1. Introduction

Imagine you are the Chief Risk Officer (CRO) of a major corporation. The Chief Executive Officer (CEO) wants your views on a major new venture. You have been inundated with reports showing that the new venture has a positive net present value and will enhance shareholder value. What sort of analysis and ideas is the CEO looking for from you?

As CRO it is your job to consider how the new venture fits into the company's portfolio. What is the correlation of the performance of the new venture with the rest of the company's business? When the rest of the business is experiencing difficulties, will the new venture also provide poor returns, or will it have the effect of dampening the ups and downs in the rest of the business?

Companies must take risks if they are to survive and prosper. The risk management function's primary responsibility is to understand the portfolio of risks that the company is currently taking and the risks it plans to take in the future. It must decide whether the risks are acceptable and, if they are not acceptable, what action should be taken.

Risk management has become progressively more important for all corporations in the last few decades. Financial institutions particularly are finding they have to increase the resources they devote to risk management. Large "rogue trader" losses such as those at Barings Bank in 1995, Allied Irish Bank in 2002, Société's´ Générale´ in 2007, and UBS in 2011 would have been avoided if procedures used by the banks for collecting data on trading positions had been more carefully developed. Huge subprime losses at banks such as Citigroup, UBS, and Merrill Lynch would have been less severe if risk management groups had been able to convince senior management that unacceptable risks were being taken.

Table 1.1: One-Year Return from Investing $100,000 in a Stock

Probability	Return
0.05	+50%
0.25	+30%
0.40	+10%
0.25	−10%
0.05	−30%

This chapter sets the scene. It starts by reviewing the classical arguments concerning the risk-return trade-offs faced by an investor who is choosing a portfolio of stocks and bonds. It then considers whether the same arguments can be used by a company in choosing new projects and managing its risk exposure. The chapter concludes that there are reasons why companies—particularly financial institutions—should be concerned with the total risk they face, not just with the risk from the viewpoint of a well-diversified shareholder.

1.1 Risk vs. Return for Investors

As all fund managers know, there is a trade-off between risk and return when money is invested. The greater the risks taken, the higher the return that can be realized. The trade-off is actually between risk and *expected return*, not between risk and actual return. The term "expected return" sometimes causes confusion. In everyday language an outcome that is "expected" is considered highly likely to occur. However, statisticians define the expected value of a variable as its average (or mean) value. Expected return is therefore a weighted average of the possible returns, where the weight applied to a particular return equals the probability of that return occurring. The possible returns and their probabilities can be either estimated from historical data or assessed subjectively.

Suppose, for example, that you have $100,000 to invest for one year. Suppose further that Treasury bills yield 5%. One alternative is to buy Treasury bills. There is then no risk and the expected return is 5%. Another alternative is to invest the $100,000 in a stock. To simplify things, we suppose that the possible outcomes

from this investment are as shown in Table 1.1. There is a 0.05 probability that the return will be +50%; there is a 0.25 probability that the return will be +30%; and so on. Expressing the returns in decimal form, the expected return per year is:

$$0.05 \times 0.50 + 0.25 \times 0.30 + 0.40 \times 0.10 + 0.25 \times (-0.10) + 0.05 \times (-0.30) = 0.10$$

This shows that, in return for taking some risk, you are able to increase your expected return per annum from the 5% offered by Treasury bills to 10%. If things work out well, your return per annum could be as high as 50%. But the worst-case outcome is a −30% return or a loss of $30,000.

1.1.1 Quantifying Risk

How do you quantify the risk you are taking when you choose an investment? A convenient measure that is often used is the standard deviation of the return over one year. This is:

$$\sqrt{E(R^2) - [E(R)]2}$$

where R is the return per annum. The symbol E denotes expected value so that $E(R)$ is the expected return per annum. In Table 1.1, as we have shown, $E(R) = 0.10$. To calculate $E(R^2)$ we must weight the alternative squared returns by their probabilities:

$$E(R^2) = 0.05 \times (0.50)^2 + 0.25 \times (0.30)^2 + 0.40 \times (0.10)^2 + 0.25 \times (-0.10)^2 + 0.05 \times (-0.30)^2 = 0.046$$

The standard deviation of the annual return is therefore:

$$\sqrt{0.046 - (0.1)^2} = 0.1897 \text{ or } 18.97\%.$$

1.1.2 Investment Opportunities

Suppose we choose to characterize every investment opportunity by its expected return and standard deviation of return. We can plot available risky investments on a chart such as Figure 1.1, where the horizontal axis is the standard deviation of the return and the vertical axis is the expected return.

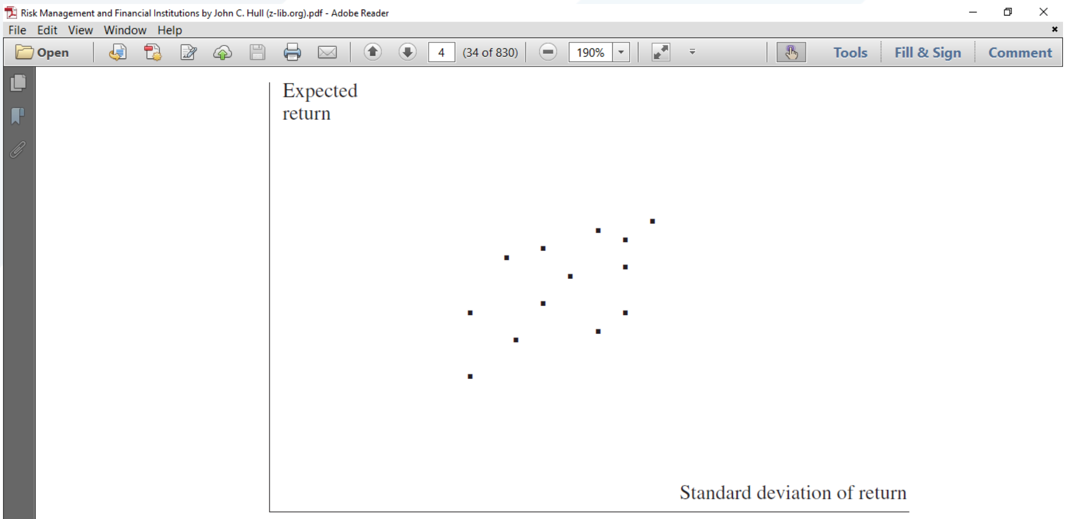


Figure 1.1 Alternative Risky Investments

Once we have identified the expected return and the standard deviation of the return for individual investments, it is natural to think about what happens when we combine investments to form a portfolio.

Consider two investments with returns R_1 and R_2. The return from putting a proportion w_1 of our money in the first investment and a proportion $w_2 = 1 - w_1$ in the second investment is:

$$w_1R_1 + w_2R_2$$

The portfolio expected return is:

$$\mu_P = w_1\mu_1 + w_2\mu_2$$

where μ_1 is the expected return from the first investment and μ_2 is the expected return from the second investment. The standard deviation of the portfolio return is given by:

$$\sigma_P = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2$$

where σ_1 and σ_2 are the standard deviations of R_1 and R_2 and ρ is the coefficient of correlation between the two.

Suppose that μ_1 is 10% per annum and σ_1 is 16% per annum, while μ_2 is 15% per annum and σ_2 is 24% per annum. Suppose also that the coefficient of correlation (ρ) between the returns is 0.2 or 20%. Table 1.2 shows the values of μ_P and σ_P for a number of different values of w_1 and w_2. The calculations show that by putting part of your money in the first investment and part in the second investment, a wide range of risk-return combinations can be achieved. These are plotted in Figure 1.2.

Table 1.2 Expected Return, μ_P, and Standard Deviation of Return, σ_P, from a Portfolio Consisting of Two Investments

w_1	w_2	μ_P	σ_P
0.0	1.0	15%	24.00%
0.2	0.8	14%	20.09%
0.4	0.6	13%	16.89%
0.6	0.4	12%	14.87%
0.8	0.2	11%	14.54%
1.0	0.0	10%	16.00%

The expected returns from the investments are 10% and 15%; the standard deviations of the returns are 16% and 24%; and the correlation between returns is 0.2.

Most investors are risk averse. They want to increase expected return while reducing the standard deviation of return. This means that they want to move as far as they can in a "northwest" direction in Figures 1.1 and 1.2. Figure 1.2 shows that forming a portfolio of the two investments we have been considering helps them do this. For example, by putting 60% in the first investment and 40% in the second, a portfolio with an expected return of 12% and a standard deviation of return equal to 14.87% is obtained. This is an improvement over the risk-return trade-off for the first investment. (The expected return is 2% higher and the standard deviation of the return is 1.13% lower.)

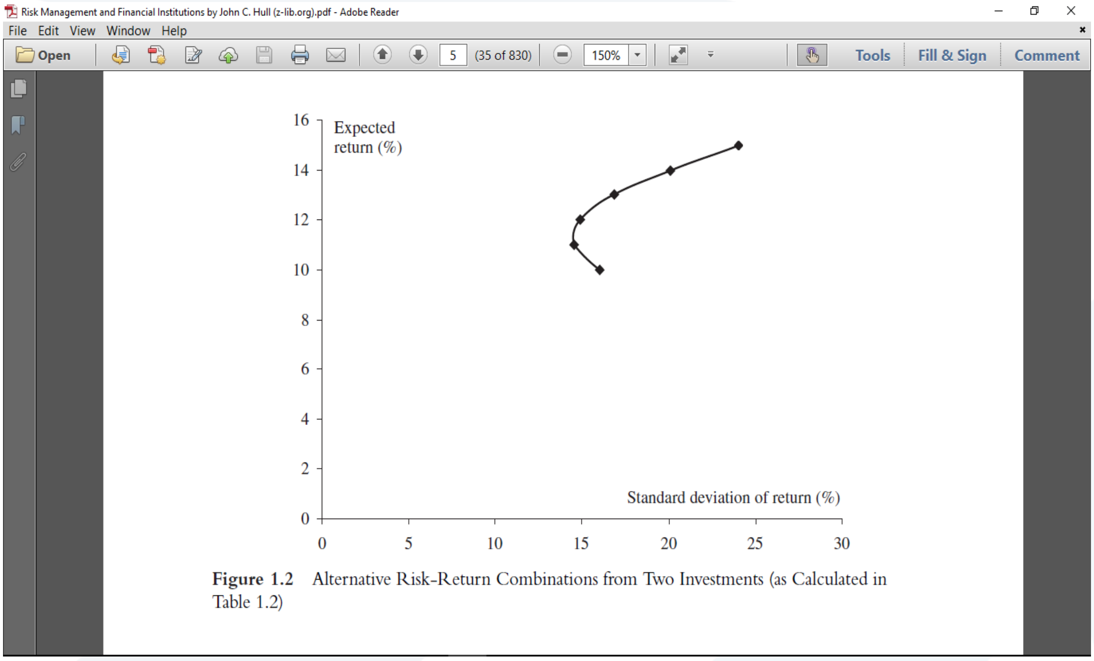


Figure 1.2 Alternative Risk-Return Combinations from Two Investments (as Calculated in Table 1.2)

1.2 The Capital Asset Pricing Model

How do investors decide on the expected returns they require for individual investments? Based on the analysis we have presented, the market portfolio should play a key role. The expected return required on an investment should reflect the extent to which the investment contributes to the risks of the market portfolio.

A common procedure is to use historical data and regression analysis to determine a best-fit linear relationship between returns from an investment and returns from the market portfolio. This relationship has the form:

$$R = a + \beta R_M + \epsilon \qquad (1.3)$$

where R is the return from the investment, R_M is the return from the market portfolio, a and β are constants, and ϵ is a random variable equal to the regression error.

Equation (1.3) shows that there are two uncertain components to the risk in the investment's return:

1. A component βR_M, which is a multiple of the return from the market portfolio.

2. A component ∈, which is unrelated to the return from the market portfolio.

The first component is referred to as *systematic risk*. The second component is referred to as *non-systematic risk*.

Consider first the non-systematic risk. If we assume that the ∈ variables for different investments are independent of each other, the non-systematic risk is almost completely diversified away in a large portfolio. An investor should not therefore be concerned about non-systematic risk and should not require an extra return above the risk-free rate for bearing non-systematic risk.

The systematic risk component is what should matter to an investor. When a large well-diversified portfolio is held, the systematic risk represented by βR_M does not disappear. An investor should require an expected return to compensate for this systematic risk.

When $\beta = 0$ there is no systematic risk and the expected return is R_F. When $\beta = 1$, we have the same systematic risk as the market portfolio, which is represented by point M, and the expected return should be $E(R_M)$. In general

$$E(R) = R_F + \beta[E(R_M) - R_F] \quad (1.4)$$

This is the *capital asset pricing model*. The excess expected return over the risk-free rate required on the investment is β times the excess expected return on the market portfolio. This relationship is plotted in Figure 1.5. The parameter β is the *beta* of the investment.

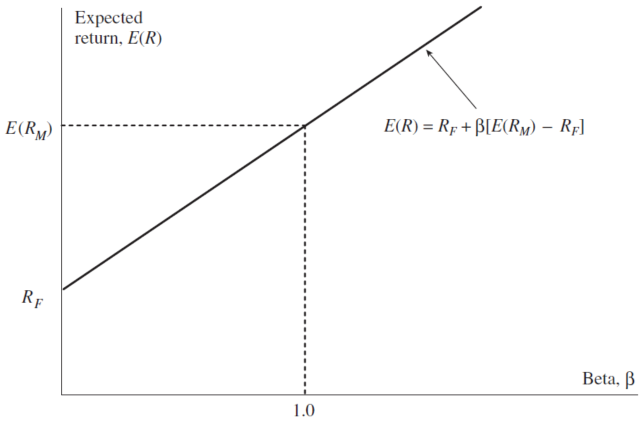


Figure 1.5 The Capital Asset Pricing Model

Example 1.1

Suppose that the risk-free rate is 5% and the return on the market portfolio is 10%. An investment with a beta of 0 should have an expected return of 5%. This is because all of the risk in the investment can be diversified away. An investment with a beta of 0.5 should have an expected return of:

$$0.05 + 0.5 \times (0.1 - 0.05) = 0.075 \text{ or } 7.5\%.$$

An investment with a beta of 1.2 should have an expected return of:

$$0.05 + 1.2 \times (0.1 - 0.05) = 0.11 \text{ or } 11\%$$

The parameter β is equal to $\rho\sigma/\sigma_M$, where ρ is the correlation between the return on the investment and the return on the market portfolio, σ is the standard deviation of the return on the investment, and σ_M is the standard deviation of the return on the market portfolio. Beta measures the sensitivity of the return on the investment to the return on the market portfolio.

We can define the beta of any investment portfolio as in equation (1.3) by regressing its returns against the returns on the market portfolio. The capital asset pricing model in equation (1.4) should then apply with the return *R* defined as the return on the portfolio.

1.2.1 Assumptions

The analysis we have presented leads to the surprising conclusion that all investors want to hold the same portfolios of assets. This is clearly not true. Indeed, if it were true, markets would not function at all well because investors would not want to trade with each other! In practice, different investors have different views on the attractiveness of stocks and other risky investment opportunities. This is what causes them to trade with each other and it is this trading that leads to the formation of prices in markets.

The reason why the analysis leads to conclusions that do not correspond with the realities of markets is that, in presenting the arguments, we implicitly made a number of assumptions. In particular:

1. We assumed that investors care only about the expected return and the standard deviation of return of their portfolio.
2. We assumed that the ϵ variables for different investments in equation (1.3) are independent. Equivalently we assumed the returns from investments are correlated with each other only because of their correlation with the market portfolio. This is clearly not true. Ford and General Motors are both in the automotive sector. There is likely to be some correlation between their returns that does not arise from their correlation with the overall stock market. This means that the ϵ for Ford and the ϵ for General Motors are not likely to be independent of each other.

3. We assumed that investors focus on returns over just one period and the length of this period is the same for all investors. This is also clearly not true. Some investors such as pension funds have very long-time horizons. Others such as day traders have very short time horizons.
4. We assumed that investors can borrow and lend at the same risk-free rate. This is approximately true in normal market conditions for a large financial institution that has a good credit rating. But it is not exactly true for such a financial institution and not at all true for small investors.
5. We did not consider tax. In some jurisdictions, capital gains are taxed differently from dividends and other sources of income. Some investments get special tax treatment and not all investors are subject to the same tax rate.
6. Finally, we assumed that all investors make the same estimates of expected returns, standard deviations of returns, and correlations between returns for available investments. To put this another way, we assumed that investors have *homogeneous expectations*. This is clearly not true. Indeed, if we lived in a world of homogeneous expectations there would be no trading.

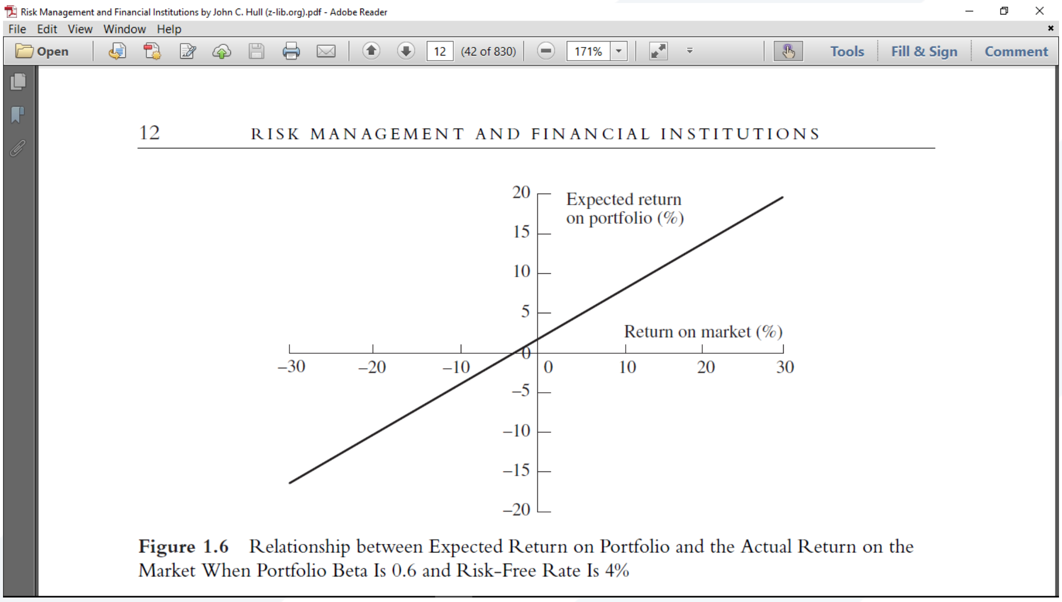
In spite of all this, the capital asset pricing model has proved to be a useful tool for portfolio managers. Estimates of the betas of stocks are readily available and the expected return on a portfolio estimated by the capital asset pricing model is a commonly used benchmark for assessing the performance of the portfolio manager.

1.2.2 Alpha

When we observe a return of R_M on the market, what do we expect the return on a portfolio with a beta of β to be? The capital asset pricing model relates the expected return on a portfolio to the expected return on the market. But it can also be used to relate the expected return on a portfolio to the actual return on the market:

$$E(R_P) = R_F + \beta(R_M - R_F)$$

where R_F is the risk-free rate and R_P is the return on the portfolio.



Example 1.2

Consider a portfolio with a beta of 0.6 when the risk-free interest rate is 4%. When the return on the market is 20%, the expected return on the portfolio is:

$$0.04 + 0.6 \times (0.2 - 0.04) = 0.136 \text{ or } 13.6\%$$

When the return on the market is 10%, the expected return from the portfolio is: $0.04 + 0.6 \times (0.1 - 0.04)$

$$= 0.076 \text{ or } 7.6\%$$

When the return from the market is −10%, the expected return from the portfolio is:

$$0.04 + 0.6 \times (-0.1 - 0.04) = -0.044 \text{ or } -4.4\%$$

The relationship between the expected return on the portfolio and the return on the market is shown in Figure 1.6.

Now, suppose that the actual return on the portfolio is greater than the expected return:

$$R_P > R_F + \beta(R_M - R_F)$$

The portfolio manager has produced a superior return for the amount of systematic risk being taken. The extra return is

$$\alpha = R_P - R_F - \beta(R_M - R_F)$$

This is commonly referred to as the *alpha* created by the portfolio manager.

Example 1.3

A portfolio manager has a portfolio with a beta of 0.8. The one-year risk-free rate of interest is 5%, the return on the market during the year is 7%, and the portfolio manager's return is 9%. The manager's alpha is

$$\alpha = 0.09 - 0.05 - 0.8 \times (0.07 - 0.05) = 0.024 \text{ or } 2.4\%$$

Portfolio managers are continually searching for ways of producing a positive alpha. One way is by trying to pick stocks that outperform the market. Another is by *market timing*. This involves trying to anticipate movements in the market as a whole and moving funds from safe investments such as Treasury bills to the stock market when an upturn is anticipated and in the other direction when a downturn is anticipated. Although the capital asset pricing model is unrealistic in some respects, the alpha and beta parameters that come out of the model are widely used to characterize investments. Beta describes the amount of systematic risk. The higher the value of beta, the greater the systematic risk being taken and the greater the extent to which returns are dependent on the performance of the market. Alpha represents the extra return made from superior portfolio management (or perhaps just good luck). An investor can make a positive alpha only at the expense of other investors who are making a negative alpha.