Portfolio Management

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Case 2—Perfect Negative Correlation (ρ = -1)

- We now examine the other extreme: two assets that move perfectly together but in exactly opposite directions. In this case the standard deviation of the portfolio is:

$$\sigma_P = \begin{bmatrix} (X_C^2\sigma_C^2 + (1 - X_C)^2\sigma_S^2 \\ - 2X_C(1 - X_C)\ \sigma_C\sigma_S) \end{bmatrix}^{1/2}$$

- Once again, the equation for standard deviation can be simplified. The term in the brackets is equivalent to either of the following two expressions:

$$(X_C\sigma_C - (1 - X_C)\sigma_S)^2$$

- Or:

$$(-X_C\sigma_C + (1 - X_C)\sigma_S)^2$$

- Thus σ_P is either:

$$\sigma_P = X_C\sigma_C - (1 - X_C)\sigma_S$$

- Or:

$$\sigma_P = -X_C\sigma_C + (1 - X_C)\sigma_S$$

- The value of σ_P for previous equations is always smaller than the value of σ_P for the case where $\rho = +1$ for all values of X_C between 0 and 1. Thus the risk on a portfolio of assets is always smaller when the correlation coefficient is -1 than when it is $+1$.

- We can go one step further. If two securities are perfectly negatively correlated (i.e., they move in exactly opposite directions), it should always be possible to find some combination of these two securities that has zero risk. By setting either one of the last two equations equal to 0, we find that a portfolio with $X_C = \sigma_S/(\sigma_S + \sigma_C)$ will have zero risk. Because $\sigma_S > 0$ and $\sigma_S + \sigma_C > \sigma_S$, this implies that $0 < X_C < 1$ or that the zero-risk portfolio will always involve positive investment in both securities.

- Now let us return to our example. Minimum risk occurs when $X_C = \frac{3}{(3+6)} = \frac{1}{3}$
- Furthermore, for the case of perfect negative correlation,

$$\overline{R_P} = 8 + 6X_C$$

$$\sigma_P = 6X_C - 3(1 - X_C)$$

- Or:

$$\sigma_P = -6X_C + 3(1 - X_C)$$

- We have now examined combinations of risky assets for perfect positive and perfect negative correlation. In Figure 5.3 we have plotted both of these relationships on the same graph. From this graph we should be able to see intuitively where portfolios of these two stocks should lie if correlation coefficients took on intermediate values.

- From the expression for the standard deviation, we see that for any value for X_C between 0 and 1, the lower the correlation, the lower the standard deviation of the portfolio. The standard deviation reaches its lowest value for ρ = -1 (curve *SBC*) and its highest value for ρ = +1 (curve *SAC*). Therefore these two curves should represent the limits within which all portfolios of these two securities must lie for intermediate values of the correlation coefficient.

- We would speculate that an intermediate correlation might produce a curve such as *SOC* in Figure 5.3. We demonstrate this by returning to our example and constructing the relationship between risk and return for portfolios of our two securities when the correlation coefficient is assumed to be 0 and +0.5.

- Table 5.2 The Expected Return and Standard Deviation of a Portfolio of Colonel Motors and Separated Edison When ρ = -1

X_C	0	0.2	0.4	0.6	0.8	1.0
$\overline{R_P}$	8.0	9.2	10.4	11.6	12.8	14.0
σ_P	3.0	1.2	0.6	2.4	4.2	6.0

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Modern Portfolio Theory and Investment Analysis by Edwin J. Elton, Martin J. Gruber, Stephen J. Brown, William N. Goetz (1).pdf - Adobe Reader

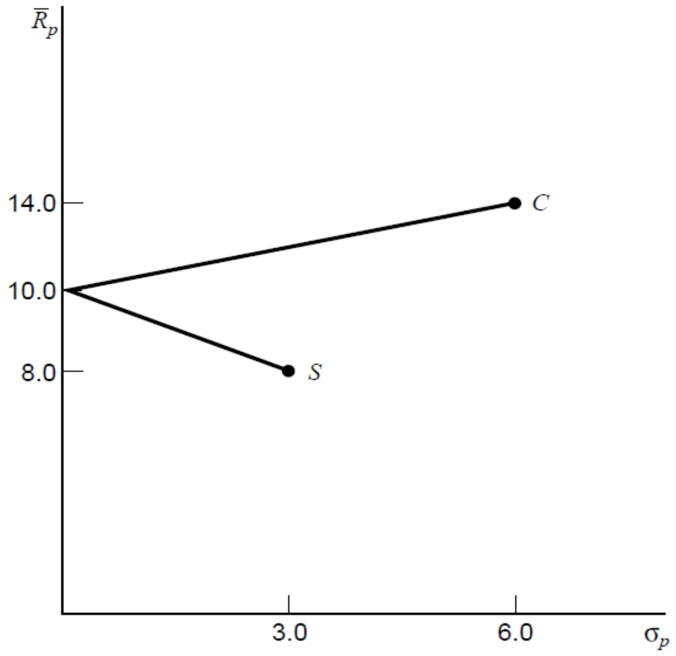

Figure 5.2 Relationship between expected return and standard deviation when $\rho = -1$.

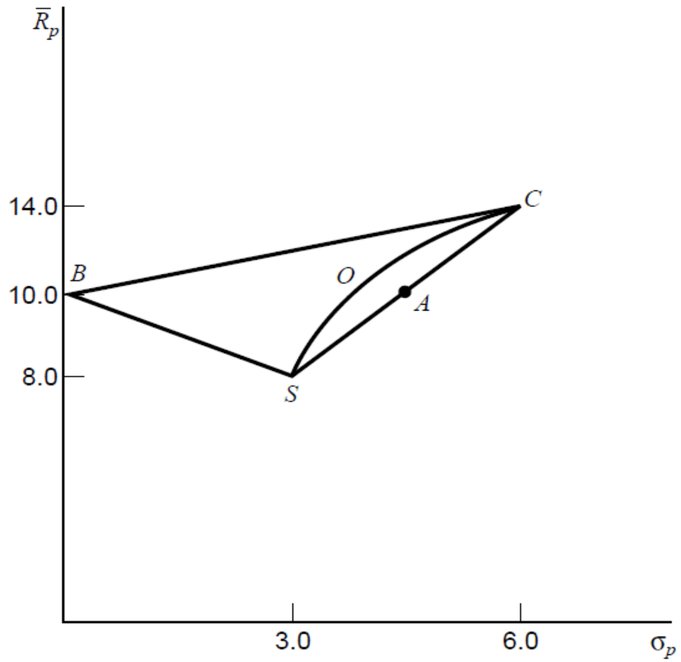

Figure 5.3 Relationship between expected return and standard deviation for various correlation coefficients.