Exercise 6: Fourier Analysis for Continuous Time Signals and Systems

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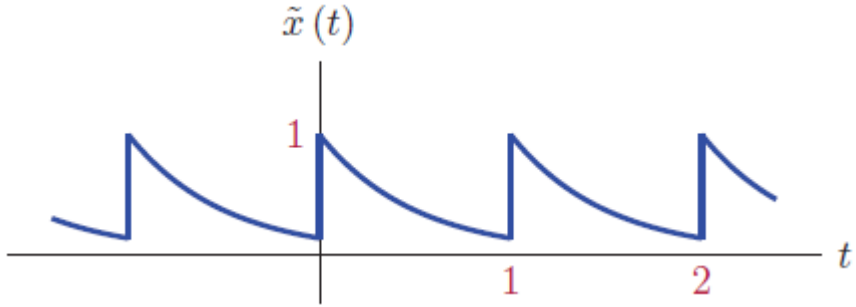
1. **Determine the TFS coefficients for the periodic signal shown. One period of the signal is $\tilde{x}(t) = e^{-2t}$ for $0 < t < 1\ s$.**

The fundamental period is $T_0 = 1$ second which corresponds to a fundamental frequency of $f_0 = 1$ Hz or $\omega_0 = 2\pi$ rad/s.



$$a_0 = \int_0^1 e^{-2t}dt = 0.4323$$

$$a_k = 2\int_0^1 e^{-2t}\cos(2\pi kt)dt = \frac{0.8647}{1+\pi^2k^2}$$

$$b_k = 2\int_0^1 e^{-2t}\sin(2\pi kt)dt = \frac{0.8647\pi k}{1+\pi^2k^2}$$

$$\tilde{x}(t) = 0.4323 + \sum_{k=1}^{\infty}\frac{0.8647}{1+\pi^2k^2}\cos(2\pi kt) + \sum_{k=1}^{\infty}\frac{0.8647\pi k}{1+\pi^2k^2}\sin(2\pi kt)$$

2. Consider the periodic signal $\tilde{x}(t)$ of exercise 1

a. Determine the EFS coefficients from the TFS coefficients obtained in exercise 1.

b. Determine the EFS coefficients by direct application of the analysis equation.

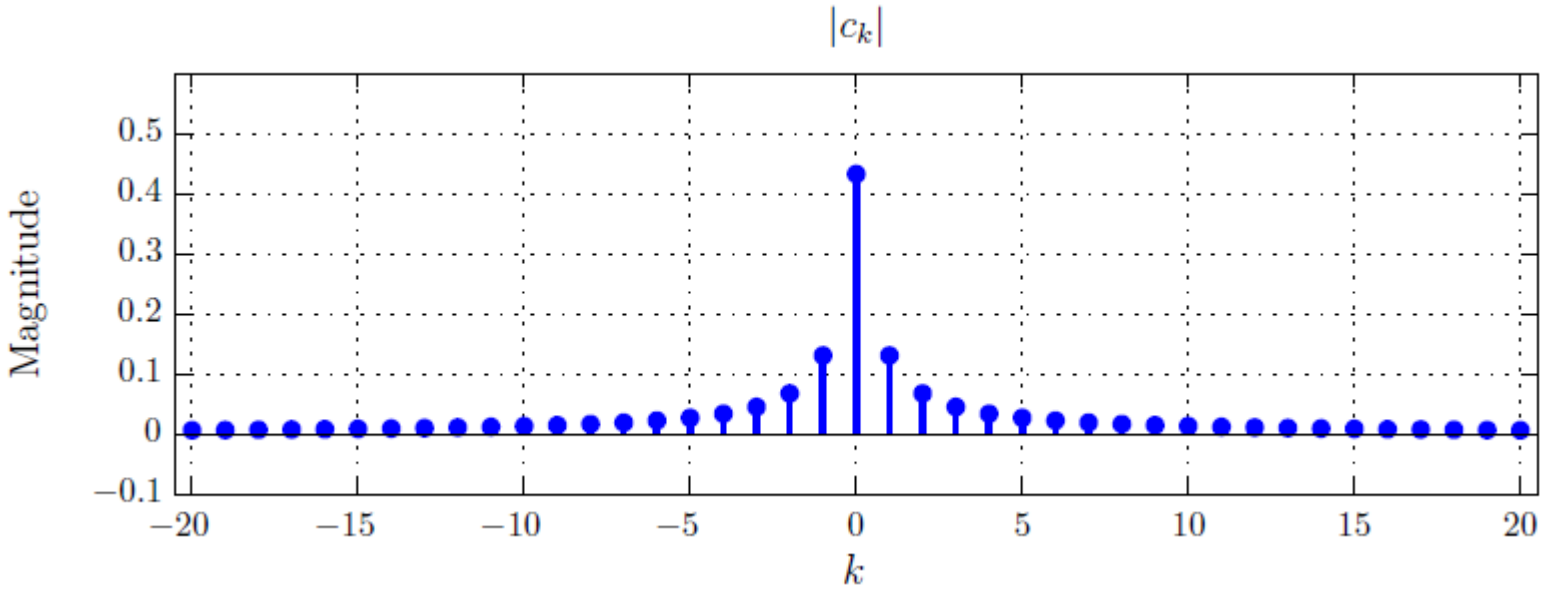
c. Sketch the line spectrum (magnitude and phase).

$$a.\ c_0 = a_0 = 0.4323$$

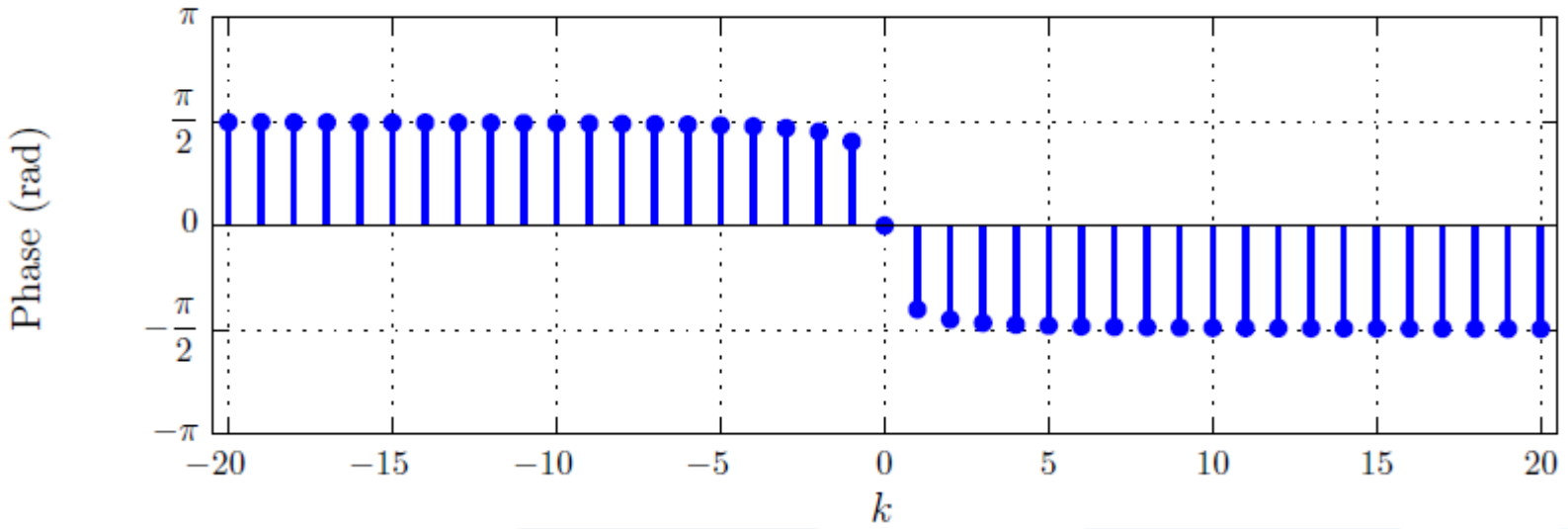
$$c_k = \frac{1}{2}(a_k - jb_k) = \frac{0.4323}{1 + j2\pi k}, \quad k > 0$$

$$c_{-k} = \frac{1}{2}(a_k + jb_k) = \frac{0.4323}{1 - j2\pi k}, \quad k > 0$$

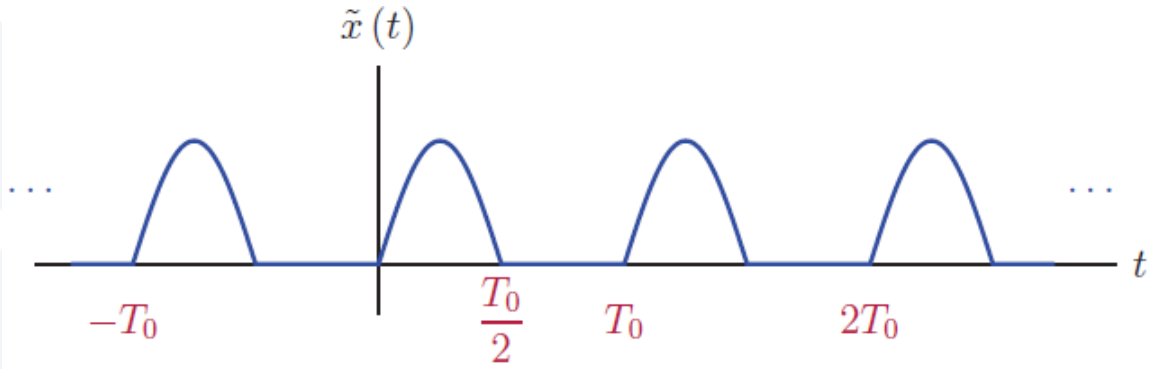
$$b.\ c_k = \int_0^1 e^{-2t} e^{-j2\pi kt} dt = \frac{0.4323}{1 + j2\pi k}, \quad \text{all } k$$



$$\theta_k = \angle\, c_k$$



3. Determine the EFS coefficients of the half-wave rectified signal shown:



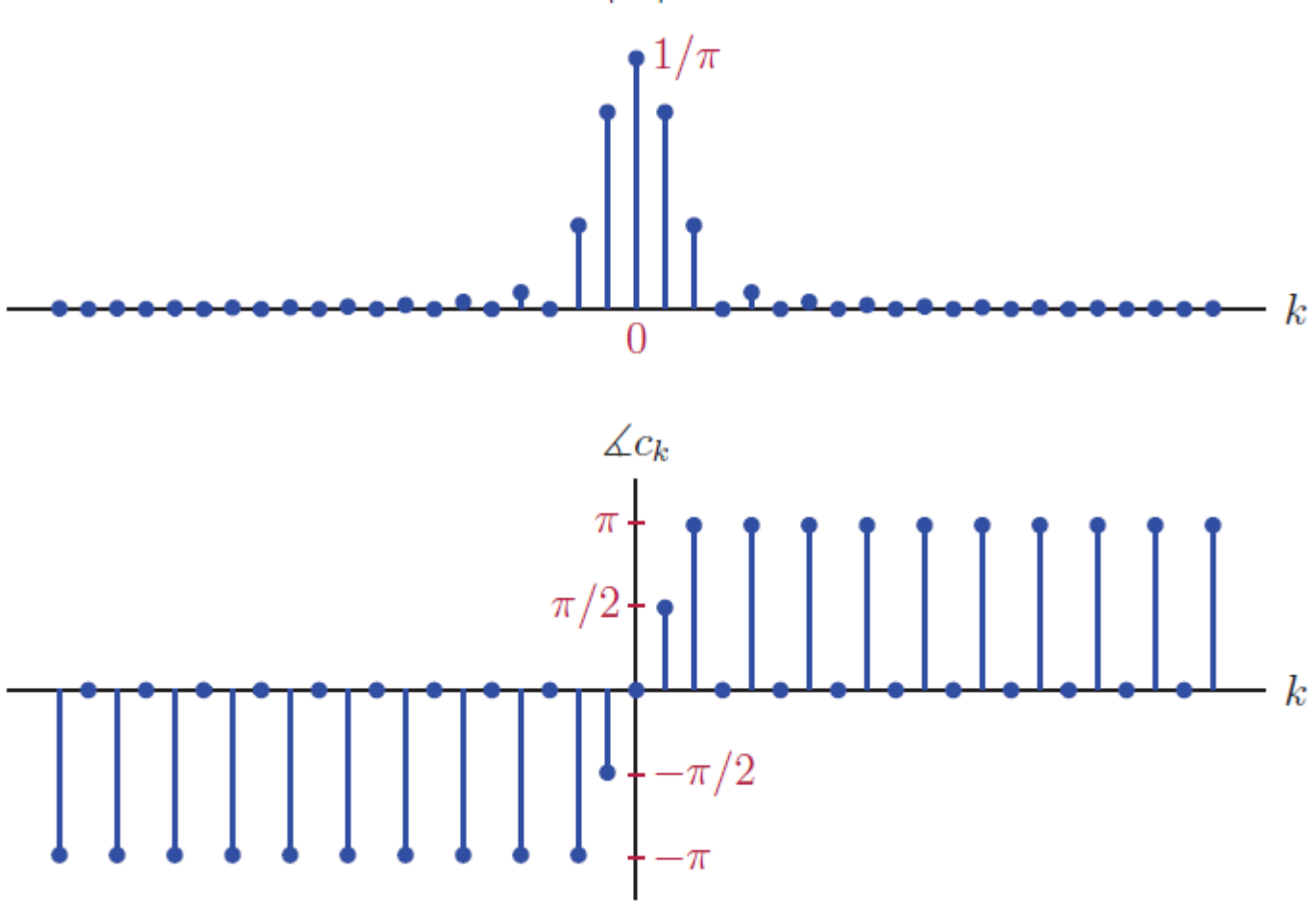
$$c_k = \frac{1}{T_0}\int_0^{T_0/2} \sin(2\pi t/T_0)\, e^{-j2\pi kt/T_0}\, dt$$

$$= \frac{1}{4\pi(k-1)}\left[e^{-j\pi(k-1)} - 1\right] - \frac{1}{4\pi(k+1)}\left[e^{-j\pi(k+1)} - 1\right]$$

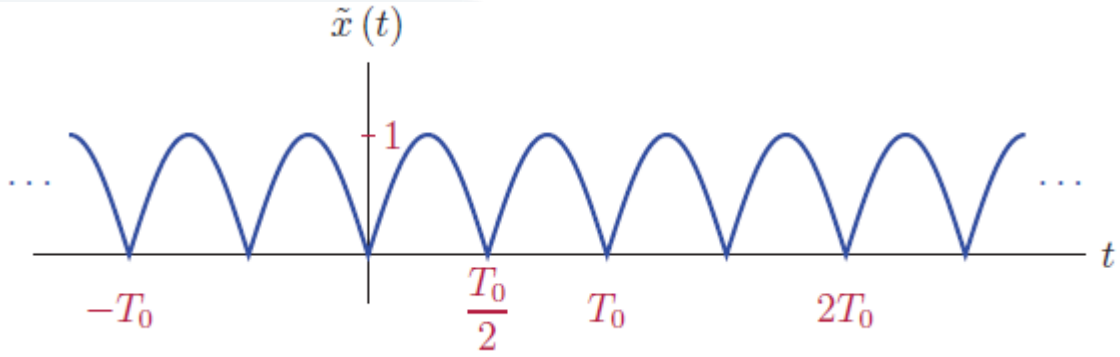
Case 1: k odd and $k \neq \pm 1$ $\quad e^{-j\pi(k-1)} = e^{-j\pi(k+1)} = 1 \Rightarrow c_k = 0$

Case 2: $k = 1$ $\quad c_1 = -\frac{j}{4}$ $\qquad$ **Case 3**: $k = -1$ $\quad c_{-1} = \frac{j}{4}$

Case 4: k even $\quad e^{-j\pi(k-1)} - 1 = e^{-j\pi(k+1)} - 1 = -2 \Rightarrow c_k = \frac{-1}{\pi(k^2-1)}$


$|c_k|$
$1/\pi$
k
0
$\angle c_k$
π
$\pi/2$
k
$-\pi/2$
$-\pi$

4. Determine the EFS coefficients of the full-wave rectified signal shown:



$$c_k = \frac{1}{T_0}\int_0^{T_0} |\sin(2\pi t/T_0)|\, e^{-j2\pi kt/T_0} dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} \sin(2\pi t/T_0) e^{-j2\pi kt/T_0} dt - \frac{1}{T_0}\int_{T_0/2}^{T_0} \sin(2\pi t/T_0) e^{-j2\pi kt/T_0} dt$$

$$c_k = \begin{cases} \dfrac{2}{\pi(1-k^2)}, & k \text{ even} \\ 0, & k \text{ odd} \end{cases}$$