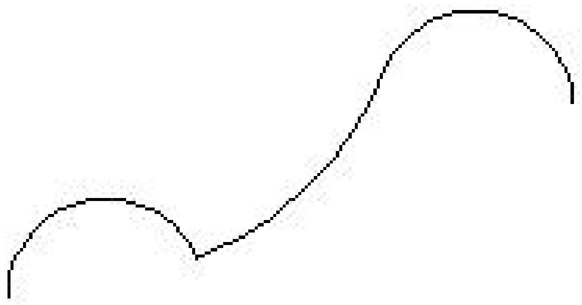
First chapter
Number system

Digital vs. Analog

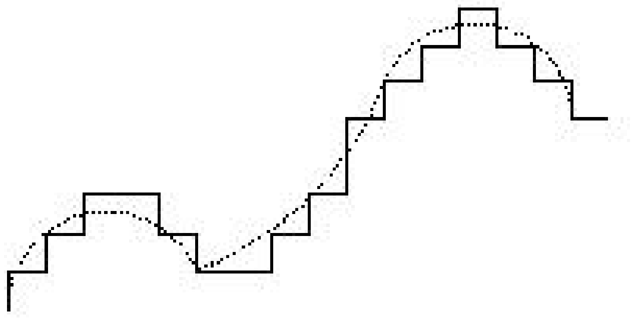
- An analog system has continuous range of values
  - A mercury thermometer
  - Vinyl records
  - Human eye

- A digital system has a set of discrete values
  - Digital Thermometer
  - Compact Disc (CD)
  - Digital camera

Benefits of using digital



Analog signal



Digital signal

- Cheap electronic circuits
- Easier to calibrate and adjust
- Resistance to noise: Clearer picture and sound

Binary System

- Discrete elements of information are represented with bits called *binary codes*.

Example: $(09)_{10} = (1001)_2$
$(15)_{10} = (1111)_2$

Question: Why are commercial products made with digital circuits as opposed to analog?

Most digital devices are programmable: By changing the program in the device, the same underlying hardware can be used for many different applications.

Binary Code

- Review the decimal number system.

  Base (Radix) is 10 - symbols (0,1, . . 9) Digits

  For Numbers > 9, add more significant digits in position to the left, e.g. 19>9.

  Each position carries a weight.

MSD

Weights: 10^3 | 10^2 | 10^1 | 10^0 • 10^{-1} | 10^{-2} | 10^{-3}

LSD

✓ If we were to write 1936.25 using a power series expansion and base 10 arithmetic:

$$1\times10^{3}+9\times10^{2}+3\times10^{1}+6\times10^{0}+2\times10^{-1}+5\times10^{-2}$$

Binary number system

- The binary number system.
  - Base is 2 - symbols (0,1) - Binary Digits (Bits)
  - For Numbers > 1, add more significant digits in position to the left, e.g. 10>1.
  - Each position carries a weight (using decimal).

MSD → Weights:

2^3	2^2	2^1	2^0	$\bullet$	2^{-1}	2^{-2}	2^{-3}

← LSD

✓ If we write 10111.01 using a decimal power series we convert from binary to decimal:

$$1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0 + 0\times 2^{-1} + 1\times 2^{-2} =$$

$$= 1\times 16 + 0\times 8 + 1\times 4 + 1\times 2 + 1\times 1 + 0\times 0.5 + 1\times 0.25 = 23.25$$

Binary number system

- $(110000.0111)_2 = (\ \ ?\ \ )_{10}$

ANS: 48.4375

- In computer work: $2^{10} = 1024$ is referred as K = kilo
  $2^{20} = 1048576$ is referred as M = mega
  $2^{30} = ?$
  $2^{40} = ?$

- What is the exact number of bytes in a 16 Gbyte memory module?

Octal/Hex number systems

- The octal number system [from Greek: OKTΩ].
  - Its base is 8 → eight digits 0, 1, 2, 3, 4, 5, 6, 7

✓ $(236.4)_8 = (158.5)_{10}$

$$2\times8^2+3\times8^1+6\times8^0+4\times8^{-1}=158.5$$

- The hexadecimal number system [from Greek: ΔEKAEΞI].
  - Its base is 16 → first 10 digits are borrowed from the decimal system and the letters A, B, C, D, E, F are used for the digits 10, 11, 12, 13, 14, 15

✓ $(D63FA)_{16} = (877562)_{10}$

$$13\times16^4+6\times16^3+3\times16^2+15\times16^1+10\times16^0=877562$$

Conversion from Decimal to Binary

- Conversion from decimal to binary:

  Let each bit of a binary number be represented by a variable whose subscript = bit positions, i.e.,

$$(110)_2 = (a_2 a_1 a_0)_2$$

Its decimal equivalent is:

$$(1\times 2^2 + 1\times 2^1 + 0\times 2^0)_{10} = (a_2\times 2^2 + a_1\times 2^1 + a_0\times 2^0)_{10}$$

It is necessary to separate the number into an integer part and a fraction: Repeatedly divide the decimal number by 2.

Conversion from Decimal to Binary

✓ Find the binary equivalent of 37.

2) 37	= 18 + 0.5	1 ← LSB
2) 18	= 9 + 0	0
2) 9	= 4 + 0.5	1
2) 4	= 2 + 0	0
2) 2	= 1 + 0	0
2) 1	= 0 + 0.5	1 ← MSB

$$37_{10} = 100101_2$$

❑ $53_{10} = \underline{\quad ? \quad}_2$ ANS: $53_{10} = 110101_2$

Conversion from Decimal to Binary

- Conversion from <u>decimal fraction</u> to <u>binary</u>:
  same method used for integers except multiplication
  is used instead of division.

✓ Convert $(0.8542)_{10}$ to binary (give answer to 6 digits).

0.8542 x 2 =	1	+	0.7084	$a_{-1} = 1$
0.7084 x 2 =	1	+	0.4168	$a_{-2} = 1$
0.4168 x 2 =	0	+	0.8336	$a_{-3} = 0$
0.8336 x 2 =	1	+	0.6672	$a_{-4} = 1$
0.6675 x 2 =	1	+	0.3344	$a_{-5} = 1$
0.3344 x 2 =	0	+	0.6688	$a_{-6} = 0$

$$(0.8542)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}a_{-5}a_{-6})_2 = (0.110110)_2$$

- $(53.8542)_{10} = (\quad ? \quad)_2$

Conversion from Decimal to Octal

- Conversion from decimal to octal:

  The decimal number is first divided by 8. The remainder is the LSB. The quotient is then divide by 8 and the remainder is the next significant bit and so on.

✓ Convert 1122 to octal.

$$8\overline{)1122} = 140 + 0.25 \quad R2 \longleftarrow \text{LSB}$$

$$8\overline{)140} = 17 + 0.5 \quad R4$$

$$8\overline{)17} = 2 + 0.125 \quad R1$$

$$8\overline{)2} = 0 + 0.25 \quad R2 \longleftarrow \text{MSB}$$

$$1122_{10} = 2142_8$$

Conversion from Decimal to Octal

- ✓ Convert $(0.3152)_{10}$ to octal (give answer to 4 digits).

0.3152 x 8 =	2	+	0.5216	$a_{-1} = 2$
0.5216 x 8 =	4	+	0.1728	$a_{-2} = 4$
0.1728 x 8 =	1	+	0.3824	$a_{-3} = 1$
0.3824 x 8 =	3	+	0.0592	$a_{-4} = 3$

$$(0.3152)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.2413)_8$$

- ❑ $(1122.3152)_{10} = (\quad ? \quad)_8$

Table 1-2
page: 8

Decimal	Hex	Binary	Octal
0	0	0000	00
1	1	0001	01
2	2	0010	02
3	3	0011	03
4	4	0100	04
5	5	0101	05
6	6	0110	06
7	7	0111	07
8	8	1000	10
9	9	1001	11
10	A	1010	12
11	B	1011	13
12	C	1100	14
13	D	1101	15
14	E	1110	16
15	F	1111	17

Conversion using Table

- Conversion from and to binary, octal, and hexadecimal plays and important part in digital computers.

since $2^3 = 8$ and $2^4 = 16$

each octal digit corresponds to 3 binary digits
and each hexa digit corresponds to 4 binary digits.

✓ $(010\ 111\ 100\ .\ 001\ 011\ 000)_2 = (274.130)_8$

✓ $(0110\ 1111\ 1101\ .\ 0001\ 0011\ 0100)_2 = (6FD.134)_{16}$

} from table

Complements

- Complements: They are used in digital computers for subtraction operation and for logic manipulation.

10’s complement and 9’s complement ⟵ *Decimal Numbers*

2’s complement and 1’s complement ⟵ *Binary Numbers*

9’s complement of $N = (10^n - 1) - N$ (N is a decimal #)

10’s complement of $N = 10^n - N$ (N is a decimal #)

1’s complement of $N = (2^n - 1) - N$ (N is a binary #)
1’s complement can be formed by changing 1’s to 0’s and 0’s to 1’s

2’s complement of a number is obtained by leaving <u>all</u> least significant 0’s and the <u>first 1</u> unchanged, and replacing 1’s with 0’s and 0’s with 1 in all higher significant digits.

Complements

- 9's complement of $N = (10^n - 1) - N$ (N is a decimal #)

✓ The 9's complement of 12345 = $(10^5 - 1) - 12345 = 87654$

✓ The 9's complement of 012345 = $(10^6 - 1) - 012345 = 987654$

- 10's complement of $N = [(10^n - 1) - N] + 1$ (N is a decimal #)

✓ The 10's complement of 739821 = $10^6 - 739821 = 260179$

✓ The 10's complement of 2500 = $10^4 - 2500 = 7500$

- Find the 9's and 10's-complement of 00000000

ANS: 99999999 and 00000000

1's and 2's Complements

- 1's complement of $N = (2^n-1) - N$ (N is a binary #)
  1's complement can be formed by changing 1's to 0's and 0's to 1's

  2's complement of a number is obtained by leaving all least significant 0's and the first 1 unchanged, and replacing 1's with 0's and 0's with 1 in all higher significant digits.

  - ✓ The 1's complement of 1101011 = 0010100
  - ✓ The 2's complement of 0110111 = 1001001

- ❑ Find the 1's and 2's-complement of 10000000

  Answer: 01111111 and   10000000

Subtraction Using Complements

- Subtraction with digital hardware using complements:

Subtraction of two n-digit unsigned numbers $M - N$ base r:

1. Add M to the r's complement of N: $M + (r^n - N)$
2. If $M \leq N$, the sum will produce an end carry and is equal to r^n that can be discarded. The result is then $M - N$.
3. If $M \geq N$, the sum will not produce an end carry and is equal to $r^n - (N - M)$

Decimal Subtraction using complements

✓ Subtract 150 – 2100 using 10's complement:

$$
\begin{array}{rr}
M = & 150 \\
\text{10's complement of } N = & +\ 7900 \\
\hline
\text{Sum} = & 8050
\end{array}
$$

There's <u>n</u>o end carry → <u>n</u>egative

Answer: – (10's complement of 8050) = – 1950

✓ Subtract 7188 – 3049 using 10's complement:

$$
\begin{array}{rr}
M = & 7188 \\
\text{10's complement of } N = & +\ 6951 \\
\hline
\text{Sum} = & 14139 \\
\text{Discard end carry } 10^4 = & -\ 10000 \\
\hline
\textit{Answer} = & 4139
\end{array}
$$

Binary Subtraction using complements

- Binary subtraction is done using the same procedure.

✓ Subtract 1010100 – 1000011 using 2's complement:

A =	1010100
2's complement of *B* =	+ 0111101
Sum =	10010001
Discard end carry =	– 10000000
Answer =	0010001

(end carry)

- Subtract 1000011 – 1010100 using 2's complement:

Answer = – 0010001

Binary Subtraction using complements

✓ Subtract 1010100 – 1000011 using 1's complement:

$$
\begin{array}{rr}
A = & 1010100 \\
\text{1's complement of } B = & +\ \underline{0111100} \\
\text{Sum} = & 10010000 \\
\text{End-around carry} = & +\ \underline{\qquad\quad 1} \\
Answer = & 0010001
\end{array}
$$

❑ Subtract 1000011 – 1010100 using 1's complement:

$Answer = -\ 0010001$

Signed binary numbers

To represent a negative binary number, the convention is to make the sign bit 1. [sign bit 0 is for positive]

01001 { 9 (unsigned binary); +9 (signed binary) }

11001 { 25 (unsigned binary); – 9 (signed binary) }

Arithmetic addition

Negative numbers must be initially in 2’s complement form and if the obtained sum is negative, it is in 2’s complement form.

+ 6	00000110
+13	00001101
+19	00010011

–6	11111010
+13	00001101
+7	00000111

- ❑ Add –6 and –13

Answer = 11101101

Transfer of Information with Registers

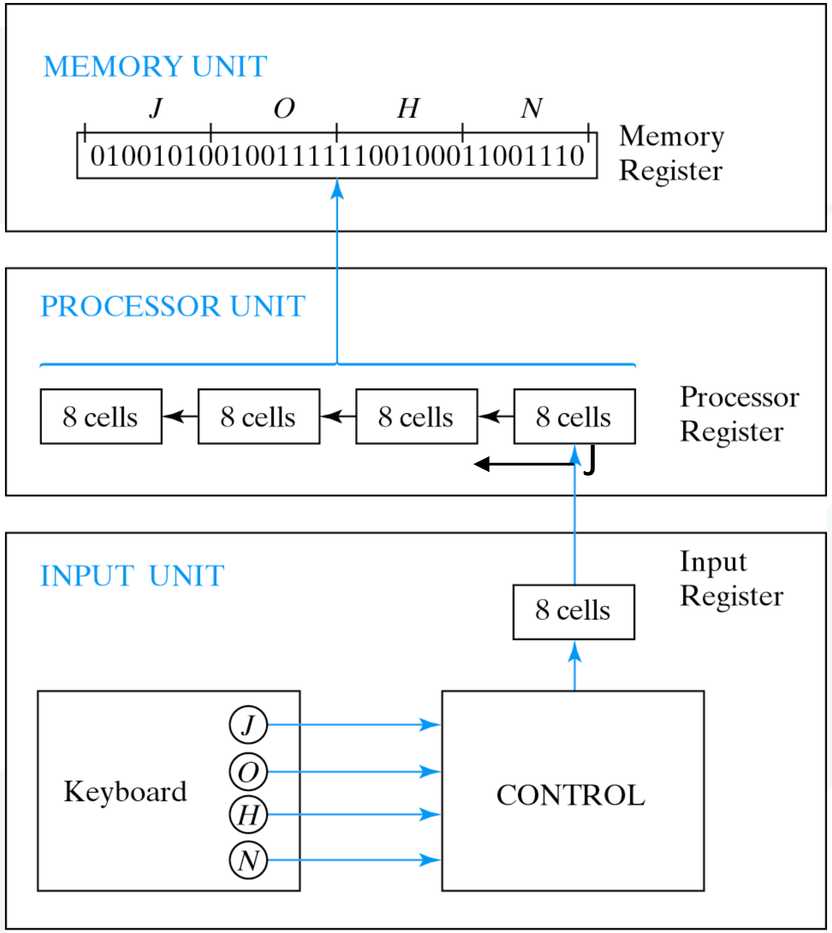


Fig. 1-1 Transfer of information with registers

Binary Information Processing

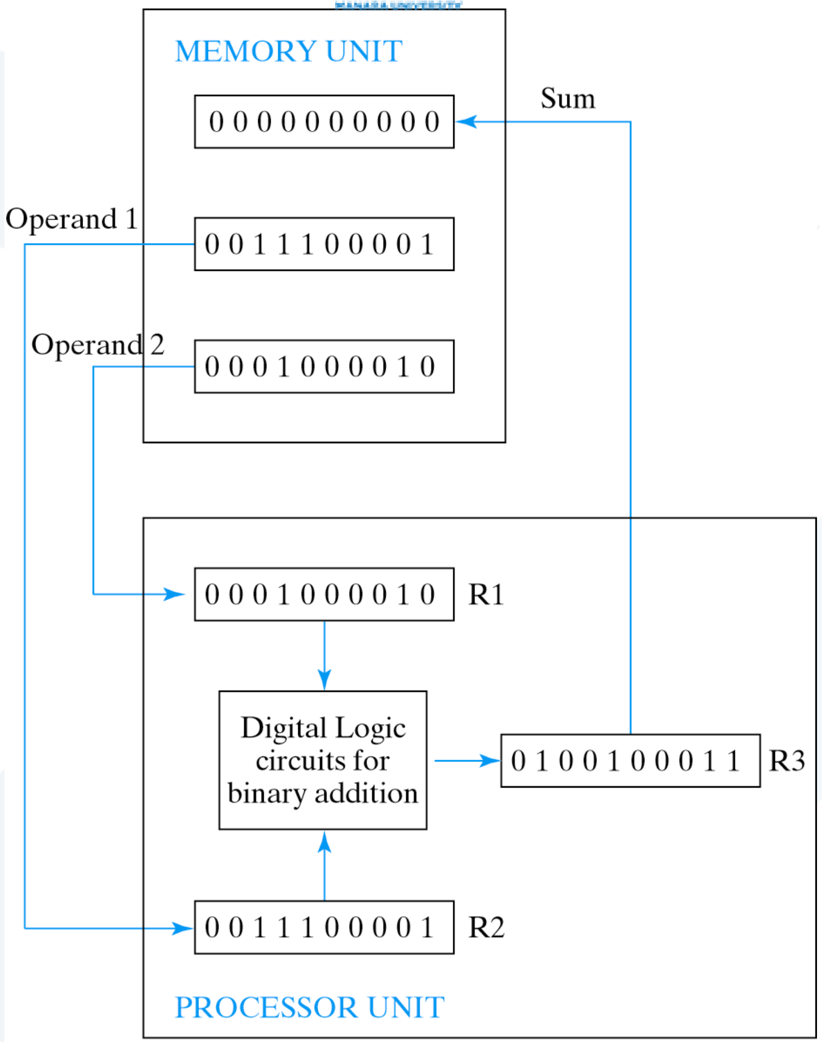


Fig. 1-2 Example of binary information processing