Financial Risk Management

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Gamma

- The *gamma*, Γ, of a portfolio measures the extent to which large changes cause problems. Gamma is the rate of change of the portfolio's delta with respect to the price of the underlying asset. It is the second partial derivative of the portfolio with respect to asset price:

$$Gamma = \frac{\partial^2 P}{\partial S^2}$$

- If gamma is small, delta changes slowly, and adjustments to keep a portfolio delta neutral need to be made only relatively infrequently. However, if gamma is large in absolute terms, delta is highly sensitive to the price of the underlying asset. It is then quite risky to leave a delta-neutral portfolio unchanged for any length of time.

- Gamma is positive for a long position in an option. The general way in which gamma varies with the price of the underlying stock is shown in Figure 8.5. Gamma is greatest for options where the stock price is close to the strike price K.

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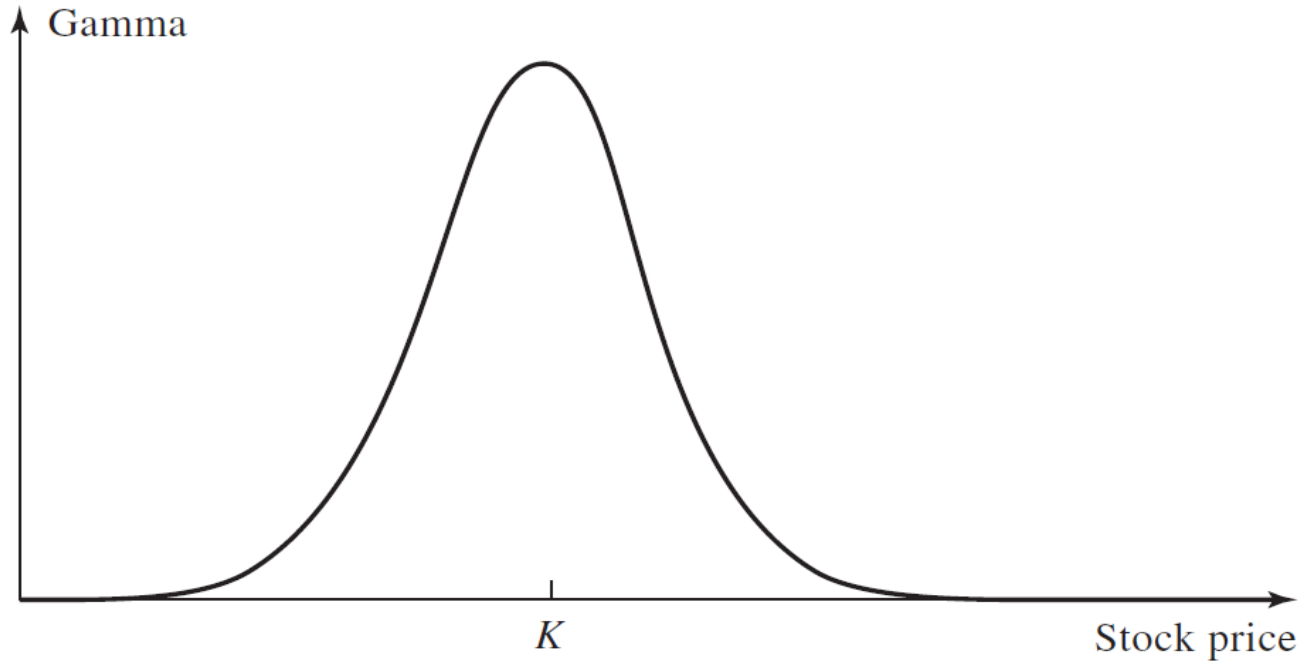

Figure 8.5 Relationship between Gamma of an Option and Price of Underlying Stock Where K Is the Option's Strike Price

Making a portfolio gamma neutral

- A linear product has zero gamma and cannot be used to change the gamma of a portfolio. What is required is a position in an instrument, such as an option, that is not linearly dependent on the underlying asset price.
- Suppose that a delta-neutral portfolio has a gamma equal to Γ, and a traded option has a gamma equal to Γ_T. If the number of traded options added to the portfolio is w_T, the gamma of the portfolio is:

$$w_T \Gamma_T + \Gamma$$

- Hence, the position in the traded option necessary to make the portfolio gamma neutral is $w_T = -\Gamma / \Gamma_T$. Including the traded option is likely to change the delta of the portfolio, so the position in the underlying asset then has to be changed to maintain delta neutrality. Note that the portfolio is gamma neutral only for a short period of time. As time passes, gamma neutrality can be maintained only if the position in the traded option is adjusted so that it is always equal to $-\Gamma / \Gamma_T$.

- Making a delta-neutral portfolio gamma neutral can be regarded as a first correction for the fact that the position in the underlying asset cannot be changed continuously when delta hedging is used. Delta neutrality provides protection against relatively small asset price moves between rebalancing. Gamma neutrality provides protection against larger movements in the asset price between hedge rebalancing.

- Suppose that a portfolio is delta neutral and has a gamma of −3,000. The delta and gamma of a particular traded call option are 0.62 and 1.50, respectively. The portfolio can be made gamma neutral by including in the portfolio a long position of $\frac{3000}{1.5} = 2000$ in the call option.
- (The gamma of the portfolio is then −3,000 + 1.5 × 2,000 = 0.) However, the delta of the portfolio will then change from zero to 2,000 × 0.62 = 1,240. A quantity, 1,240, of the underlying asset must therefore be sold to keep it delta neutral.

Vega

- Another source of risk in derivatives trading is the possibility that volatility will change. The volatility of a market variable measures our uncertainty about the future value of the variable. In option valuation models, volatilities are often assumed to be constant, but in practice they do change through time. Spot positions and forwards do not depend on the volatility of underlying asset prices, but options and more complicated derivatives do. Their values are liable to change because of movements in volatility as well as because of changes in the asset price and the passage of time.

- The *vega* of a portfolio, V , is the rate of change of the value of the portfolio with respect to the volatility, σ, of the underlying asset price.

$$V = \frac{\partial P}{\partial \sigma}$$

- If vega is high in absolute terms, the portfolio's value is very sensitive to small changes in volatility. If vega is low in absolute terms, volatility changes have relatively little impact on the value of the portfolio.

- The vega of a portfolio can be changed by adding a position in a traded option. If V is the vega of the portfolio and V_T is the vega of a traded option, a position of $-V/V_T$ in the traded option makes the portfolio instantaneously vega neutral. Unfortunately, a portfolio that is gamma neutral will not, in general, be vega neutral, and vice versa. If a hedger requires a portfolio to be both gamma and vega neutral, at least two traded derivatives dependent on the underlying asset must usually be used.

- **Example**
- Consider a portfolio dependent on the price of a single asset that is delta neutral, with a gamma of −5,000 and a vega of −8,000. The options shown in the table below can be traded. The portfolio could be made vega neutral by including a long position in 4,000 of Option 1. This would increase delta to 2,400 and require that 2,400 units of the asset be sold to maintain delta neutrality. The gamma of the portfolio would change from −5,000 to −3,000.

	Delta	Gamma	Vega
Portfolio	0	−5,000	−8,000
Option 1	0.6	0.5	2.0
Option 2	0.5	0.8	1.2

- To make the portfolio gamma and vega neutral, both Option 1 and Option 2 can be used. If w_1 and w_2 are the quantities of Option 1 and Option 2 that are added to the portfolio, we require that the solution to these equations is $w_1 = 400, w_2 = 6{,}000$.

$$-5{,}000 + 0.5w_1 + 0.8w_2 = 0$$
$$-8{,}000 + 2.0w_1 + 1.2w_2 = 0$$

- The portfolio can therefore be made gamma and vega neutral by including 400 of Option 1 and 6,000 of Option 2. The delta of the portfolio after the addition of the positions in the two traded options is 400 × 0.6 + 6,000 × 0.5 = 3,240. Hence, 3,240 units of the underlying asset would have to be sold to maintain delta neutrality.