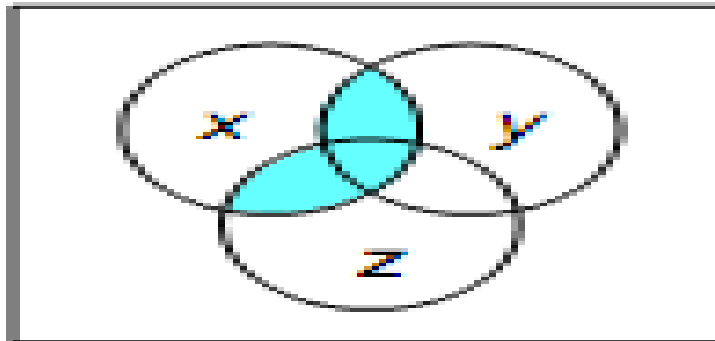
(d) $x \cdot y$



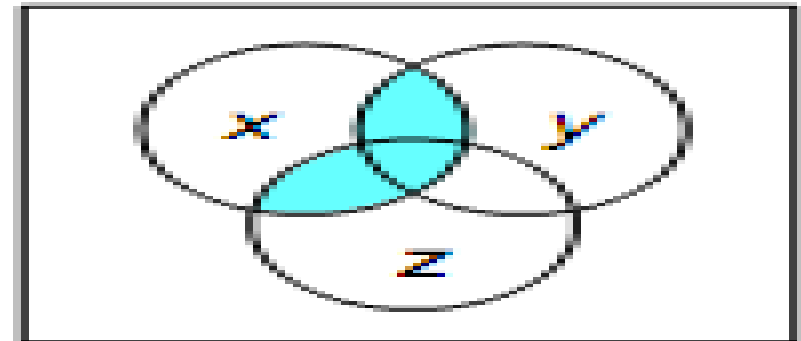
(b) $y + z$



(e) $x \cdot z$



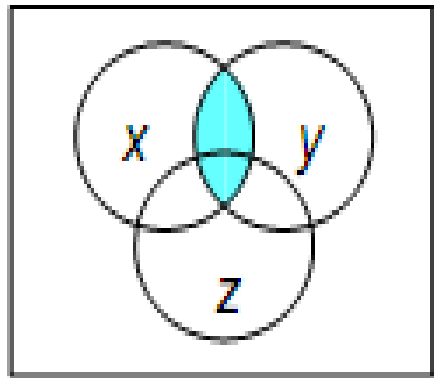
(c) $x \cdot (y + z)$



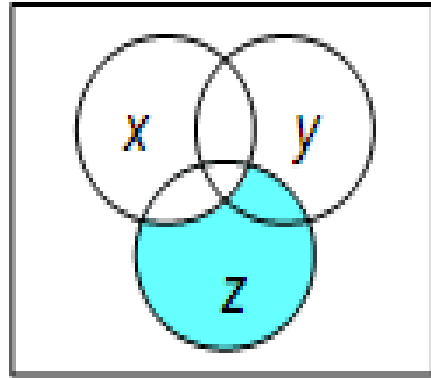
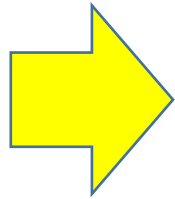
(f) $x \cdot y + x \cdot z$

$$= x \cdot y + x \cdot z$$

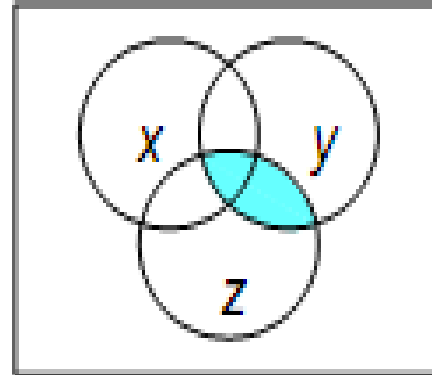
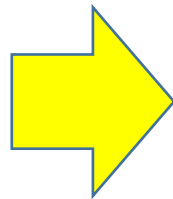
Verification of $x \cdot y + \bar{x} \cdot z + y \cdot z = x \cdot y + \bar{x} \cdot z$



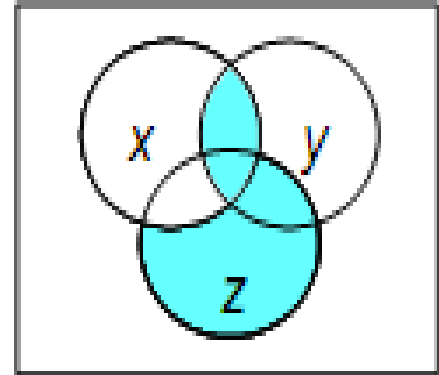
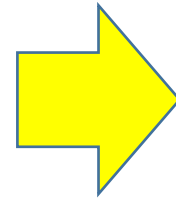
$x \cdot y$



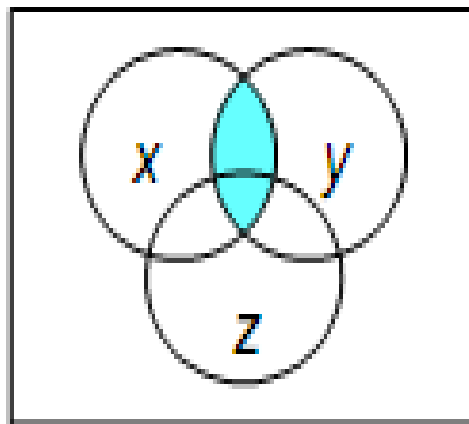
$\bar{x} \cdot z$



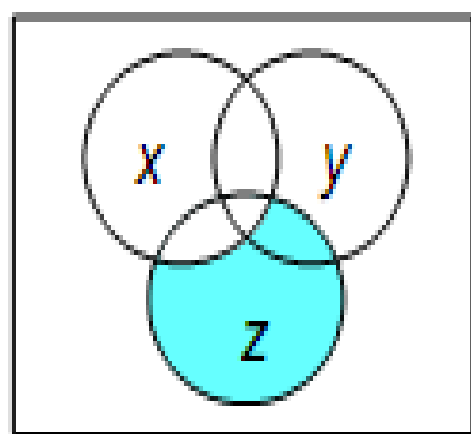
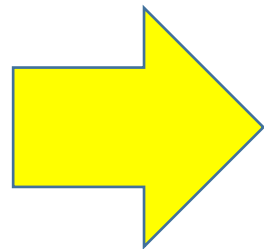
$y \cdot z$



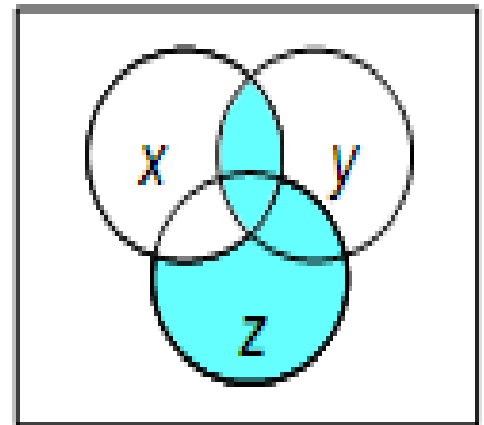
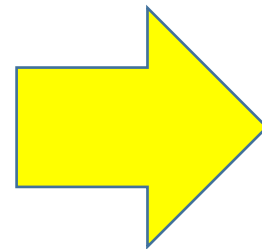
$x \cdot y + \bar{x} \cdot z + y \cdot z$



$x \cdot y$



$\bar{x} \cdot z$



$x \cdot y + \bar{x} \cdot z$

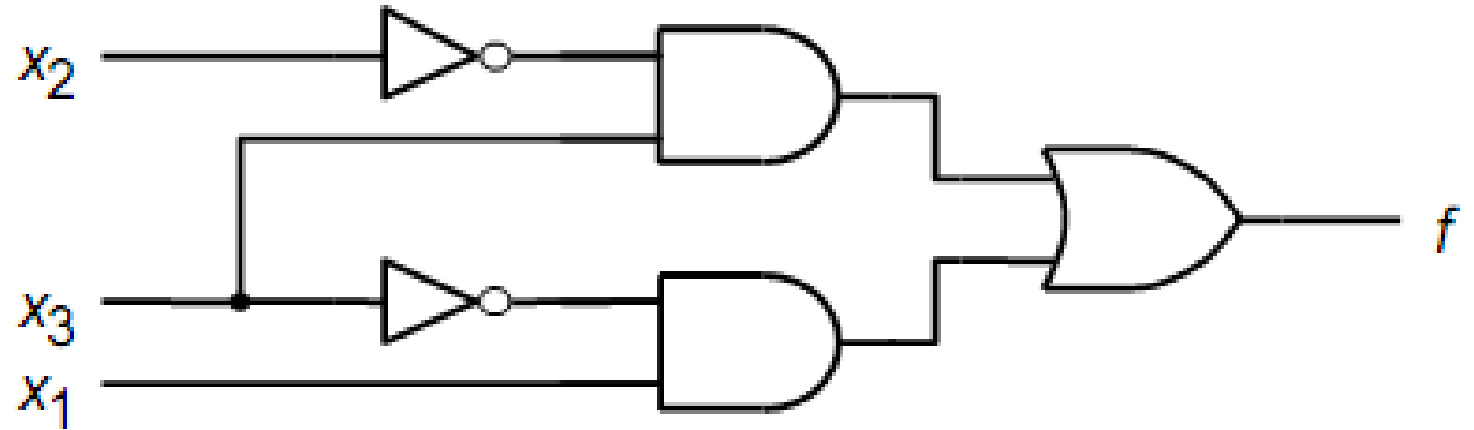
Synthesis of digital circuits

Three-variable minterms and maxterms.

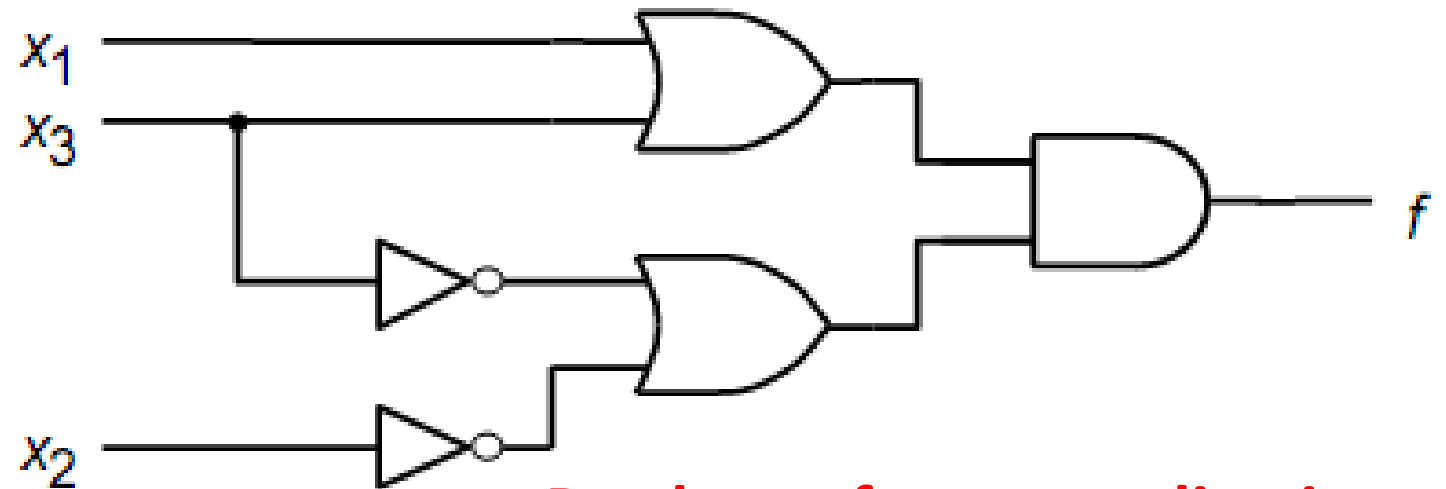
Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1 \bar{x}_2 \bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1 \bar{x}_2 x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1 x_2 \bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1 x_2 x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1 \bar{x}_2 \bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1 \bar{x}_2 x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1 x_2 \bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

Example: For the three variable function given by the following truth table, determine the minterms, maxterms, canonical SOP, canonical POS, minterm list or on-set, maxterm list or off-set, minimal SOP and minimal POS by algebraic manipulations.

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

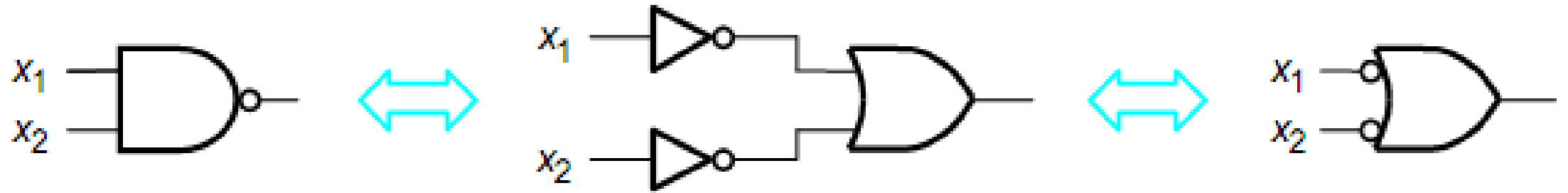


Sum-of-products realizations

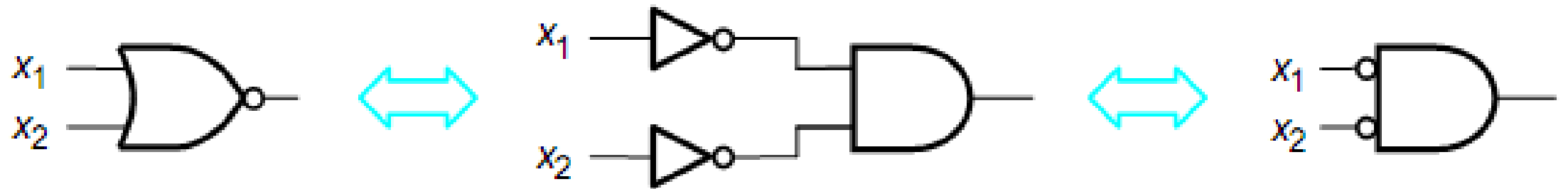


Product-of-sums realizations

DeMorgan's equivalents of NAND and NOR gates.

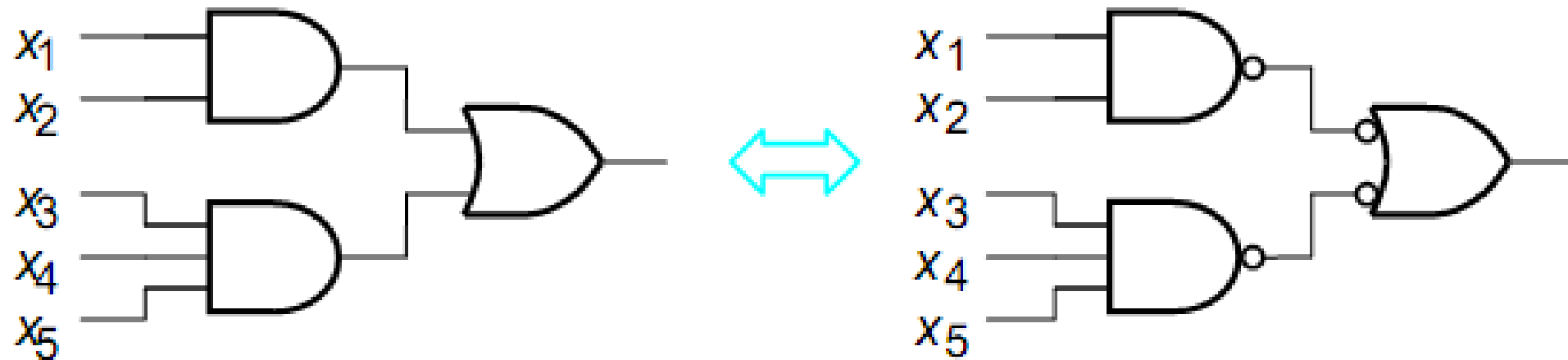


$$(a) \overline{x_1 x_2} = \bar{x}_1 + \bar{x}_2$$

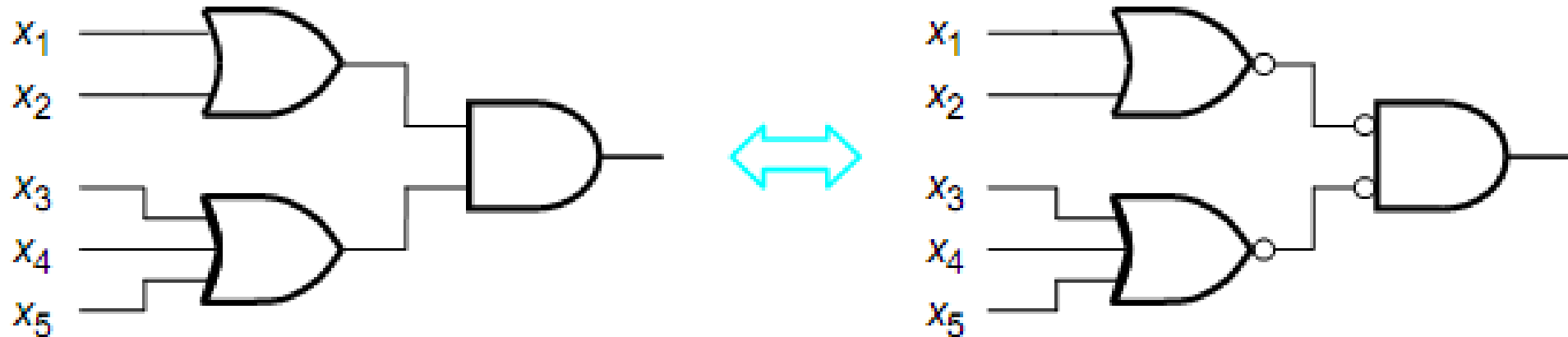


$$(b) \overline{x_1 + x_2} = \bar{x}_1 \bar{x}_2$$

- Converting a AND-OR realization of an SOP to a NAND-NAND realization



- Converting a OR-AND realization of a Product of Sums (POS) to a NOR-NOR realization



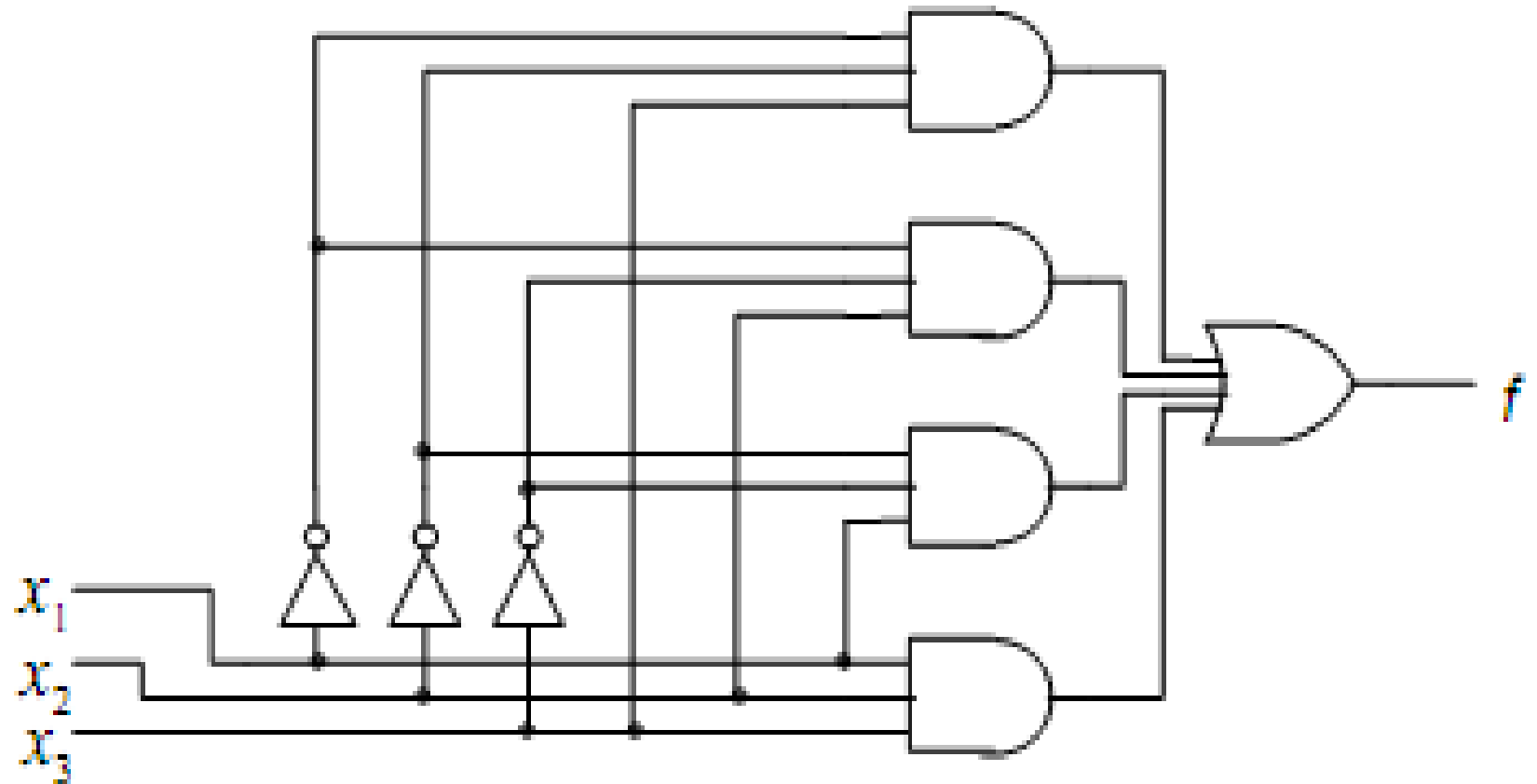
realization of an SOP

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$f(x_1, x_2, x_3) = \sum m(1, 2, 4, 7)$$

$$= (\bar{x}_1 \cdot \bar{x}_2 \cdot x_3) + (\bar{x}_1 \cdot x_2 \cdot \bar{x}_3) + (x_1 \cdot \bar{x}_2 \cdot \bar{x}_3) + (x_1 \cdot x_2 \cdot x_3)$$

Sum-of-products realizations



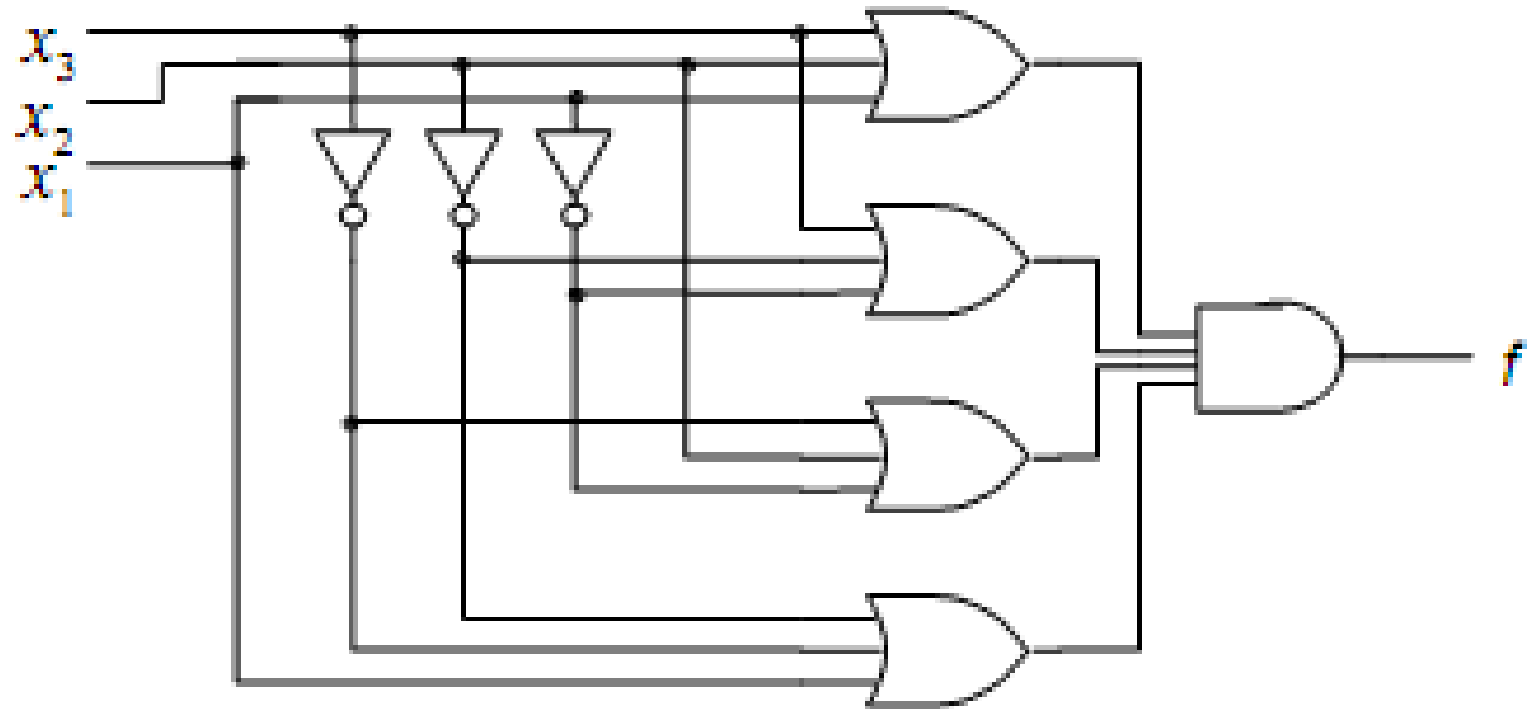
realization of an POS

$$f(x_1, x_2, x_3) = \prod M(0, 3, 5, 6)$$

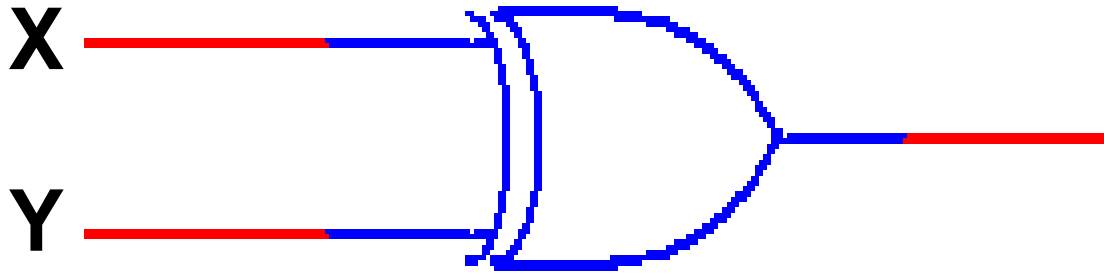
$$f(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3) \\ (\bar{x}_1 + x_2 + \bar{x}_3)(x_1 + x_2 + \bar{x}_3)$$

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Product-of-sums realizations



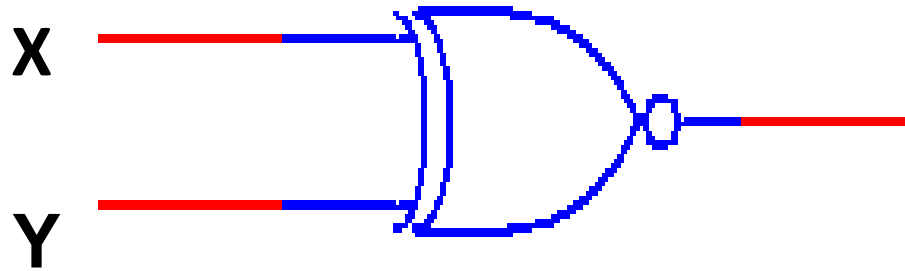
XOR Gate – Exclusive OR



$$Z = \bar{X}Y + X\bar{Y} = X \oplus Y$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

XNOR Gate – Exclusive NOR



$$Z = \overline{X} \overline{Y} + X Y = \overline{X \oplus Y}$$

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	1

Logic Design with XOR & XNOR

Example

Algebraically manipulate the logic expression for F_1 so that XOR and XNOR gates can be used to implement the function. Other AOI gates can be used as needed.

$$F_1 = X\bar{Y}Z + \bar{X}YZ + \bar{X}\bar{Y}\bar{Z} + X\bar{Y}\bar{Z}$$

Solution

$$F_1 = X \bar{Y} Z + \bar{X} Y Z + \bar{X} \bar{Y} \bar{Z} + X \bar{Y} Z$$

$$F_1 = Z (X \bar{Y} + \bar{X} Y) + \bar{Y} (\bar{X} \bar{Z} + X Z)$$

$$F_1 = Z (X \oplus Y) + \bar{Y} (\overline{X \oplus Z})$$

