Explain how the value and price of a forward contract are determined at expiration, during the life of the contract, and at initiation.

Because neither party to a forward transaction pays to enter the contract at initiation, the forward contract price must be set so the contract has zero value at initiation. To understand how this price is set, consider an asset that has no storage costs and no benefits to holding it so that the net cost of carry is simply the opportunity cost of the invested funds, which we assume to be the risk-free rate.

Under these conditions the current forward price of an asset to be delivered at time T, $F_0(T)$, must equal the spot price of the asset, S_0, compounded at the risk-free rate for a period of length T (in years) and we can write:

$$F_0(T) = S_0 (1 + Rf)^T \text{ which is equivalent to: } \frac{F_0(T)}{S_0} = (1 + R_f)^T$$

If the forward price were $F_0(T)+$, a price greater than $S_0 (1 + Rf)^T$, an arbitrageur could take a short position in the forward contract, promising to sell the asset at time T at $F_0(T)+$, and buy the asset at S_0, with funds borrowed at Rf, which requires no cash investment in the position. At time T, the arbitrageur would deliver the asset and receive $F_0(T)+$, repay the loan at a cost of $S_0(1 + Rf)^T$, and keep the positive difference between $F_0(T)+$ and $S_0(1 + Rf)^T$.

If the forward price were $F_0(T)–$, a price less than $S_0(1 + Rf)^T$, a profit could be earned with the opposite transactions, short selling the asset for S_0, investing the proceeds at Rf, and taking a long position in the forward. At time T, the arbitrageur would receive $S_0(1 + Rf)^T$ from investing the proceeds of the short sale, pay $F_0(T)–$ to purchase the asset and cover the short asset position, and keep the difference between $S_0 (1 + Rf)^T$ and $F_0(T)–$. This process is the mechanism that ensures $F_0(T)$ is the (no-arbitrage) price in a forward contract that has zero value at T = 0.

When the forward is priced at its no-arbitrage price the value of the forward at initiation,

$$V_0(T) = S_0 - \frac{F_0(T)}{(1 + R_f)^T} = 0, because\ S_0 = \frac{F_0(T)}{(1 + R_f)^T}$$

During its life, at time t < T, the value of the forward contract is the spot price of the asset minus the present value of the forward price,

$$V_t(T) = S_t - \frac{F_0(T)}{(1 + R_f)^{T-t}}$$

At expiration at time *T*, the discounting term is $(1 + Rf)^0 = 1$ and the payoff to a long forward is $S_T – F_0(T)$, the difference between the spot price of the asset at expiration and the price of the forward contract.

Describe monetary and nonmonetary benefits and costs associated with holding the underlying asset and explain how they affect the value and price of a forward contract.

We can denote the present value of any costs of holding the asset from time 0 to expiration at time *T* as PV_0 (cost) and the present value of any cash flows from the asset and any convenience yield over the holding period as PV_0 (benefit).

Consider first a case where there are costs of holding the asset but no benefits. The asset can be purchased now and held to time *T* at a total cost of $[S_0 + PV_0$ (cost)] $(1 + Rf)^T$, so this is the no-arbitrage forward price. Any other forward price will create an arbitrage opportunity at the initiation of the forward contract.

In a case where there are only benefits of holding the asset over the life of the forward contract, the cost of buying the asset and holding it until the expiration of the forward at time *T* is $[S_0 – PV_0$ (benefit)] $(1 + Rf)^T$. Again, any forward price that is not the no-arbitrage forward price will create an arbitrage opportunity. Note the no-arbitrage forward price is lower the greater the present value of the benefits and higher the greater the present value of the costs incurred over the life of the forward contract.

If an asset has both storage costs and benefits from holding the asset over the life of the forward contract, we can combine these into a more general formula and express the no-arbitrage forward price (that will produce a value of zero for the forward at initiation) as:

$$[S_0 + PV_0\ (cost) - PV_0\ (benefits)](1 + R_f)^T = F_0(T)$$

Both the present values of the costs of holding the asset and the benefits of holding the asset decrease as time passes and the time to expiration (T – t) decreases, so the value of the forward at any point in time $t < T$ is:

$$V_t(T) = S_t + PV_t\ (cost) - PV_t\ (benefits) - \frac{F_0(T)}{(1 + R_f)^{T-t}}$$

At expiration $t = T$ the costs and benefits of holding the asset until expiration are zero, as is $T - t$, so that the payoff on a long forward position at time T is, again, simply $S_T - F_0(T)$, the difference between the spot price of the asset at expiration and the forward price.

Define a forward rate agreement and describe its uses.

A **forward rate agreement** (FRA) is a derivative contract that has a future interest rate, rather than an asset, as its underlying. The point of entering into an FRA is to lock in a certain interest rate for borrowing or lending at some future date. One party will pay the other party the difference (based on an agreed-upon notional contract value) between the fixed interest rate specified in the FRA and the market interest rate at contract settlement.

Libor is most often used as the underlying rate. U.S. dollar Libor refers to the rates on Eurodollar time deposits, interbank U.S. dollar loans in London.

Consider an FRA that will, in 30 days, pay the difference between 90-day Libor and the 90-day rate specified in the FRA (the contract rate). A company that expects to borrow 90-day funds in 30 days will have higher interest costs if 90-day Libor 30 days from now increases. A long position in the FRA (pay fixed, receive floating) will receive a payment that will offset the increase in borrowing costs from the increase in 90-day Libor.

Conversely, if 90-day Libor 30 days from now decreases over the next 30 days, the long position in the FRA will make a payment to the short in the amount that the company's borrowing costs have decreased relative to the FRA contract rate.

FRAs are used by firms to hedge the risk of (remove uncertainty about) borrowing and lending they intend to do in the future. A company that intends to borrow funds in 30 days could take a long position in an FRA, receiving a payment if future 90-day Libor (and its borrowing cost) increases, and making a payment if future 90-day Libor (and its borrowing cost) decreases, over the 30-day life of the FRA. Note a perfect hedge means not only that the firm's borrowing costs will not be higher if rates increase, but also that the firm's borrowing costs will not be lower if interest rates decrease.

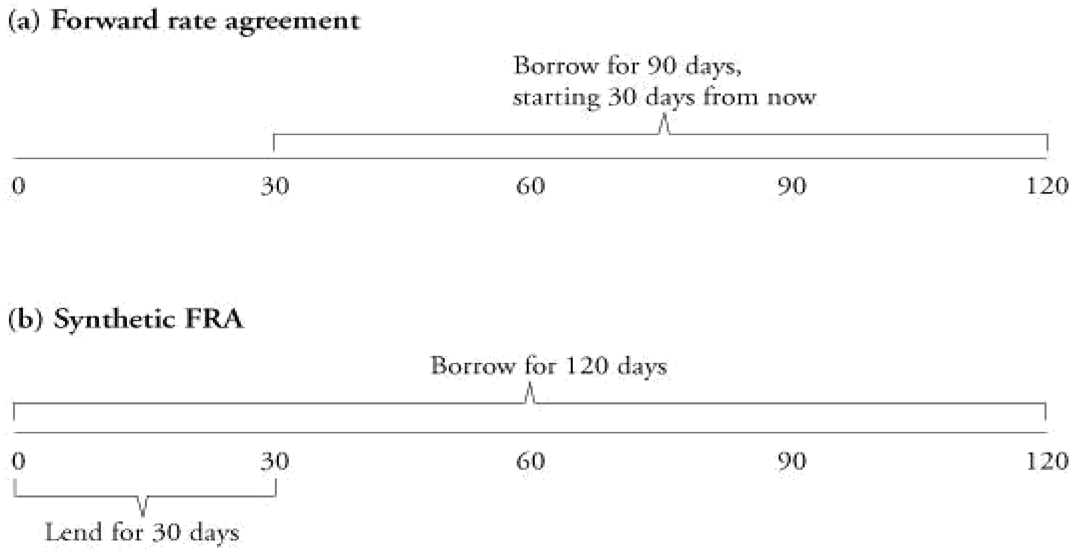
For a firm that intends to have funds to lend (invest) in the future, a short position in an FRA can hedge its interest rate risk. In this case, a decline in rates would decrease the return on funds loaned at the future date, but a positive payoff on the FRA would augment these returns so that the return from both the short FRA and loaning the funds is the no-arbitrage rate that is the *price* of the FRA at initiation.

Rather than enter into an FRA, a bank can create the same payment structure with two Libor loans, a **synthetic FRA**. A bank can borrow money for 120 days and lend that amount for 30 days. At the end of 30 days, the bank receives funds from the repayment of the 30-day loan it made, and has use of these funds for the next

90 days at an effective rate determined by the original transactions. The effective rate of interest on this 90-day loan depends on both 30-day Libor and 120-day Libor at the time the money is borrowed and loaned to the third party. This rate is the contract rate on a 30-day FRA on 90-day Libor. The resulting cash flows will be the same with either the FRA or the synthetic FRA.

Figure 1 illustrates these two methods of "locking in" a future lending or borrowing rate (i.e., hedging the risk from uncertainty about future interest rates).

Figure 1: 30-day FRA on 90-day Libor



Note that the no-arbitrage price of an FRA is determined by the two transactions in the synthetic FRA, borrowing for 120 days and lending for 30 days.