Exercise 2: Signal Representation and Modeling

Ramez Koudsieh

CECC507: Signals and Systems

Manara University

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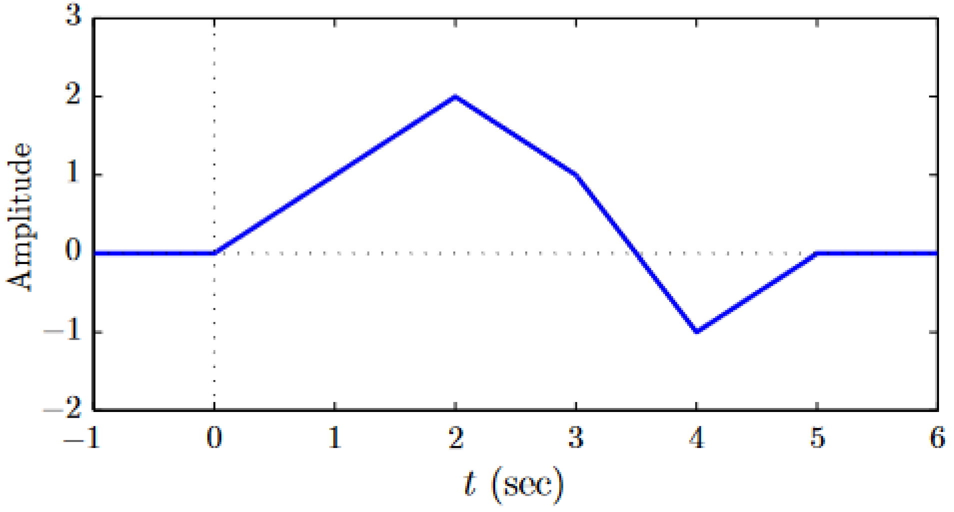
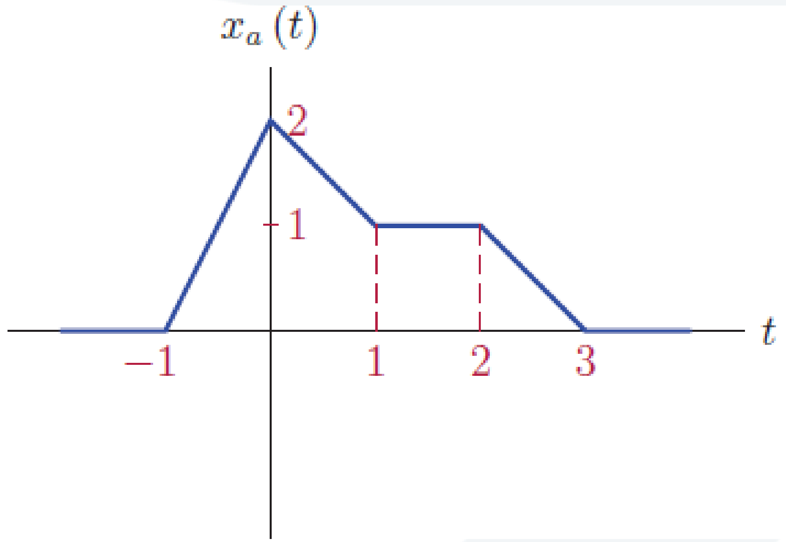
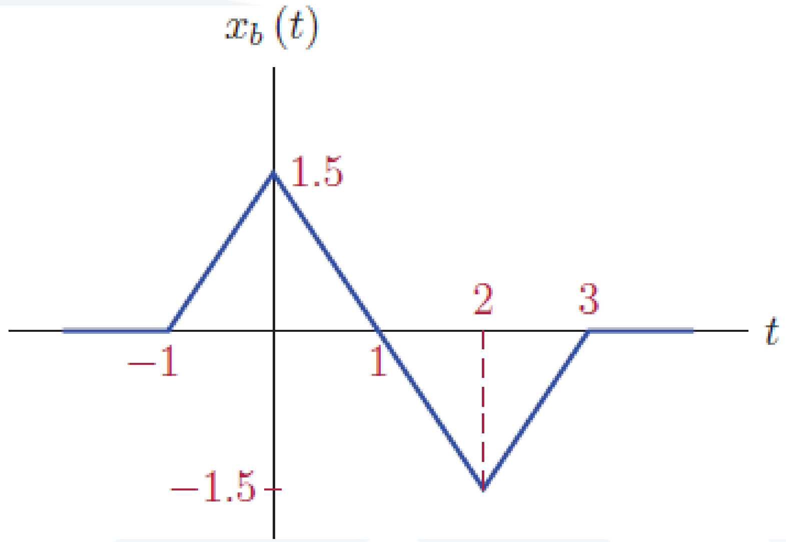
1. Sketch and label each of the signals defined below:

$$x(t) = \begin{cases} 0, & t < 0 \text{ or } t > 5 \\ t, & 0 < t < 2 \\ -t + 4, & 2 < t < 3 \\ -2t + 7, & 3 < t < 4 \\ t - 5, & 4 < t < 5 \end{cases}$$



2. Using the two signals $x_a(t)$ and $x_b(t)$, compute and sketch the signals specified below:

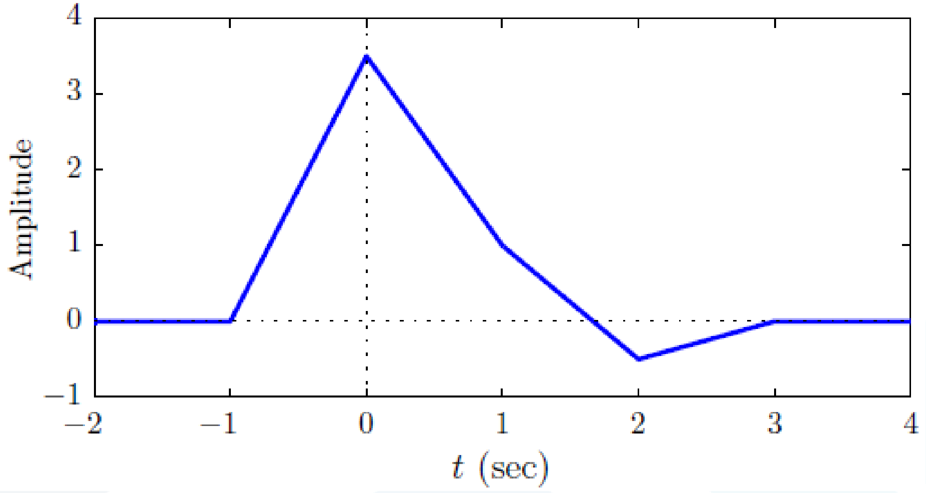




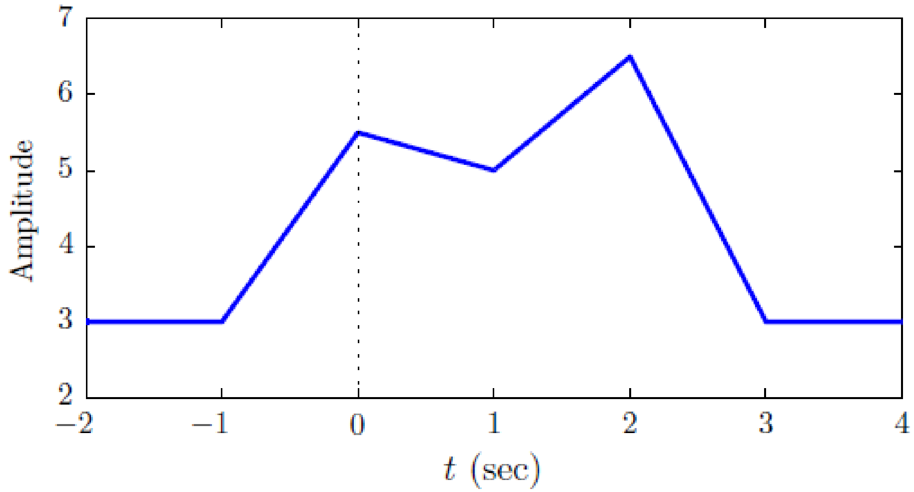
a. $g_1(t) = x_a(t) + x_b(t)$

b. $g_2(t) = 2x_a(t) - x_b(t) + 3$

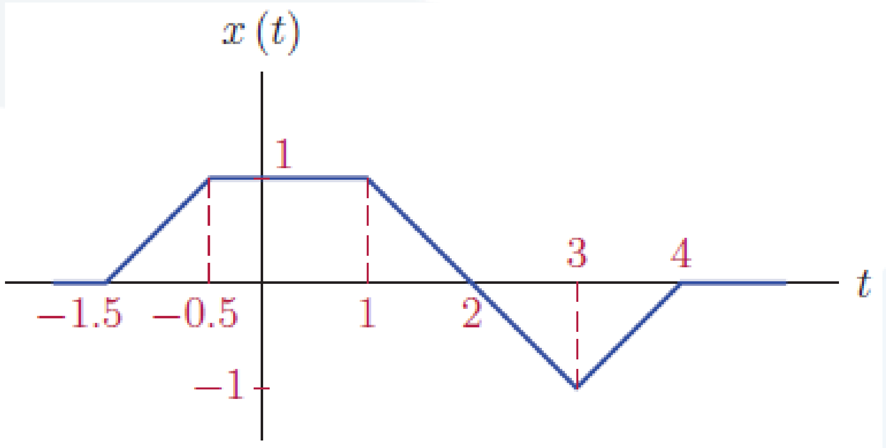
$$g_1(t) = \begin{cases} 0, & t < -1 \text{ or } t > 3 \\ 3.5t + 3.5, & -1 < t < 0 \\ -2.5t + 3.5, & 0 < t < 1 \\ -1.5t + 2.5, & 1 < t < 2 \\ 0.5t - 1.5, & 2 < t < 3 \end{cases}$$



$$g_2(t) = \begin{cases} 3, & t < -1 \text{ or } t > 3 \\ 2.5t + 5.5, & -1 < t < 0 \\ -0.5t + 5.5, & 0 < t < 1 \\ 1.5 + 3.5, & 1 < t < 24 \\ -3.5t + 13.5, & 2 < t < 3 \end{cases}$$



3. For the signal $x(t)$ shown, compute the following:

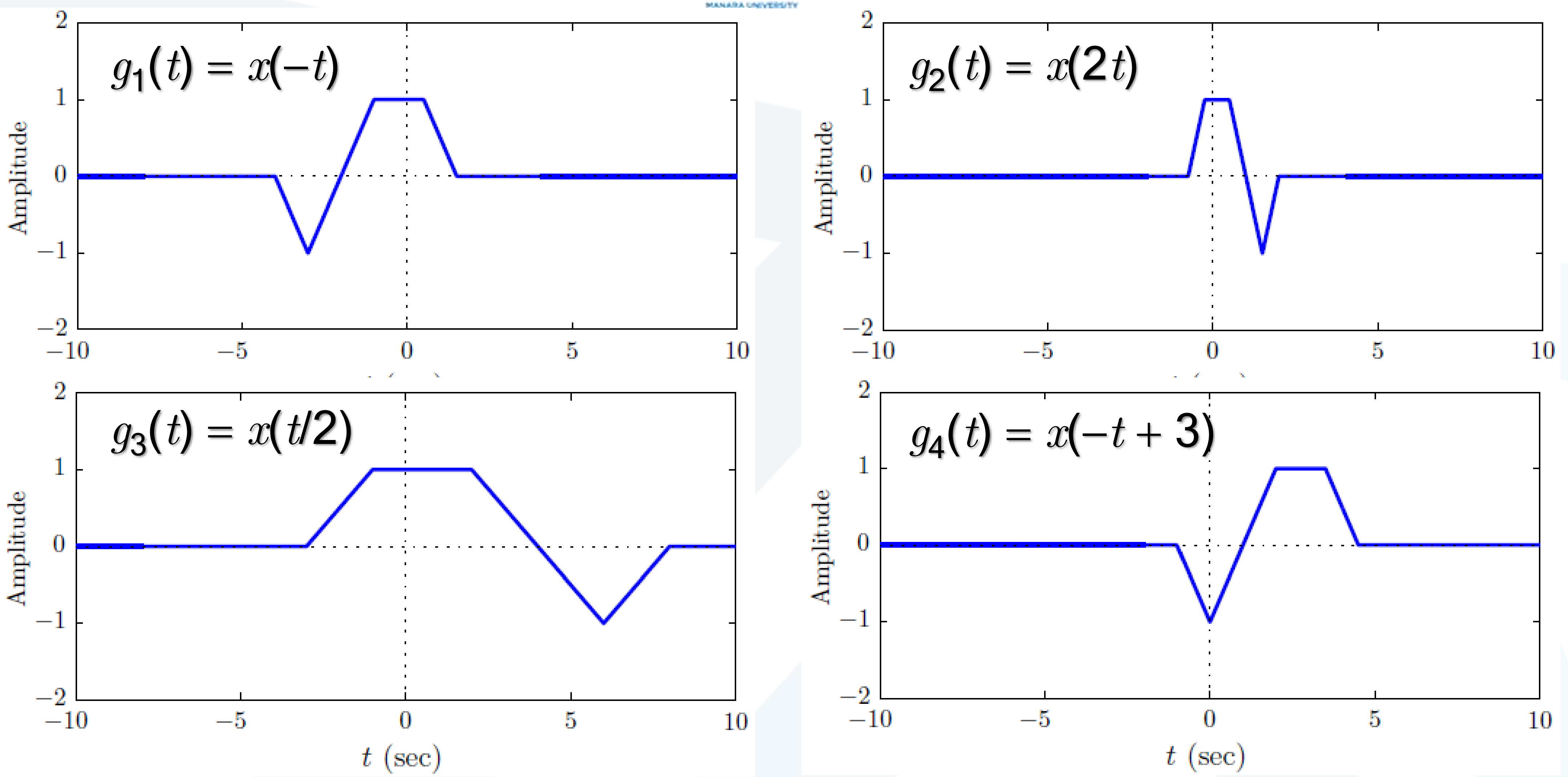


a. $g_1(t) = x(-t)$

b. $g_2(t) = x(2t)$

c. $g_3(t) = x(t/2)$

d. $g_4(t) = x(-t+3)$



g1(t) = x(−t)
g2(t) = x(2t)
g3(t) = x(t/2)
g4(t) = x(−t + 3)
Amplitude
2
1
0
−1
−2
−10
−5
0
5
10
t (sec)

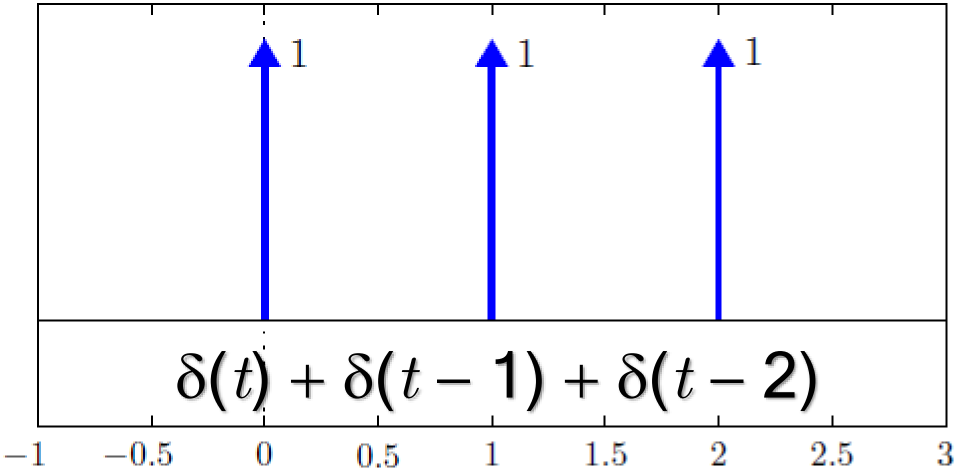
4. Sketch each of the following functions:

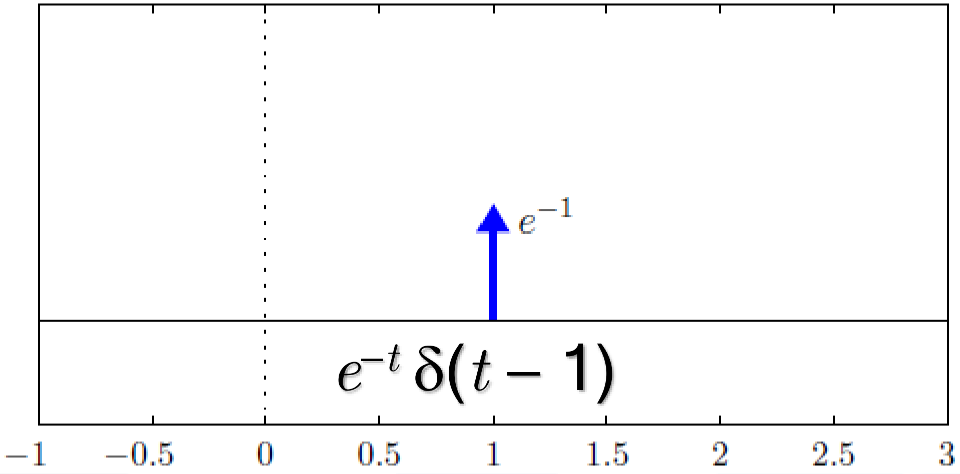
a. $\delta(t) + \delta(t-1) + \delta(t-2)$

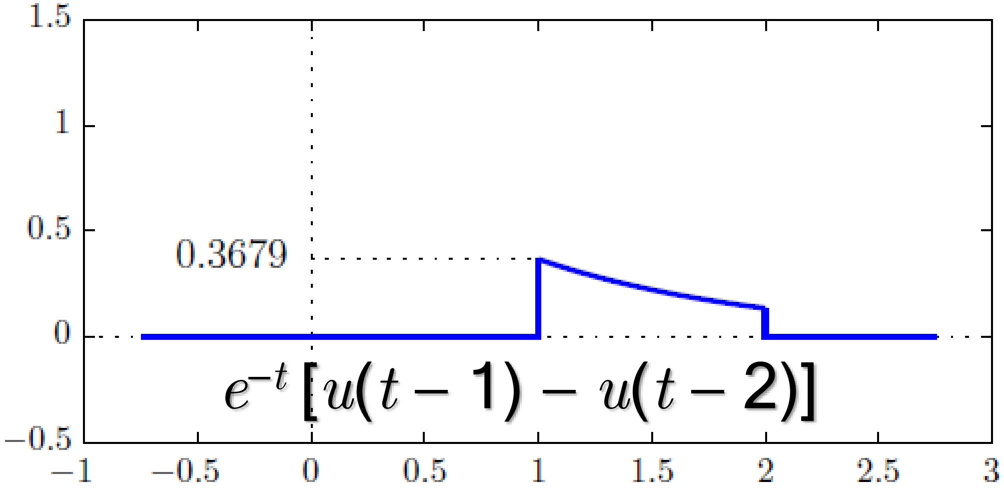
b. $e^{-t}\,\delta(t-1)$

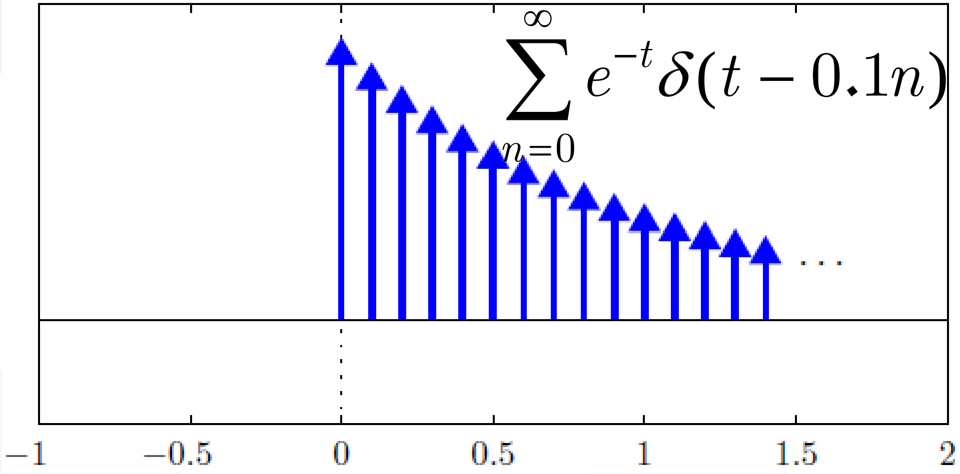
c. $e^{-t}\,[u(t-1) - u(t-2)]$

d. $\sum_{n=0}^{\infty} e^{-t}\delta(t - 0.1n)$


1
1
1
$\delta(t) + \delta(t-1) + \delta(t-2)$
−1
−0.5
0
0.5
1
1.5
2
2.5
3


e^{-1}
$e^{-t}\delta(t-1)$
−1
−0.5
0
0.5
1
1.5
2
2.5
3


1.5
1
0.5
0
−0.5
0.3679
$e^{-t}[u(t-1) - u(t-2)]$
−1
−0.5
0
0.5
1
1.5
2
2.5
3


$\sum_{n=0}^{\infty} e^{-t}\delta(t-0.1n)$
...
−1
−0.5
0
0.5
1
1.5
2

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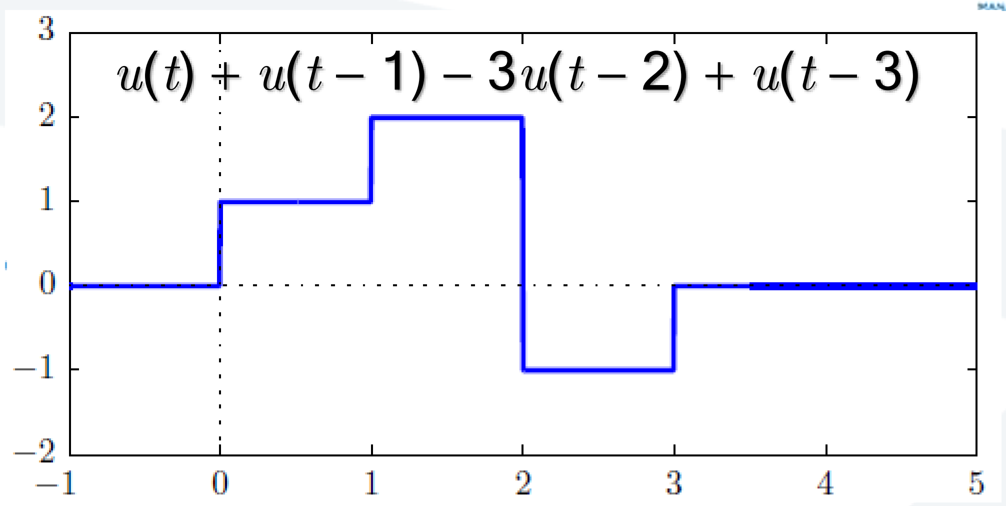
5. Sketch each of the following functions in the time interval $-1 \leq t \leq 5$:

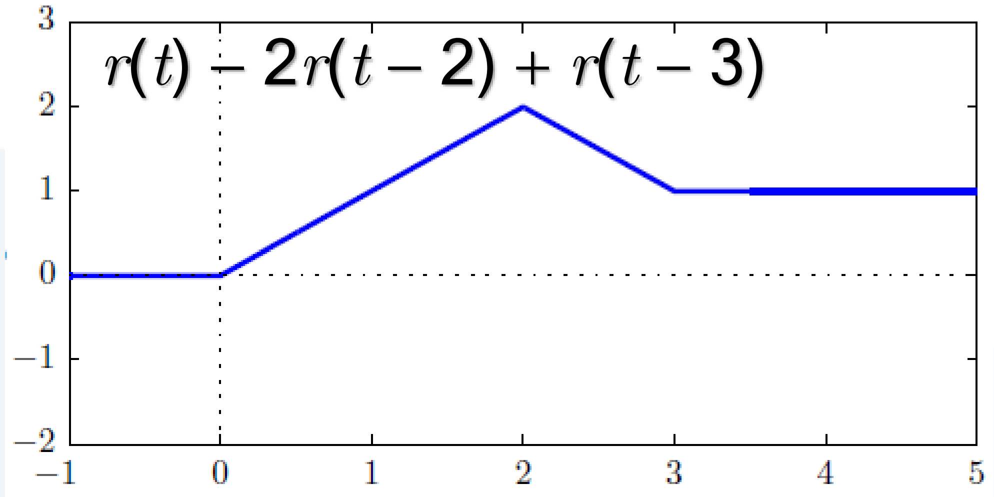
a. $u(t) + u(t-1) - 3u(t-2) + u(t-3)$

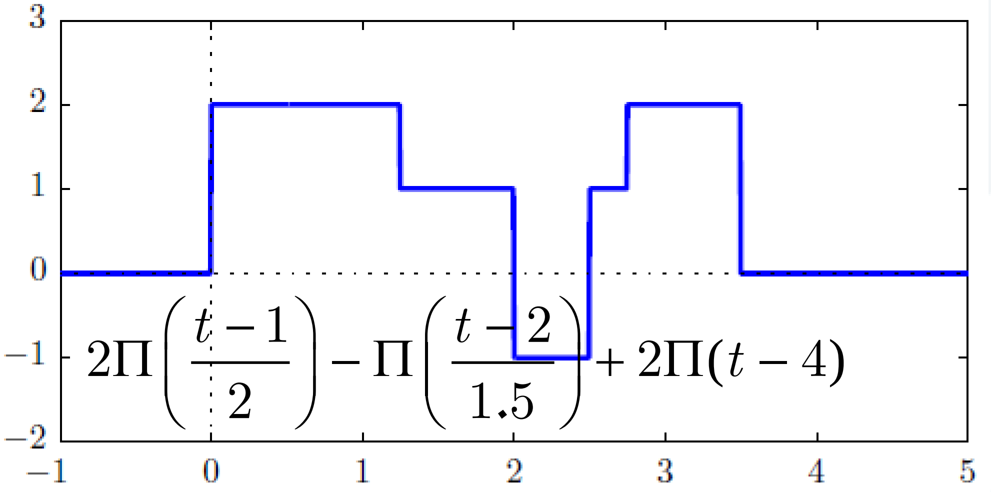
b. $r(t) - 2r(t-2) + r(t-3)$

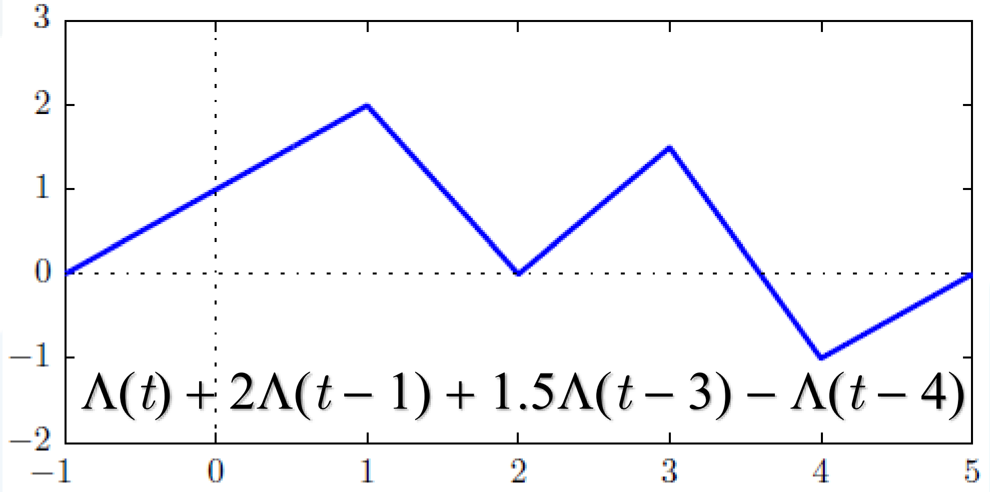
c. $2\Pi\left(\frac{t-1}{2}\right) - \Pi\left(\frac{t-2}{1.5}\right) + 2\Pi(t-4)$

d. $\Lambda(t) + 2\Lambda(t-1) + 1.5\Lambda(t-3) - \Lambda(t-4)$

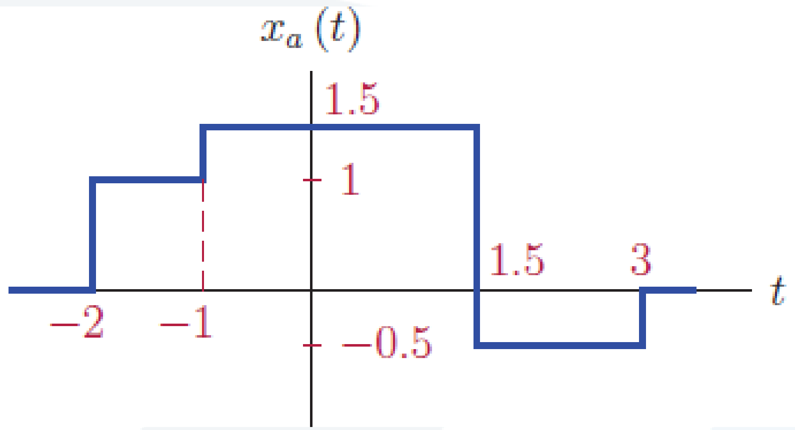

$u(t) + u(t-1) - 3u(t-2) + u(t-3)$


$r(t) - 2r(t-2) + r(t-3)$


$2\Pi\left(\frac{t-1}{2}\right) - \Pi\left(\frac{t-2}{1.5}\right) + 2\Pi(t-4)$


$\Lambda(t) + 2\Lambda(t-1) + 1.5\Lambda(t-3) - \Lambda(t-4)$

6. Express the signal shown using scaled and time shifted unit step functions:

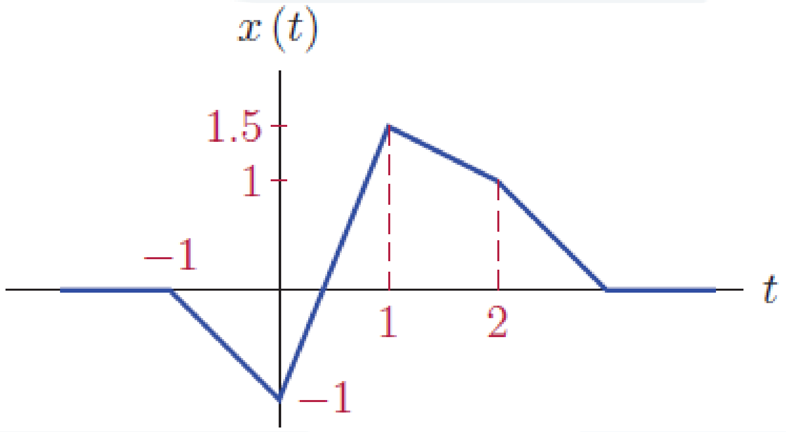


$$x_a(t) = u(t+2) + 0.5u(t+1) - 2u(t-1.5) + 0.5u(t-3)$$

7. Express the signal shown:

a. Unit-ramp functions

b. Unit-triangle functions



$$x(t) = -r(t+1) + 3.5r(t) - 3r(t-1) - 0.5r(t-2) + r(t-3)$$

$$x(t) = -\Lambda(t) + 1.5\Lambda(t-1) + \Lambda(t-2)$$

8. Determine if each signal below is periodic or not. If the signal is periodic, determine the fundamental period and the fundamental frequency.

a. $x(t) = 2\sin(\sqrt{20}t)$ $\quad 2\pi f_0 = \sqrt{20} \Rightarrow f_0 = \sqrt{5}/\pi \text{ Hz}, \quad T_0 = 1/f_0 = \pi/\sqrt{5} \text{ sec}$

b. $x(t) = \cos^2(3t - \pi/4)$ $\quad 2\pi f_0 = 6 \Rightarrow f_0 = 3/\pi \text{ Hz}, \quad T_0 = 1/f_0 = \pi/3 \text{ sec}$

c. $x(t) = e^{-|t|}\cos(2t)$ $\quad$ X

d. $x(t) = e^{j(2t+\pi/5)}$ $\quad 2\pi f_0 = 2 \Rightarrow f_0 = 1/\pi \text{ Hz}, \quad T_0 = 1/f_0 = \pi \text{ sec}$

e. $x(t) = e^{(-2+j5)t}$ $\quad$ X

9. Determine the normalized energy of each of the signals

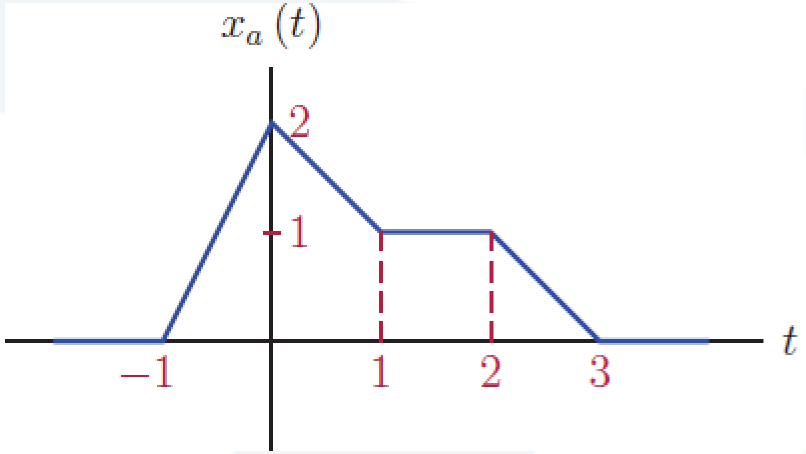
a. $x(t) = e^{-2|t|}$

b. $x(t) = e^{-2t}u(t)$

$$a.\quad E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{0} e^{-4t} dt + \int_{0}^{\infty} e^{-4t} dt = \frac{1}{2}$$

$$b.\quad E_x = \int_{0}^{\infty} e^{-4t} dt = \frac{1}{4}$$

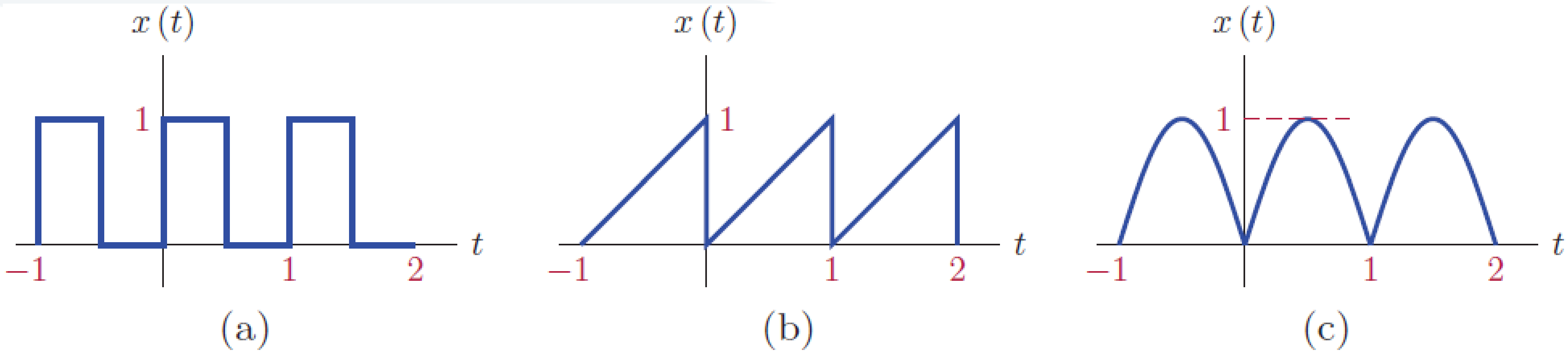
10. Determine the normalized energy of the signal below



$$x_a(t) = \begin{cases} 0, & t < -1 \text{ or } t > 3 \\ 2t+2, & -1 < t < 0 \\ -t+2, & 0 < t < 1 \\ 1, & 1 < t < 2 \\ -t+3, & 2 < t < 3 \end{cases}$$

$$E_x = \int_{-1}^{0} (2t+2)^2 dt + \int_{0}^{1} (-t+2)^2 dt + \int_{1}^{2} (1)^2 dt + \int_{2}^{3} (-t+3)^2 dt = 5$$

11. Determine the normalized average power of the signals below



$$P_x = \int_0^{1/2} (1)^2 dt = \frac{1}{2}$$

$$P_x = \int_0^{1} (t)^2 dt = \frac{1}{3}$$

$$P_x = \int_0^{1} \sin^2(\pi t) dt = \frac{1}{2}$$

12. For each of the signals listed below, find the even and odd components $x_e(t)$ and $x_o(t)$. In each case, sketch the original signal and its two components

a. $x(t) = e^{-3|t|} \cos(t)$

b. $x(t) = e^{-3|t|} \sin(t)$

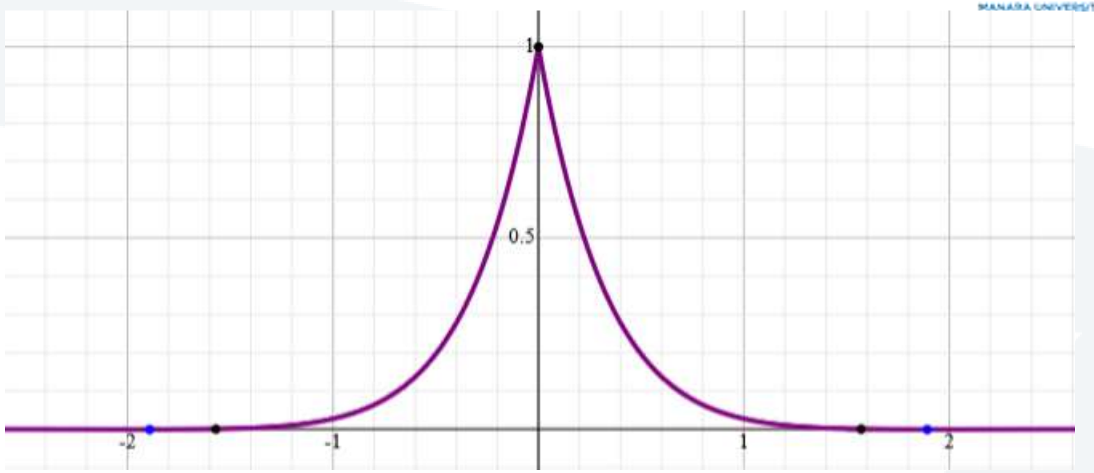
c. $x(t) = e^{-2|t-1|}$

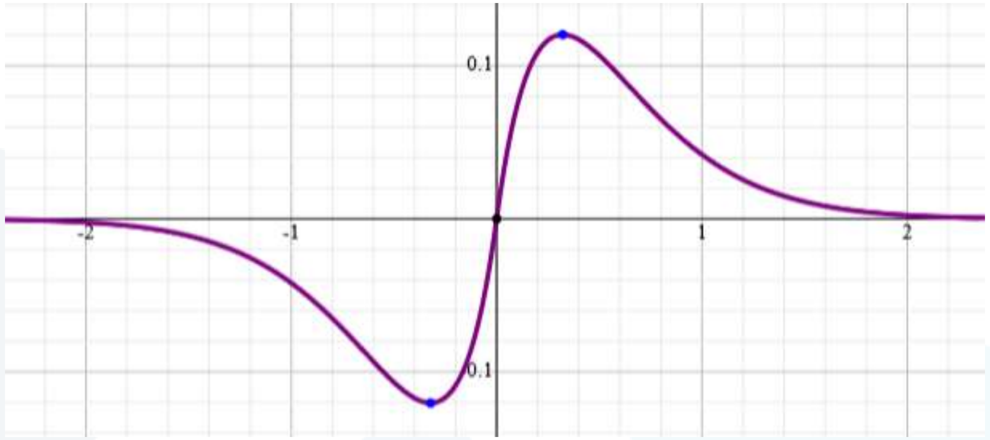
$$x_e(t) = \tfrac{1}{2} e^{-3|t|} \cos(t) + \tfrac{1}{2} e^{-3|-t|} \cos(-t) = e^{-3|t|} \cos(t)$$

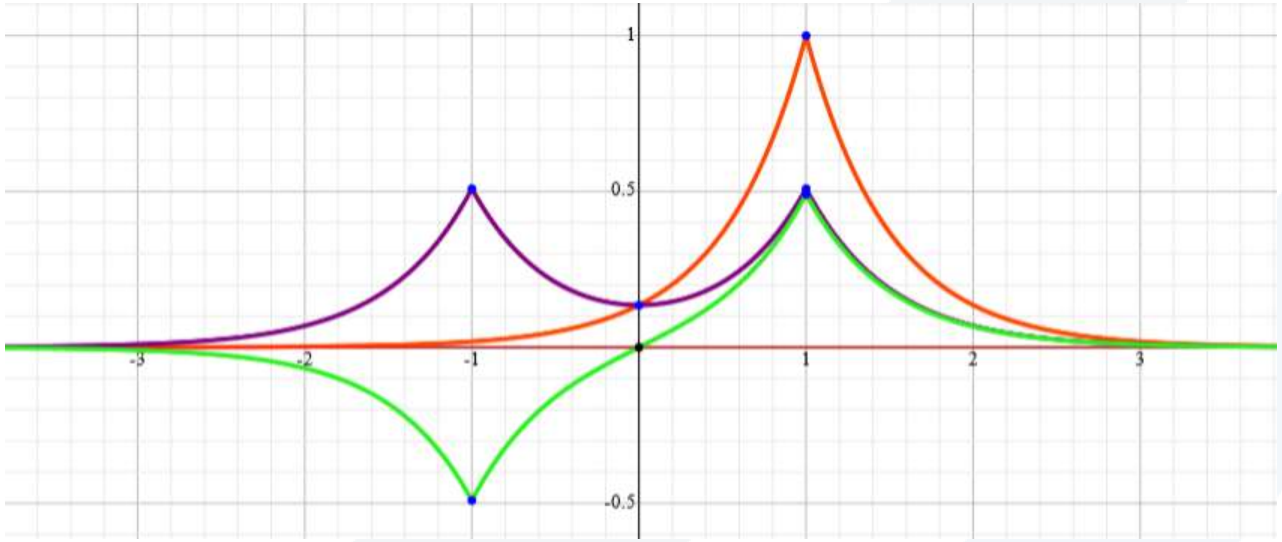
$$x_o(t) = \tfrac{1}{2} e^{-3|t|} \cos(t) - \tfrac{1}{2} e^{-3|-t|} \cos(-t) = 0$$

$$x_e(t) = \tfrac{1}{2} e^{-3|t|} \sin(t) + \tfrac{1}{2} e^{-3|-t|} \sin(-t) = 0$$

$$x_o(t) = \tfrac{1}{2} e^{-3|t|} \sin(t) - \tfrac{1}{2} e^{-3|-t|} \sin(-t) = e^{-3|t|} \sin(t)$$







$$x_e(t) = \frac{1}{2}e^{-2|t-1|} + \frac{1}{2}e^{-2|-t-1|}$$

$$x_o(t) = \frac{1}{2}e^{-2|t-1|} - \frac{1}{2}e^{-2|-t-1|}$$