Portfolio Management

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COMBINATIONS OF TWO RISKY ASSETS REVISITED: SHORT SALES NOT ALLOWED

- Recall that the expected return on a portfolio of two assets is given by:

$$\overline{R_P} = X_A\overline{R_A} + X_B\overline{R_B}$$

$\overline{R_P}$ is the expected return on the portfolio.
X_A is the fraction of the portfolio held in asset *A*.
X_B is the fraction of the portfolio held in asset *B*.
$\overline{R_A}$ is the expected return on asset *A*.
$\overline{R_B}$ is the expected return on asset *B*.

- In addition, because we require the investor to be fully invested, the fraction he invests in *A* plus the fraction he invests in *B* must equal 1, or: $X_A + X_B = 1$ (equation 2).
- We can rewrite this expression as: $X_B = 1 - X_A$
- So we can express the expected return on a portfolio of two assets as:

$$\overline{R_P} = X_A\overline{R_A} + (1 - X_A)\overline{R_B}$$

- Notice that the expected return on the portfolio is a simple weighted average of the expected returns on the individual securities and that the weights add to 1. The same is not necessarily true of the risk (standard deviation of the return) of the portfolio.

- Recall that the standard deviation of the return on a portfolio is given by:

$$\sigma_P = \left[(X_A^2\sigma_A^2 + X_B^2\sigma_B^2 + 2X_A X_B \sigma_{AB}) \right]^{1/2}$$

σ_P is the standard deviation of the return on the portfolio.
σ_A^2 is the variance of the return on security *A*.
σ_B^2 is the variance of the return on security *B*.
σ_{AB} is the covariance between the returns on security *A* and security *B*.

- If we substitute Equation (2) into this expression, we obtain:

$$\sigma_P = \begin{bmatrix} (X_A^2\sigma_A^2 + (1 - X_A)^2\sigma_B^2 \\ + 2X_A(1 - X_A)\sigma_{AB}) \end{bmatrix}^{1/2}$$

- Recalling that $\sigma_{AB} = \rho_{AB}\sigma_A\sigma_B$, where ρ_{AB} is the correlation coefficient between securities *A* and *B*, the previous equation becomes:

$$\sigma_P = \begin{bmatrix} (X_A^2\sigma_A^2 + (1 - X_A)^2\sigma_B^2 \\ + 2X_A(1 - X_A)\rho_{AB}\sigma_A\sigma_B) \end{bmatrix}^{1/2}$$

- We know that a correlation coefficient has a maximum value of +1 and a minimum value of -1. A value of +1 means that two securities will always move in perfect unison, whereas a value of -1 means that their movements are exactly opposite to each other.
- We start with an examination of these extreme cases, then we turn to an examination of some intermediate values for the correlation coefficients. As an aid in interpreting results, we examine a specific example as well as general expressions for risk and return.

- For the example, we consider two stocks: a large manufacturer of automobiles (“Colonel Motors”) and an electric utility company operating in a large city (“Separated Edison”). Assume the stocks have the following characteristics:

	Expected Return	Standard Deviation
Colonel Motors (C)	14%	6%
Separated Edison (S)	8%	3%

- As you might suspect, the car manufacturer has a bigger expected return and a bigger risk than the electric utility.

Case 1—Perfect Positive Correlation ($\rho = +1$)

- Let the subscript C stand for Colonel Motors and the subscript S stand for Separated Edison. If the correlation coefficient is +1, then the equation for the risk on the portfolio simplifies to:

$$\sigma_P = \begin{bmatrix} (X_C^2\sigma_C^2 + (1 - X_C)^2\sigma_S^2 \\ + 2X_C(1 - X_C)\,\sigma_C\sigma_S) \end{bmatrix}^{1/2}$$

- Note that the term in square brackets has the form $X^2 + 2XY + Y^2$ and thus can be written as:

$$(X_C\sigma_C + (1 - X_C)\sigma_S)^2$$

- Because the standard deviation of the portfolio is equal to the positive square root of this expression, we know that:

$$\sigma_P = X_C\sigma_C + (1 - X_C)\sigma_S$$

- while the expected return on the portfolio is:

$$\overline{R_P} = X_C\overline{R_C} + (1 - X_C)\overline{R_S}$$

- Thus with the correlation coefficient equal to +1, both risk and return of the portfolio are simply linear combinations of the risk and return of each security.

- Solving for X_C in the expression for standard deviation yields:

$$X_C = \frac{\sigma_P - \sigma_S}{\sigma_C - \sigma_S}$$

- Thus:

$$\overline{R_P} = \frac{\sigma_P - \sigma_S}{\sigma_C - \sigma_S}\overline{R_C} + (1 - \frac{\sigma_P - \sigma_S}{\sigma_C - \sigma_S})\overline{R_S}$$

Table 5.1 The Expected Return and Standard Deviation of a Portfolio of Colonel Motors and Separated Edison When $\rho = +1$

X_C	0	0.2	0.4	0.5	0.6	0.8	1.0
$\bar{R}_P$	8.0	9.2	10.4	11	11.6	12.8	14.0
σ_P	3.0	3.6	4.2	4.5	4.8	5.4	6.0

- Table 5.1 presents the return on a portfolio for selected values of X_C, and Figure 5.1 presents a graph of this relationship. Note that the relationship is a straight line. The equation of the straight line could easily be derived as follows. Utilizing the equation presented earlier for σ_P to solve for X_C yields:

$$X_C = \frac{\sigma_P}{3} - 1$$

- Substituting this expression for X_C into the equation for $\overline{R_P}$ and rearranging yields:

$$\overline{R_P} = 2 + 2\,\sigma_P$$

Modern Portfolio Theory and Investment Analysis by Edwin J. Elton, Martin J. Gruber, Stephen J. Brown, William N. Goetz (1).pdf - Adobe Reader

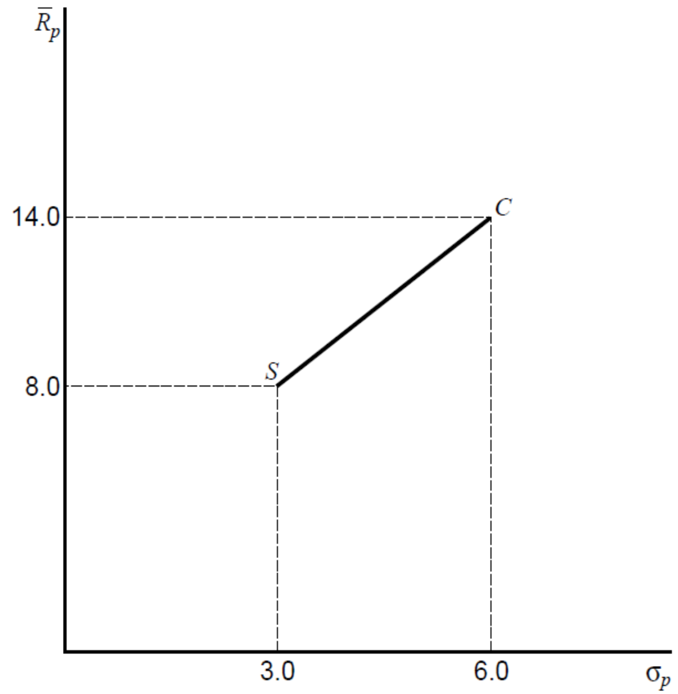

Figure 5.1 Relationship between expected return and standard deviation when $\rho = +1$.

- In the case of perfectly correlated assets, the return and risk on the portfolio of the two assets is a weighted average of the return and risk on the individual assets. There is no reduction in risk from purchasing both assets. This can be seen by examining Figure 5.1 and noting that combinations of the two assets lie along a straight line connecting the two assets. Nothing has been gained by diversifying rather than purchasing the individual assets.