Financial Derivatives

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First Term

2021-2022

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Basics of derivative pricing and valuation

Here the focus is on the pricing and valuation of derivatives based on a no-arbitrage condition. The derivation of the price in a forward contract and calculating the value of a forward contract over its life are important applications of no-arbitrage pricing. We should also understand the equivalence of interest rates swaps to a series of forward rate agreements and how each factor that affects option values affects puts and calls.

Explain how the concepts of arbitrage, replication, and risk neutrality are used in pricing derivatives

For most risky assets, we estimate current value as the discounted present value of the expected price of the asset at some future time. Because the future price is subject to risk (uncertainty), the discount rate includes a risk premium along with the risk-free rate.

There may be costs of owning an asset, such as storage and insurance costs. For financial assets, these costs are very low and not significant. The other important cost of holding an asset is the opportunity cost of the funds that are invested in the asset, which we usually measure as the asset cost times the risk-free rate, compounded over the holding period.

There may also be benefits to holding the asset, either monetary or non-monetary. Dividend payments on a stock or interest payments on a bond are examples of monetary benefits of owning an asset. Non-monetary benefits of holding an asset are sometimes referred to as its **convenience yield.** The convenience yield is difficult to measure and is only significant for some assets, primarily commodities.

If an asset is difficult to sell short in the market, owning it may convey benefits in circumstances where selling the asset is advantageous. For example, a shortage of the asset may drive prices up, making sale of the asset in the short term profitable. While the ability to look at a painting or sculpture provides non-monetary benefits to its owner, this is unlikely with corn or other commodities.

The net cost of holding an asset, considering both the costs and benefits of holding the asset, is referred to as the **cost of carry.** Taking into account all of these costs and benefits, we can describe the present value of an asset, based on its expected future price, as:

$S_0 = E(S_T) / (1 + Rf + \text{risk premium})^T$ + PV (benefits of holding the asset for time T) – PV (costs of holding the asset for time T),

where:

S_0 is the current spot price of the asset and $E(S_T)$ is the expected value of the asset at time T, the end of the expected holding period.

We assume that investors are **risk-averse** so they require a positive premium (higher return) on risky assets. An investor who is **risk-neutral** would require no risk premium and, as a result, would discount the expected future value of an asset at the risk-free rate.

In contrast to this model of calculating the current value of a risky asset, the valuation of derivative securities is based on a **no-arbitrage** condition. Arbitrage refers to a transaction wherein an investor purchases one asset (or portfolio of assets) at one price and simultaneously sells an asset or portfolio that has the same future payoff, regardless of future events, at a higher price, realizing a risk-free gain on the transaction.

While arbitrage opportunities may be rare, the reasoning is that when they do exist they will be exploited rapidly. Therefore, we can use the no-arbitrage condition to determine the current value (spot price) of an asset or portfolio of assets that have the same future payoffs regardless of future events. Because there are transactions costs of exploiting an arbitrage opportunity, small differences in price may persist because the arbitrage gain is less than the transactions cost of exploiting it.

In markets for traditional securities, we don’t often encounter two assets that have the same future payoffs. With derivative securities, however, the risk of the derivative is entirely based on the risk of the underlying asset, so we can construct a portfolio of the underlying asset and a derivative based on it that has no uncertainty about its value at some future date (i.e., a hedged portfolio).

Because the future payoff is certain, we can calculate the present value of the portfolio as the future payoff discounted at the risk-free rate. This will be the current value of the portfolio under the no-arbitrage condition, which will force the return on a risk-free (hedged) portfolio to the risk-free rate. This structure, with a long position in the asset and a short position in the derivative security, can be represented as:

asset position at time 0 + short position in a forward contract at time 0 = (payoff on the asset at time T + payoff on the short forward at time T) / $(1 + Rf)^T$

Because the payoff at time *T* (expiration of the forward) is from a fully hedged position, its time *T* value is certain. To prevent arbitrage, the above equality must hold. If the net cost of buying the asset and selling the forward at time *t* is less than the present value (discounted at Rf) of the certain payoff at time *T*, an investor can borrow the funds (at Rf) to buy the asset, sell the forward at time *t*, and earn a risk-free return in excess of Rf.

If the net cost is greater than the present value of the certain payoff at time *T*, an arbitrageur could sell the hedged position (short the asset, invest the proceeds at Rf, and buy the forward). At expiration, the asset can be purchased with the maturity payment on the loan and the excess of that repayment over the forward price is a gain with no net investment over the period.

When the equality holds we say the derivative is currently at its no-arbitrage price. Because we know Rf, the spot price of the asset, and the certain payoff at time *T*, we can solve for the no-arbitrage price of the derivative based on the no arbitrage price of the forward.

Note the investor's risk aversion has not entered into our valuation of the derivative as it did when we described the valuation of a risky asset. For this reason, the determination of the no-arbitrage derivative price is sometimes called **risk-neutral pricing,** which is the same as no-arbitrage pricing or the price under a no-arbitrage condition.

Because we can create a risk-free asset (or portfolio) from a position in the underlying asset that is hedged with a position in a derivative security, we can duplicate the payoff on a derivative position with the risk-free asset and the underlying asset or duplicate the payoffs on the underlying asset with a position in the risk-free asset and the derivative security. This process is called **replication** because we are replicating the payoffs on one asset or portfolio with those of a different asset or portfolio.

As an example of replication and risk-neutral pricing, consider a long position in a stock and a short position in a forward contract at 50 on the stock. Regardless of the price of the stock at the settlement of the forward contract, the stock will be delivered for the forward price of 50. As 50 will be received at the forward settlement date, the value today is 50 discounted at the risk-free rate for the time until settlement of the forward contract. For a share of stock and a short forward at 50 with six months until settlement, we can write:

$S – F(50) = 50/(1 + Rf)^{0.5}$

and replicate a long forward position as

$F(50) = S – 50/(1 + Rf)^{0.5}$

That is, we can replicate the long forward position by purchasing a share of stock and borrowing the present value of 50 at the risk-free rate so the value at the maturity of the loan will be the stock price minus 50. Alternatively, we could replicate a short forward position by selling a share of stock short and lending the present value of 50 at the risk-free rate.

Another example of risk-neutral pricing is that combining a risky bond with a credit protection derivative replicates a risk-free bond. So we can write:

risky bond + credit protection = bond valued at the risk-free rate

and see that the no-arbitrage price of credit protection is the value of the bond if it were risk-free minus the price of the risky bond.

As a final example of risk-neutral pricing and replication, consider an investor who buys a share of stock, sells a call on the stock at 40, and buys a put on the stock at 40 with the same expiration date as the call. The investor will receive 40 at option expiration regardless of the stock price because:

- If the stock price is 40 at expiration, the put and the call are both worthless at expiration.
- If the stock price > 40 at expiration, the call will be exercised, the stock will be delivered for 40, and the put will expire worthless.
- If the stock price is < 40 at expiration, the put will be exercised, the stock will be delivered for 40, and the call will expire worthless.

Thus, for a six-month call and put we can write:

- stock + put – call = $40/(1+Rf)^{0.5}$ and equivalently
- call = stock + put – $40/(1+Rf)^{0.5}$ and
- put = call + $40/(1+Rf)^{0.5}$ – stock

These replications will be introduced later in this reading as the *put-call parity* relationship.

Distinguish between value and price of forward and futures contracts

Recall that the *value* of futures and forward contracts is zero at initiation. As the expected future price of the underlying asset changes, the value of the futures or forward contract position may increase or decrease with the gains or losses in value of the long position in the contract just opposite to the gains or losses in the short position on the contract.

In contrast to the value of a futures or forward position, the *price* of a futures or forward contract refers to the futures or forward price specified in the contract.

As an example of this difference, consider a long position in a forward contract to buy the underlying asset in the future at $50, which is the forward contract price. At initiation of the contract, the value is zero but the contract price is $50. If the expected future value of the underlying asset increases, the value of the long contract position will increase (and the value of the short position will decrease by a like amount).

The contract *price* at which the long forward can purchase the asset in the future remains the same. If a new forward contract were now created it would have a zero value, but a higher forward price that reflects the higher expected future value of the underlying asset.