Explain why forward and futures prices differ

Forwards and futures serve the same function in gaining exposure to or hedging specific risks, but differ in their degree of standardization, liquidity, and, in many instances, counterparty risk. From a pricing and valuation perspective, the most important distinction is that futures gains and losses are settled each day and the margin balance is adjusted accordingly.

If gains put the margin balance above the initial margin level, any funds in excess of that level can be withdrawn. If losses put the margin value below the minimum margin level, funds must be deposited to restore the account margin to its initial (required) level. Forwards, typically, do not require or provide funds in response to fluctuations in value during their lives.

While this difference is theoretically important in some contexts, in practice it does not lead to any difference between the prices of forwards and futures that have the same terms otherwise. If interest rates are constant, or even simply uncorrelated with futures prices, the prices of futures and forwards are the same. A positive correlation between interest rates and the futures price means that (for a long position) daily settlement provides funds (excess margin) when rates are high and they can earn more interest, and requires funds (margin deposits) when rates are low and opportunity cost of deposited funds is less. Because of this, futures prices will be higher than forward prices when interest rates and futures prices are positively correlated, and they will be lower than forward prices when interest rates and futures prices are negatively correlated.

Explain the exercise value, time value, and moneyness of an option.

Moneyness refers to whether an option is *in the money or out of the money*. If immediate exercise of the option would generate a positive payoff, it is in the money. If immediate exercise would result in a loss (negative payoff), it is out of the money. When the current asset price equals the exercise price, exercise will generate neither a gain nor loss, and the option is *at the money*.

The following describes the conditions for a **call option** to be in, out of, or at the money. S is the price of the underlying asset and X is the exercise price of the option.

- *In-the-money call options.* If S – X > 0, a call option is in the money. S – X is the amount of the payoff a call holder would receive from immediate exercise, buying a share for *X* and selling it in the market for a greater price *S*.
- *Out-of-the-money call options.* If S – X < 0, a call option is out of the money.
- *At-the-money call options*. If S = X, a call option is said to be at the money.

The following describes the conditions for a **put option** to be in, out of, or at the money.

- *In-the-money put options.* If X – S > 0, a put option is in the money. X – S is the amount of the payoff from immediate exercise, buying a share for *S* and exercising the put to receive *X* for the share.
- *Out-of-the-money put options.* When the stock's price is greater than the strike price, a put option is said to be out of the money. If X – S < 0, a put option is out of the money.
- *At-the-money put options*. If S = X, a put option is said to be at the money.

We define the **intrinsic value** (or **exercise value**) of an option the maximum of zero and the amount that the option is in the money. That is, the intrinsic value is the amount an option is in the money, if it is in the money, or zero if the option is at or out of the money. The intrinsic value is also the exercise value, the value of the option if exercised immediately.

Prior to expiration, an option has time value in addition to any intrinsic value. The **time value** of an option is the amount by which the **option premium** (price) exceeds the intrinsic value and is sometimes called the *speculative value* of the option. This relationship can be written as:

$$option\ premium\ =\ intrinsic\ value\ +\ time\ value$$

At any point during the life of an option, its value will typically be greater than its intrinsic value. This is because there is some probability that the underlying asset price will change in an amount that gives the option a positive payoff at expiration greater than the (current) intrinsic value. Recall that an option's intrinsic value (to a buyer) is the amount of the payoff at expiration and is bounded by zero.

When an option reaches expiration, there is no time remaining and the time value is zero. This means the value at expiration is either zero, if the option is at or out of the money, or its intrinsic value, if it is in the money.

Identify the factors that determine the value of an option and explain how each factor affects the value of an option.

There are six factors that determine option prices.

1. **Price of the underlying asset.** For call options, the higher the price of the underlying, the greater its intrinsic value and the higher the value of the option. Conversely, the lower the price of the underlying, the less its intrinsic value and the lower the value of the call option.
2. **The exercise price.** A higher exercise price decreases the values of call options and a lower exercise price increases the values of call options.
   A higher exercise price increases the values of put options and a lower exercise price decreases the values of put options.
3. **The risk-free rate of interest.** An increase in the risk-free rate will increase call option values, and a decrease in the risk-free rate will decrease call option values.

An increase in the risk-free rate will decrease put option values, and a decrease in the risk-free rate will increase put option values.

4. **Volatility of the underlying.** Volatility is what makes options valuable. If there were no volatility in the price of the underlying asset (its price remained constant), options would always be equal to their intrinsic values and time or speculative value would be zero. An increase in the volatility of the price of the underlying asset increases the values of both put and call options and a decrease in volatility of the price of the underlying decreases both put values and call values.
5. **Time to expiration.** Because volatility is expressed per unit of time, longer time to expiration effectively increases expected volatility and increases the value of a call option. Less time to expiration decreases the time value of a call option so that at expiration its value is simply its intrinsic value. For most put options, longer time to expiration will increase option values for the same reasons.
6. **Costs and benefits of holding the asset.** If there are benefits of holding the underlying asset (dividend or interest payments on securities or a convenience yield on commodities), call values are decreased and put values are increased. The reason for this is most easily understood by considering cash benefits. When a stock pays a dividend, or a bond pays interest, this reduces the value of the asset. Decreases in the value of the underlying asset decrease call values and increase put values.

Explain put–call parity for European options.

Our derivation of **put-call parity** for European options is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a call with exercise price *X* and a pure-discount, riskless bond that pays *X* at maturity (option expiration). The payoff for a fiduciary call at expiration is *X* when the call is out of the money, and X + (S – X) = S when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is (X – S) + S = X when the put is in the money, and *S* when the put is out of the money.

If at expiration S is greater than or equal to X:

- The protective put pays S on the stock while the put expires worthless, so the payoff is S.
- The fiduciary call pays X on the bond portion while the call pays (S – X), so the payoff is X + (S – X) = S.

If at expiration X is greater than S:

- The protective put pays S on the stock while the put pays (X – S), so the payoff is S + (X – S) = X.
- The fiduciary call pays X on the bond portion while the call expires worthless, so the payoff is X.

In either case, the payoff on a protective put is the same as the payoff on a fiduciary call. Our no-arbitrage condition holds that portfolios with identical payoffs regardless of future conditions must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + X / (1 + Rf)^T = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$S = c - p + X / (1 + Rf)^T$$

$$p = c - S + X / (1 + Rf)^T$$

$$c = S + p - X / (1 + Rf)^T$$

$$X / (1 + Rf)^T = S + p - c$$

Example: Call option valuation using put-call parity

Suppose that the current stock price is $52, and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of $50. The put price is $1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, $50 call. Estimate the price of the 3-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

Call = put + stock – present value (X)

Call = $1.50 + $52 - $\frac{\$50}{1.05^{0.25}}$ = $4.11

This means that if a 3-month, $50 call is available, it should be priced at (within transactions costs of) $4.11 per share.