مقرر الإشارات والنظم

قسم هندسة الروبوت والأنظمة الذكية

د. السموءل صالح

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محاضرة العملي الأسبوع ٥

الفصل الثاني ٢٠٢٢/٢٠٢١

Fourier series

Trigonometric Fourier series: TFS

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \cos(k\omega_0 t)\, dt\,, \quad \text{for} \quad k = 1, \ldots, \infty$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \sin(k\omega_0 t)\, dt\,, \quad \text{for} \quad k = 1, \ldots, \infty$$

$$a_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t)\, dt \qquad \text{(dc component)}$$

$$\tilde{x}(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

Fourier series

Homework

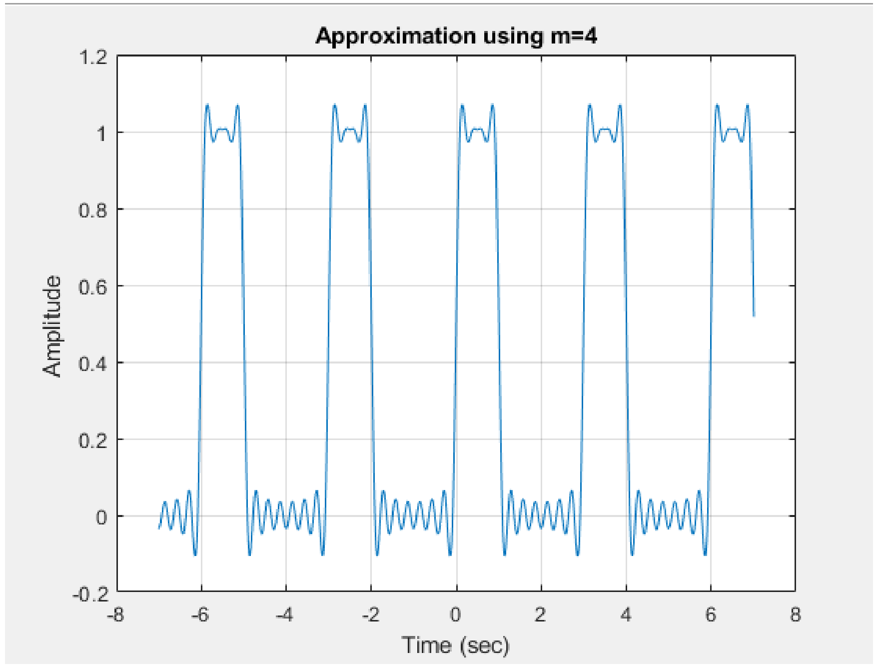
Periodic Continuous-Time Signals

```
m = 10; % Number of harmonics to be used.
t = [ -7:0.01:7]; % Create a vector of time instants.
f0 = ..........................................; % Fundamental frequency is f0 = 1/3 Hz.
omega0 = 2*pi*f0;
a0= ..........................................% Recall that a0 =1/3.
% We will start by setting x(t)=a0.
% Start the loop to compute the contribution of each harmonic.
 for k = 1:m,
 ak = ..................................
 bk = ............................................
 x = a0 +ak*cos(k*omega0 *t)+bk*sin (k*omega0 *t);
 end;
```

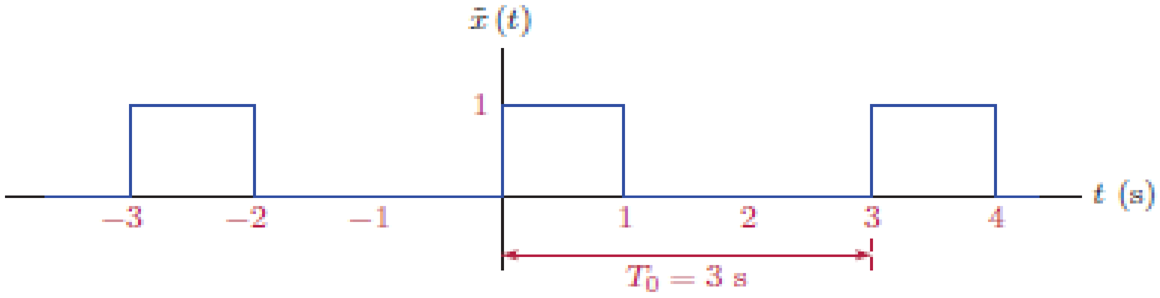
```
% Graph the resulting approximation to x(t).
clf;
plot(t,x); grid;
title('Approximation using m=4');
xlabel ('Time (sec)');
ylabel ('Amplitude ');
```

Fourier series

Periodic Continuous-Time Signals



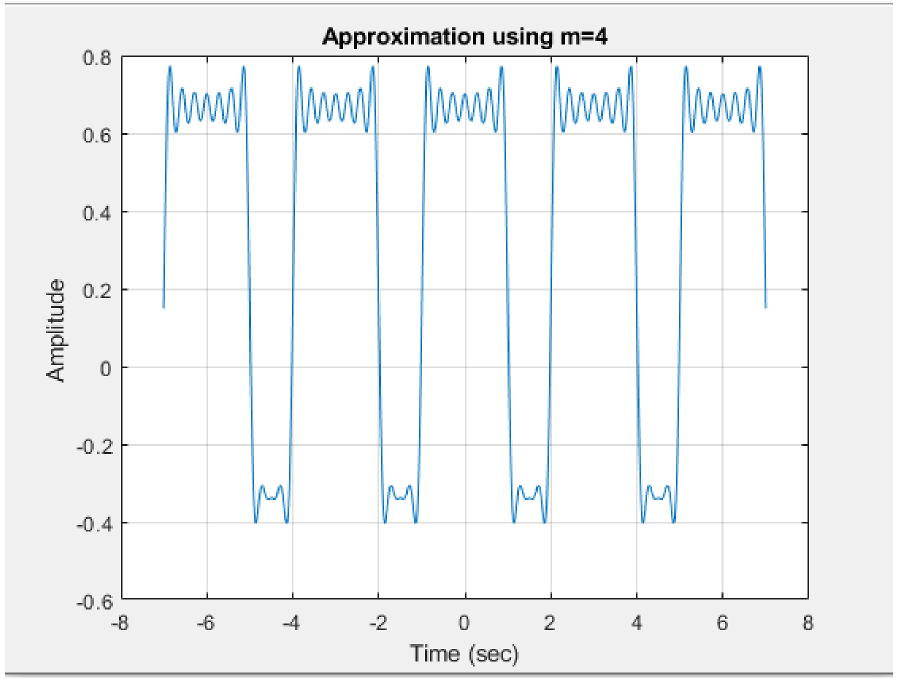
Homework: group1

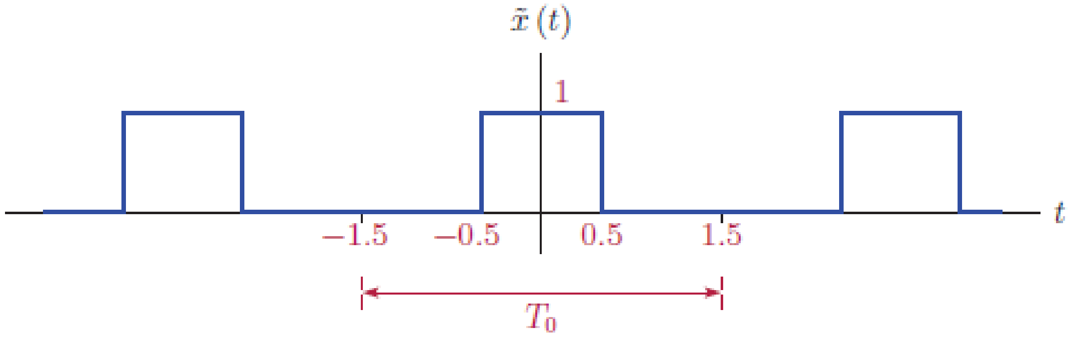


Fourier series

Homework: group2

Periodic Continuous-Time Signals

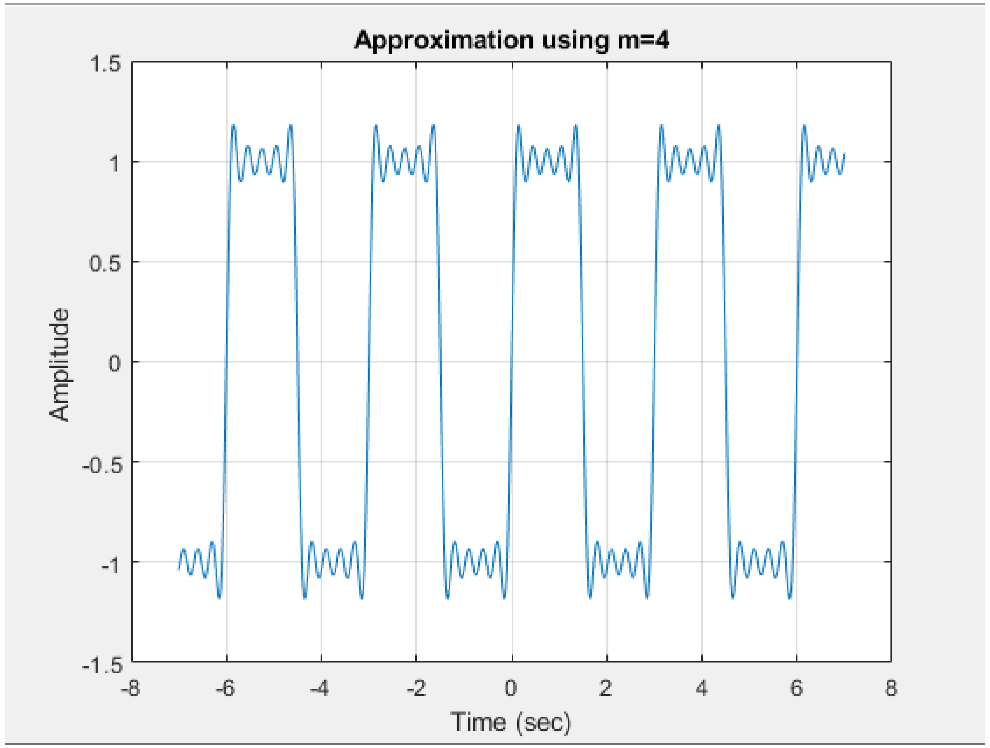


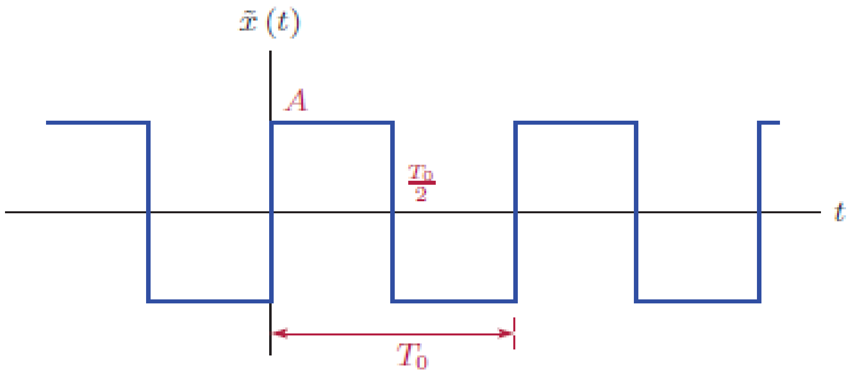


Fourier series

Homework: group3

Periodic Continuous-Time Signals





Fourier series

Periodic Continuous-Time Signals

Exponential Fourier series : EFS

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \tilde{x}(t) \, e^{-jk\omega_0 t} \, dt$$

$$c_k = \frac{1}{2}(a_k - jb_k)$$

$$c_{-k} = \frac{1}{2}(a_k + jb_k)$$

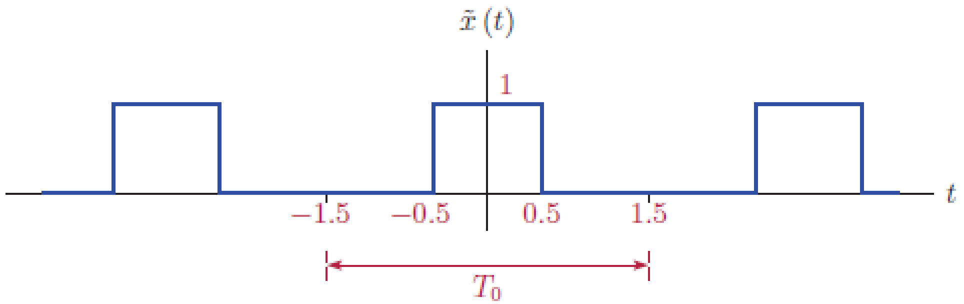
In general, the coefficients of the EFS representation of a periodic signal $\tilde{x}(t)$ are complex-valued. They can be graphed in the form of a line spectrum if each coefficient is expressed in polar complex form with its magnitude and phase:

$$c_k = |c_k| \, e^{j\theta_k} \tag{4.67}$$

Fourier series

Periodic Continuous-Time Signals

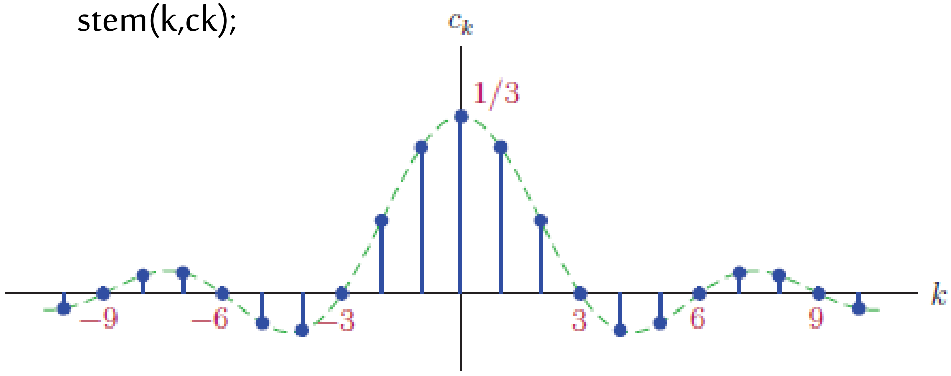
مثال ١



$$c_k = \frac{1}{T_0}\int_{-0.5}^{+0.5}(1)e^{-jkw_0 t}dt = \frac{\sin\left(\frac{\pi k}{T_0}\right)}{\pi k} = \frac{1}{T_0}\,sinc\,\left(\frac{\pi k}{T_0}\right)$$

Homework: group 4

```
T0= 3;
k = [ -25:25]; % Create a vector of index values .
k = k+eps; % Avoid division by zero.
ck =.......................................% Compute the EFS coefficients.
clf;
stem(k,ck);
```



A line graph of the set of coefficients c_k

استبدل حدود التكامل علماً ان قيمة الدور ٣ وأعد الحساب مرتين:

-1 , +1

-0.25 , +0.25

ماذا تلاحظ :

Fourier Transform

Non-Periodic Continuous-Time Signals

(Inverse transform)

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

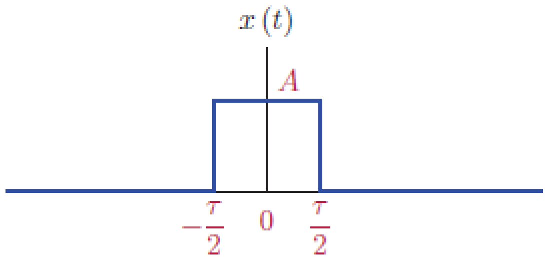
Forward transform)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Fourier Transform

Non-Periodic Continuous-Time Signals

مثال ١



أوجد تحويل فورييه للنبضة المستطيلة القياسية المبينة في الشكل :

من قانون تحويل فورييه :

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt = \int_{-\infty}^{-T/2} x(t)e^{-j2\pi ft}dt + \int_{-T/2}^{+T/2} x(t)e^{-j2\pi ft}dt + \int_{+T/2}^{+\infty} x(t)e^{-j2\pi ft}dt$$

يكون الحدان الأول والثالث من العلاقة السابقة معدومين لأن الاشارة s(t) معدومة في المجالين الموضحين فيصبح التكامل :

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt = \int_{-T/2}^{+T/2} x(t)e^{-j2\pi ft}dt = \int_{-T/2}^{+T/2} A\, e^{-j2\pi ft}dt = \frac{A}{-j2\pi f}\left[e^{-j2\pi ft}\right]_{-T/2}^{+T/2}$$

بالتعويض بحدود التكامل :

$$X(f) = \frac{A}{-j2\pi f}\left[e^{-\frac{j2\pi fT}{2}} - e^{+-\frac{j2\pi fT}{2}}\right] = \frac{A}{\pi f}\left[sin(\pi fT)\right]$$

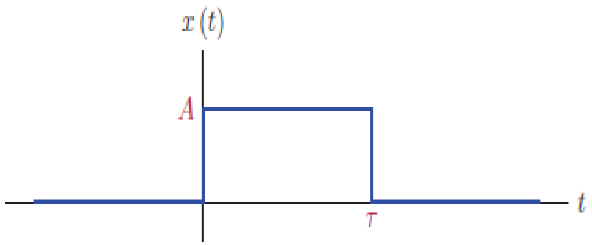
نضرب الحد السابق ب T ونقسم عليها :

$$X(f) = \frac{TA}{T\pi f}\left[sin(\pi fT)\right] = \text{AT sinc}\,(\pi fT) = \text{AT sinc}\left(\frac{w}{2}T\right)$$

Fourier Transform

Non-Periodic Continuous-Time Signals

مثال 2



$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt = \int_{-\infty}^{0} x(t)e^{-j2\pi ft}dt + \int_{0}^{T} x(t)e^{-j2\pi ft}dt + \int_{T}^{+\infty} x(t)e^{-j2\pi ft}dt$$

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt = 0 + \int_{0}^{T} x(t)e^{-j2\pi ft}dt + 0 = \int_{0}^{T} A\,e^{-j2\pi ft}dt = \frac{A}{-j2\pi f}[e^{-j2\pi ft}]_0^T = \frac{A}{-j2\pi f}[e^{-j2\pi fT} - 1]$$

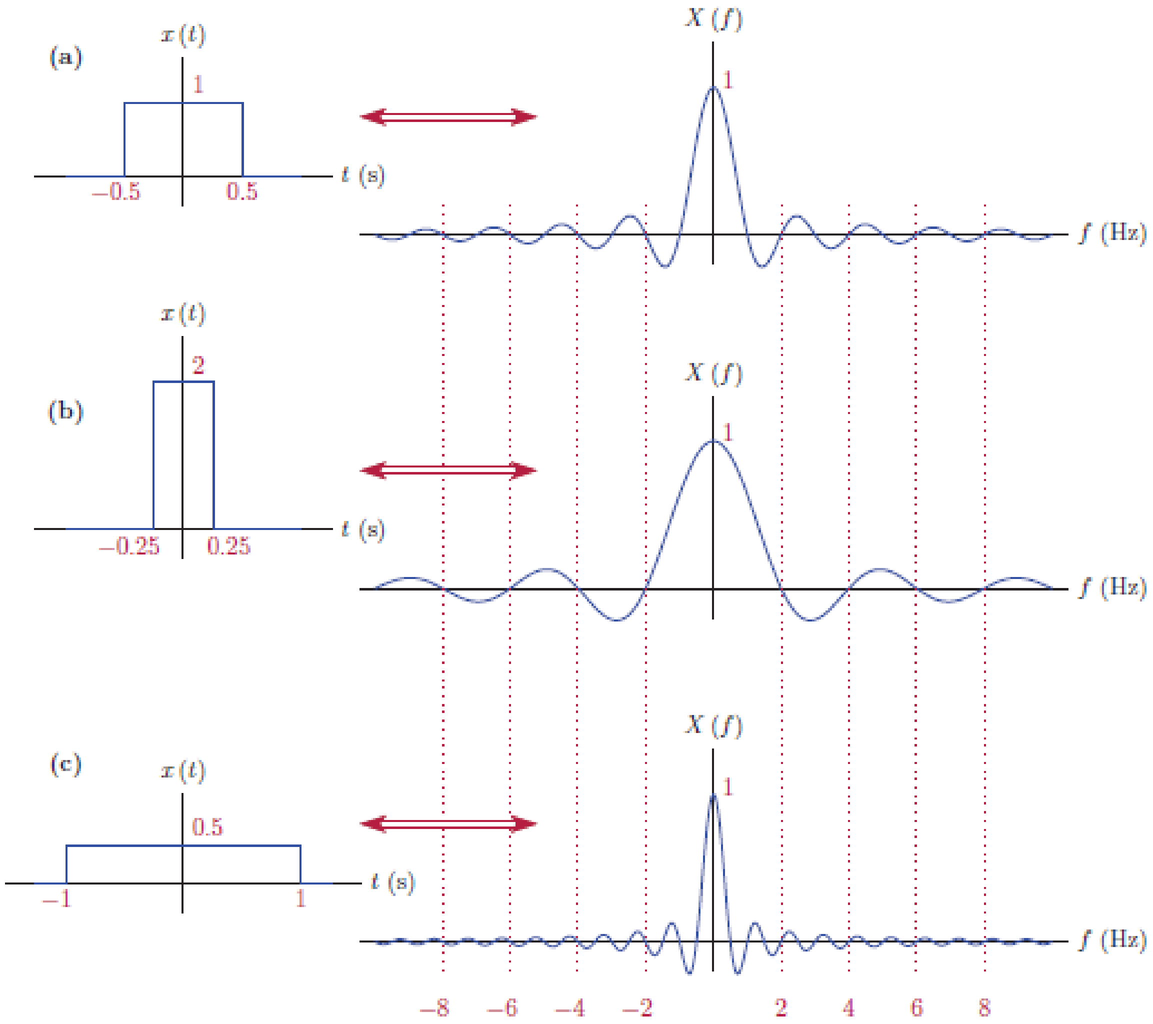
$$X(f) = \frac{A}{j2\pi f}\left[1 - e^{-j2\pi fT}\right] = \frac{A}{j2\pi f}\left[1 - e^{-j\pi fT}.e^{-j\pi fT}\right] = \frac{Ae^{-j\pi fT}}{j2\pi f}\left[e^{+j\pi fT} - e^{-j\pi fT}\right] = \frac{\mathrm{A}e^{-j\pi fT}}{\pi f} sin(\pi fT)$$

$$X(f) = \frac{TAe^{-j\pi fT}}{T\pi f}[sin(\pi fT)] = \mathrm{AT}e^{-j\pi fT}\mathrm{sinc}\,(\pi fT)$$

Fourier Transform

Non-Periodic Continuous-Time Signals

AT sinc $(\boldsymbol{\pi f T})$



Fourier Transform

Non-Periodic Continuous-Time Signals

- القيم الأكبر لمطال الطيف عند الترددات الأقرب للصفر أي ان التوافقيات ذات الترددات الاخفض لها الأهمية العليا من المكونات الترددية للإشارة وتتناقص مطالات التوافقيات ذات الترددات العالية
- تحدث نقاط التقاطع مع الصفر عند عدد صحيح من 1/T حيث T هو عرض النبضة المستطيلة وبالتالي كلما قل العرض تصبح النقاط الصفرية متباعدة ويتمدد الطيف بالاتجاهين أي ان مطالات التوافقيات ذات التردد الاعلى تزداد أهميتها
- اما يزيادة عرض النبضة فتتقارب نقاط التقاطع مع الصفر ويتقلص الطيف في الاتجاهين أي أن التوافقيات ذات الترددات الأعلى تصبح بمطالات صغيرة ومهملة

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Linearity

Linearity of the Fourier transform:

$$\alpha_1 \, x_1(t) + \alpha_2 \, x_2(t) \overset{\mathcal{F}}{\longleftrightarrow} \alpha_1 \, X_1(\omega) + \alpha_2 \, X_2(\omega) \tag{4.161}$$

Proof: Using the forward transform equation given by Eqn. (4.127) with the time domain signal $[\alpha_1 \, x_1(t) + \alpha_2 \, x_2(t)]$ leads to:

$$\begin{aligned}
\mathcal{F}\{\alpha_1 \, x_1(t) + \alpha_2 \, x_2(t)\} &= \int_{-\infty}^{\infty} [\alpha_1 \, x_1(t) + \alpha_2 \, x_2(t)] \, e^{-j\omega t} \, dt \\
&= \int_{-\infty}^{\infty} \alpha_1 \, x_1(t) \, e^{-j\omega t} \, dt + \int_{-\infty}^{\infty} \alpha_2 \, x_2(t) \, e^{-j\omega t} \, dt \\
&= \alpha_1 \int_{-\infty}^{\infty} x_1(t) \, e^{-j\omega t} \, dt + \alpha_2 \int_{-\infty}^{\infty} x_2(t) \, e^{-j\omega t} \, dt \\
&= \alpha_1 \, \mathcal{F}\{x_1(t)\} + \alpha_2 \, \mathcal{F}\{x_2(t)\}
\end{aligned} \tag{4.162}$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Time Shifting

$$A rect(\frac{t \pm t0}{T})$$

$$\Downarrow$$

$$\text{AT sinc}\left(\frac{\boldsymbol{w}}{\boldsymbol{2}}\boldsymbol{T}\right)\boldsymbol{e^{\pm j2\pi f t_0}}$$

it can be shown that

$$x(t-\tau) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)\, e^{-j\omega\tau} \tag{4.218}$$

Proof: Applying the Fourier transform integral in Eqn. (4.127) to $x(t-\tau)$ we obtain

$$\mathcal{F}\{x(t-\tau)\} = \int_{-\infty}^{\infty} x(t-\tau)\, e^{-j\omega t}\, dt \tag{4.219}$$

Let $\lambda = t - \tau$ in the integral of Eqn. (4.219) so that

$$\mathcal{F}\{x(t-\tau)\} = \int_{-\infty}^{\infty} x(\lambda)\, e^{-j\omega(\lambda+\tau)}\, d\lambda \tag{4.220}$$

The exponential function in the integral of Eqn. (4.220) can be written as a product of two exponential functions to obtain

$$\begin{aligned}\mathcal{F}\{x(t-\tau)\} &= \int_{-\infty}^{\infty} x(\lambda)\, e^{-j\omega\lambda}\, e^{-j\omega\tau}\, d\lambda \\ &= e^{-j\omega\tau} \int_{-\infty}^{\infty} x(\lambda)\, e^{-j\omega\lambda}\, d\lambda \\ &= e^{-j\omega\tau}\, X(\omega)\end{aligned} \tag{4.221}$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Frequency Shifting

For a transform pair

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

it can be shown that

$$x(t)\, e^{j\omega_0 t} \overset{\mathcal{F}}{\longleftrightarrow} X(\omega - \omega_0)$$

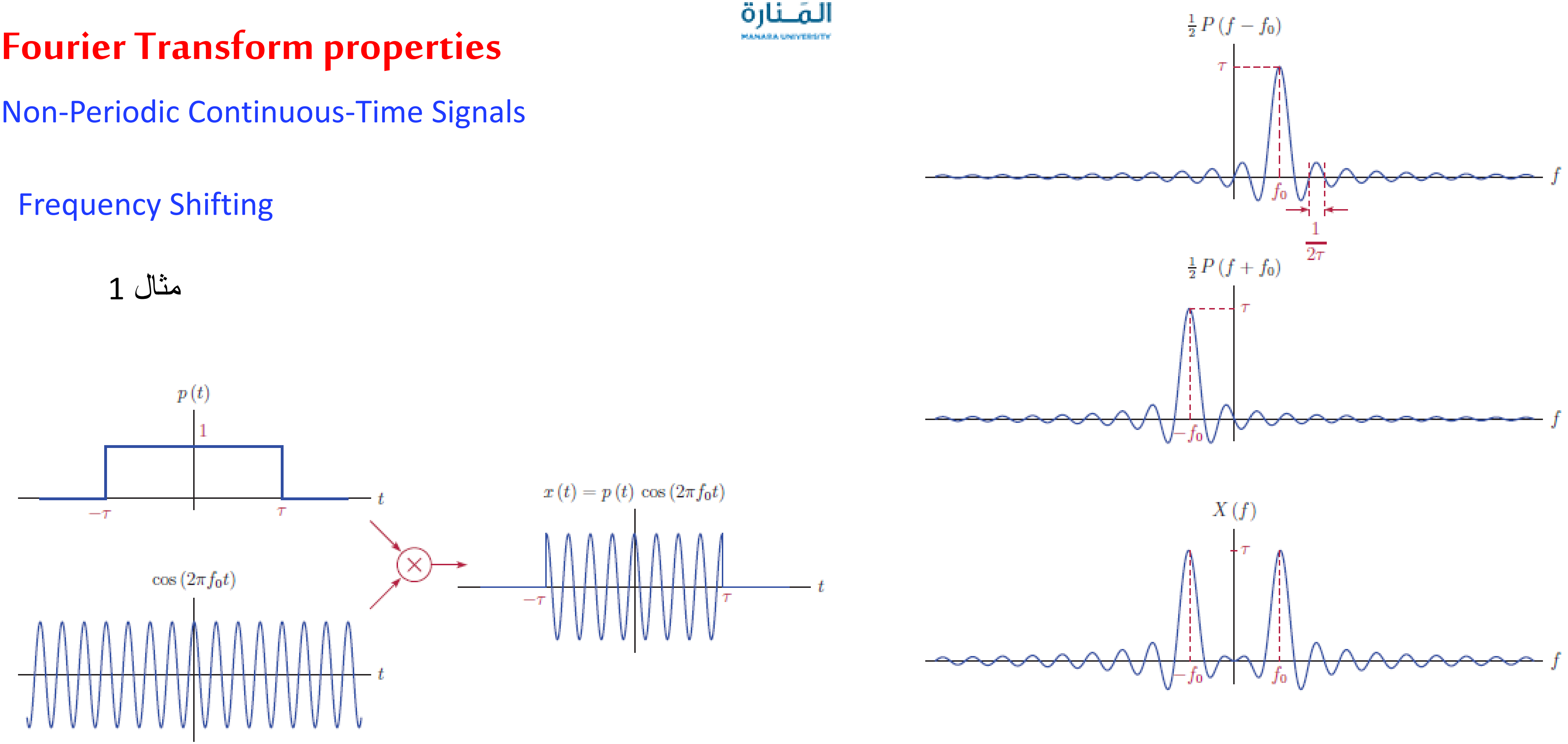
$$\begin{aligned}\mathcal{F}\left\{ x(t)\, e^{j\omega_0 t} \right\} &= \int_{-\infty}^{\infty} x(t)\, e^{j\omega_0 t}\, e^{-j\omega t}\, dt \\ &= \int_{-\infty}^{\infty} x(t)\, e^{-j(\omega-\omega_0)t}\, dt \\ &= X(\omega - \omega_0)\end{aligned}$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Frequency Shifting

مثال 1



Fourier Transform properties

Non-Periodic Continuous-Time Signals

Homework: optional

Frequency Shifting

أثبت صحة مايلي:

$$x(t)\sin(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\left[X(\omega-\omega_0)\,e^{-j\pi/2} + X(\omega+\omega_0)\,e^{j\pi/2}\right]$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Time & frequency scaling

For a transform pair

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

it can be shown that

$$x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

The parameter a is any non-zero and real-valued constant.

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Time & frequency scaling : **proof**

Proof: The Fourier transform of $x(at)$ is

$$\mathcal{F}\{x(at)\} = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt \tag{4.240}$$

A new independent variable λ will be introduced through the variable change $at = \lambda$. Let us substitute

$$t = \frac{\lambda}{a} \quad \text{and} \quad dt = \frac{d\lambda}{a} \tag{4.241}$$

in the integral of Eqn. (4.241). We need to consider the cases of $a > 0$ and $a < 0$ separately. If $a > 0$, then $t \to \pm\infty$ implies $\lambda \to \pm\infty$. Therefore, the limits of the integral remain unchanged under the variable change, and we have

$$\begin{aligned} \mathcal{F}\{x(at)\} =& \frac{1}{a} \int_{-\infty}^{\infty} x(\lambda)\, e^{-j\omega\lambda/a}\, d\lambda \\ =& \frac{1}{a} X\left(\frac{\omega}{a}\right), \quad a > 0 \end{aligned} \tag{4.242}$$

If $a < 0$, however, the same substitution leads to

$$\mathcal{F}\{x(at)\} = \frac{1}{a} \int_{\infty}^{-\infty} x(\lambda)\, e^{-j\omega\lambda/a}\, d\lambda \tag{4.243}$$

$$\begin{aligned} \mathcal{F}\{x(\lambda)\} &= -\frac{1}{a} \int_{-\infty}^{\infty} x(\lambda)\, e^{-j\omega\lambda/a}\, d\lambda \\ &= -\frac{1}{a} X\left(\frac{\omega}{a}\right), \quad a < 0 \end{aligned}$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

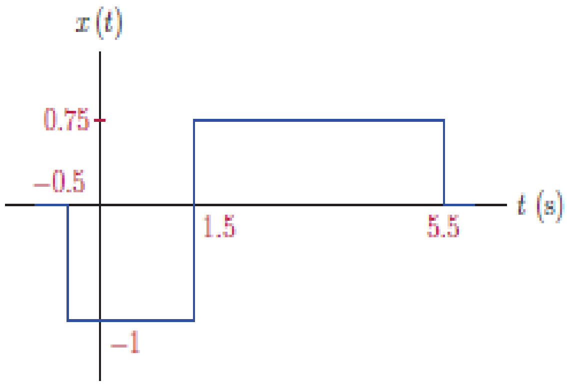
Time & frequency scaling

$$FT\{rect(t)\} = sinc(\pi f)$$

$$FT\{x(at)\} = \frac{1}{a} X(\frac{w}{a})$$

$$FT\{x(t-t0)\} = X(w)e^{-jwt0}$$

مثال 1



$$x(t) = -rect\left(\frac{t-0.5}{2}\right) + 0.75rect\left(\frac{t-3.5}{4}\right)$$

$$x1(t) = -rect\left(\frac{t}{2}\right) + 0.75rect\left(\frac{t}{4}\right)$$

$$X1(w) = -2\ sinc\ (w) + 3sinc\ (2w)$$

$$X(w) = -2\ sinc\ (w)e^{-j0.5w} + 3sinc\ (2w)e^{-j3.5w}$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Differentiation in time

For a given transform pair

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

it can be shown that

$$\frac{d^n}{dt^n}[x(t)] \overset{\mathcal{F}}{\longleftrightarrow} (j\omega)^n X(\omega)$$

$$\frac{d}{dt}[x(t)] = \frac{d}{dt}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega\right] \qquad (4.250)$$

Swapping the order of differentiation and integration on the right side of Eqn. (4.250) we can write

$$\begin{aligned}\frac{d}{dt}[x(t)] &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{d}{dt}\left[X(\omega)\, e^{j\omega t}\right] d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} [j\omega\, X(\omega)]\, e^{j\omega t}\, d\omega \\ &= \mathcal{F}^{-1}\{j\omega\, X(\omega)\} \qquad (4.251)\end{aligned}$$

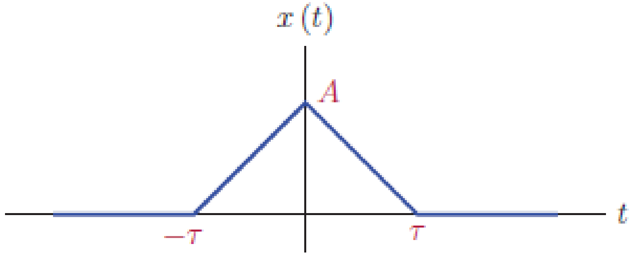
which leads us to the conclusion

$$\frac{d}{dt}[x(t)] \overset{\mathcal{F}}{\longleftrightarrow} j\omega\, X(\omega) \qquad (4.252)$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

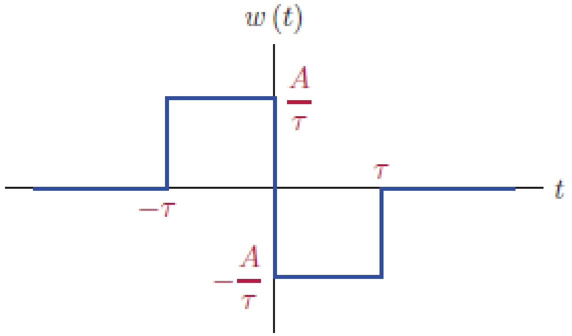
Differentiation in time



$$x(t) = \begin{cases} A\left(1 - \left|\frac{t}{T}\right|\right), & -T \leq t \leq T \\ 0 & others \end{cases}$$

$$x(t) = A.tri\left(\frac{t}{T}\right)$$

$$w(t) = \frac{dx(t)}{dt} \rightarrow \qquad w(t) = \begin{cases} -\frac{A}{T}, & 0 \leq t \leq T \\ +\frac{A}{T}, & -T \leq t \leq 0 \end{cases}$$



$$w(t) = \frac{A}{T}\left(rect\left(\frac{t+\frac{T}{2}}{T}\right) - rect\left(\frac{t-\frac{T}{2}}{T}\right)\right)$$

$$W(f) = Asinc\,(\pi fT)e^{+\frac{j2\pi fT}{2}} - Asinc\,(\pi fT)e^{-\frac{j2\pi fT}{2}}) = 2jAsinc\,(\pi fT)\sin\,(\pi fT)$$

$$W(f) = FT\{w(t)\} = FT\left\{\frac{dx(t)}{dt}\right\} = (j2\pi f)X(f)$$

$$X(f) = \frac{W(f)}{j2\pi f} = 2jAsinc\,(\pi fT)\sin\,(\pi fT)/j2\pi f$$

$$X(f) = ATsinc\,(\pi fT)\frac{\sin(\pi fT)}{T\pi f} = \text{AT}\,(sinc\,(\pi fT))^2$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Homework:optional

Differentiation in the frequency domain

For a transform pair

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

it can be shown that

$$(-jt)^n \, x(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{d^n}{d\omega^n}\left[X(\omega)\right]$$

If we choose to use f instead or ω, then

$$(-j2\pi t)^n \, x(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{d^n}{df^n}\left[X(f)\right]$$

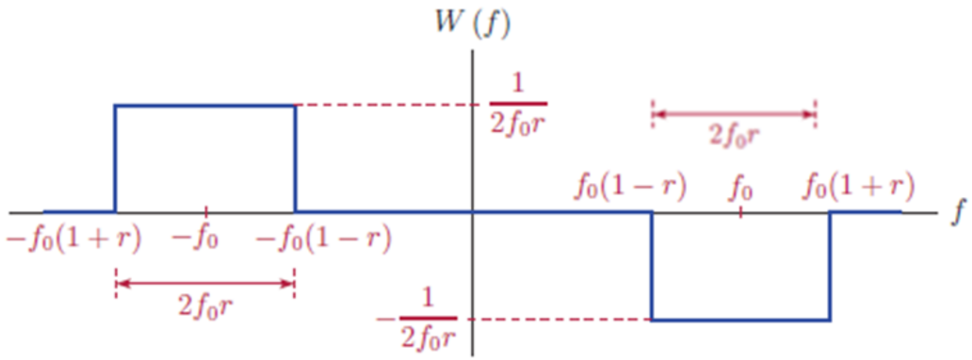
أثبت صحة مايلي:

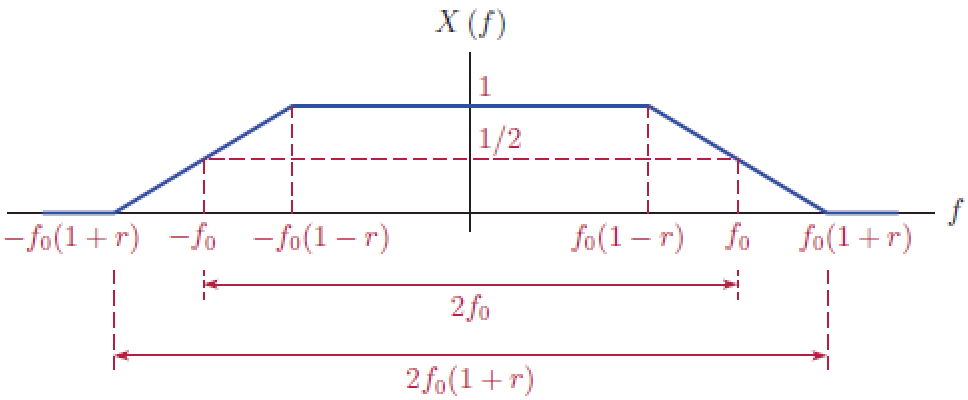
Fourier Transform properties

Non-Periodic Continuous-Time Signals

Differentiation in the frequency domain

$$W(f) = \frac{dX(f)}{df}$$





$$W(f) = \frac{1}{2f_0r}\left(rect\left(\frac{f+f_0}{2f_0r}\right) - rect\left(\frac{f-f_0}{2f_0r}\right)\right)$$

$$X(f) = \begin{cases} 1, & |f| \le f_0(1-r) \\ \frac{1}{2r}\left[-\frac{|f|}{f_0} + 1 + r\right], & f_0(1-r) < |f| \le f_0(1+r) \\ 0, & |f| > f_0(1+r) \end{cases}$$

$$w(t) = sinc\,(2\pi f_0 rt)e^{-j2\pi f_0 t} - sinc\,(2\pi f_0 rt)e^{+j2\pi f_0 t}) =$$

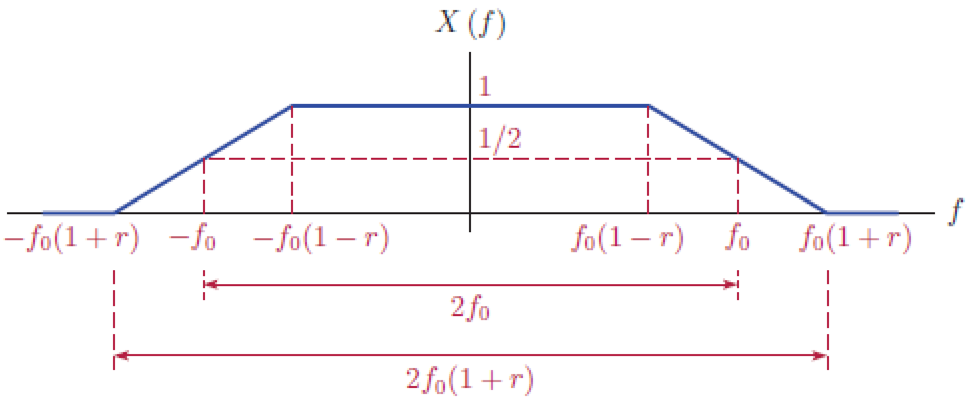
$$-2jsinc(2\pi f_0 rt)\sin(2\pi f_0 t)$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Differentiation in the frequency domain

$$w(t) = IFT\{W(f)\} = IFT\left\{\frac{dX(f)}{df}\right\} = (-j2\pi t)x(t)$$



$$X(f) = \begin{cases} 1, & |f| \le f_0(1-r) \\ \frac{1}{2r}\left[-\frac{|f|}{f_0} + 1 + r\right], & f_0(1-r) < |f| \le f_0(1+r) \\ 0, & |f| > f_0(1+r) \end{cases}$$

$$x(t) = -\frac{w(t)}{(j2\pi t)} = 2f_0 \frac{sinc(2\pi f_0 rt)\sin(2\pi f_0 t)}{2f_0 \pi t}$$

$$= 2f_0 \text{sinc}\,(2\pi f_0 rt)\text{sinc}(2f_0 \pi t)$$

Fourier Transform properties

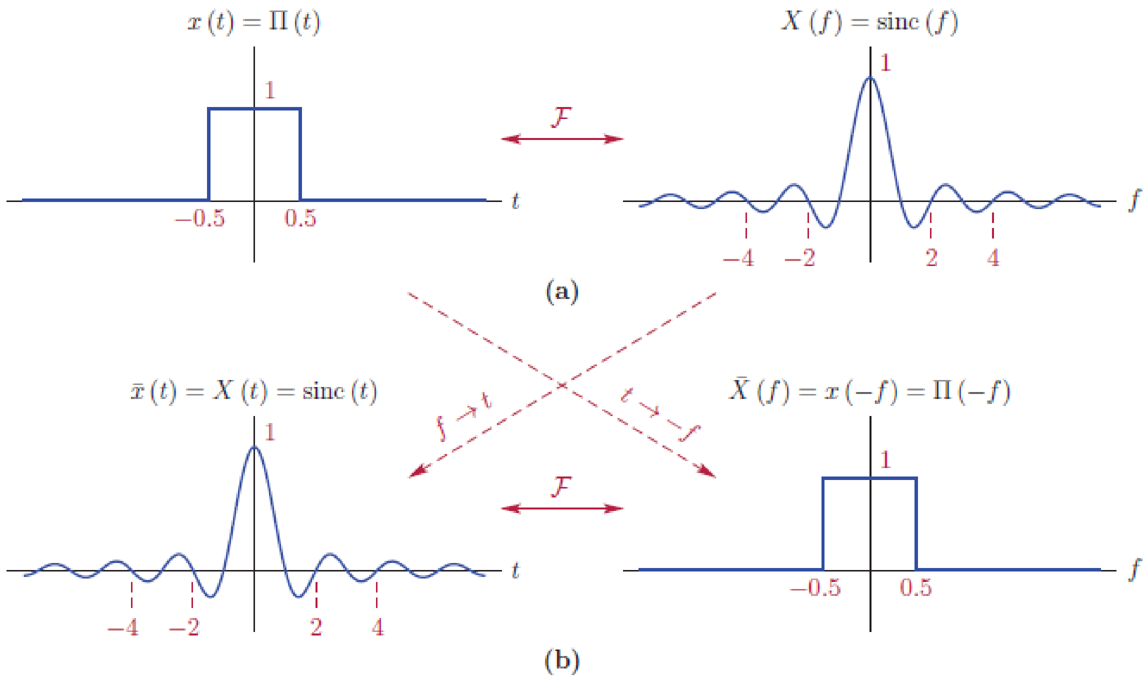
Non-Periodic Continuous-Time Signals

Duality

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f) \quad \text{implies that} \quad X(t) \overset{\mathcal{F}}{\longleftrightarrow} x(-f)$$

مثال ١

Fourier transform of the sinc function



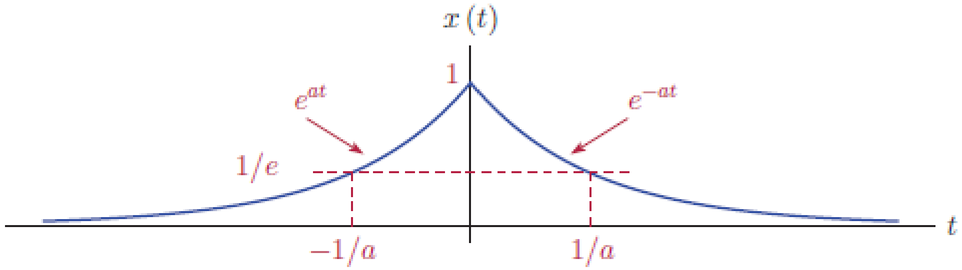
Fourier Transform properties

Non-Periodic Continuous-Time Signals

Duality

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega) \quad \text{implies that} \quad X(t) \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\, x(-\omega)$$

مثال ٢



$$\boldsymbol{x(t) = e^{-a\,|t|}}$$

$$\boldsymbol{X(w) = \frac{1}{a - jw} + \frac{1}{a + jw} = \frac{2a}{a^2 + w^2}}$$

اذا علمت ان تحويل فورييه للإشارة الاسية ثنائية الجانب كماهو معطى جانباً :

اوجد تحويل فورييه للتابع:

$$X(t) = \frac{1}{3 + 2t^2}$$

$$X(t) = \frac{1}{2\left(\frac{3}{2} + t^2\right)} = \frac{1}{4}\sqrt{\frac{2}{3}}\left(\frac{2 * \sqrt{\frac{3}{2}}}{\frac{3}{2} + t^2}\right) : \quad a = \sqrt{\frac{3}{2}}$$

$$X(t) = \frac{1}{4}\sqrt{\frac{2}{3}}\left(\frac{2a}{\boldsymbol{a^2 + t^2}}\right) \rightarrow FT\left(X(t)\right) = 2\pi x(w) = \frac{\pi}{\sqrt{6}} e^{-\sqrt{\frac{3}{2}}|w|}$$

Fourier Transform properties

Non-Periodic Continuous-Time Signals

Integration

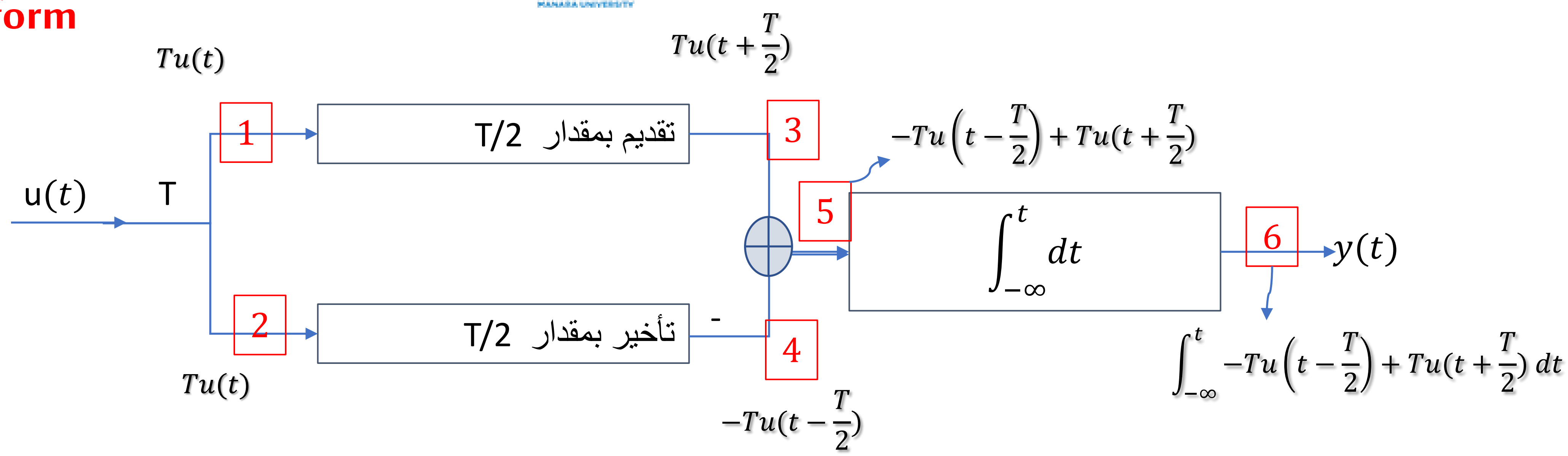
For a transform pair

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

it can be shown that

$$\int_{-\infty}^{t} x(\lambda)\, d\lambda \overset{\mathcal{F}}{\longleftrightarrow} \frac{X(\omega)}{j\omega} + \pi X(0)\, \delta(\omega)$$

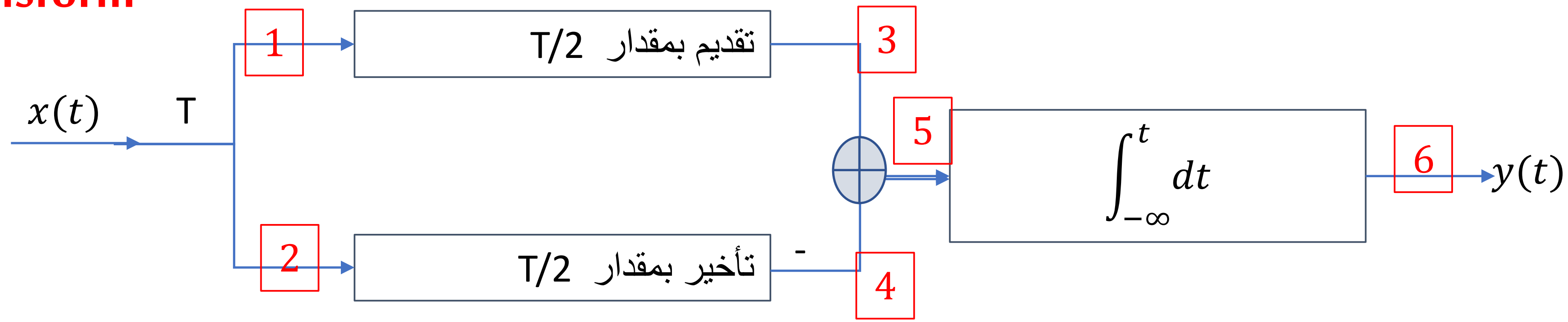
Fourier Transform



لديك المخطط الصندوقي المبين في الشكل :

١. حدد الإشارات المبينة حسب النقاط المرقمة وارسمها باعتبار إشارة الدخل هي $x(t) = u(t)$
٢. حدد إشارة الخرج y(t)
٣. أوجد الاستجابة الزمنية
٤. حول المخطط الى المجال الترددي

Fourier Transform



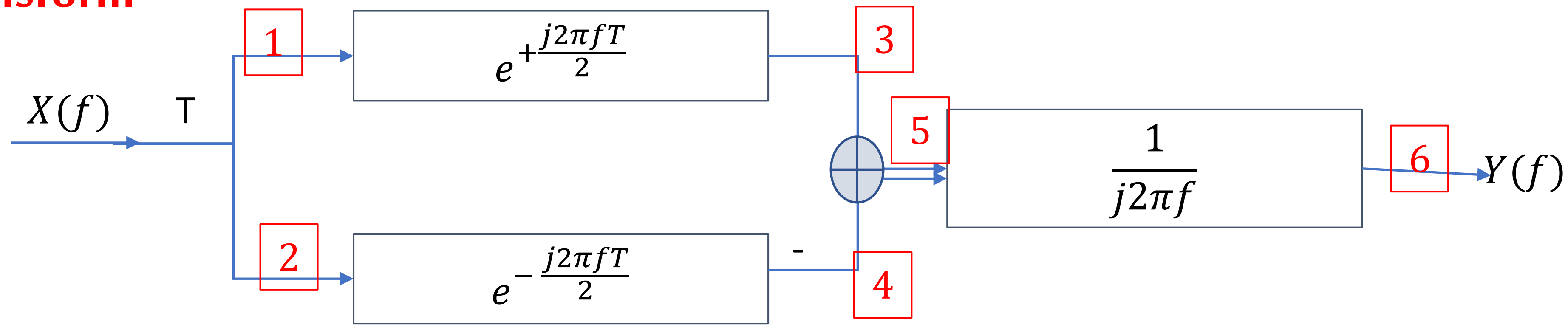
$$y(t) = T\int_{-\infty}^{t}\left[u\left(t+\frac{T}{2}\right)\right]dt - T\int_{-\infty}^{t}\left[u\left(t-\frac{T}{2}\right)\right]dt$$

$$Y(f) = FT\left\{T\int_{-\infty}^{t}[\,u(t+T/2)dt\right\} - FT\left\{T\int_{-\infty}^{t}[\ u(t-T/2)]dt\right\}$$

$$Y(F) = \frac{T}{j2\pi f}X(F)e^{j2\pi f\frac{T}{2}} - \frac{T}{j2\pi f}X(F)e^{-j2\pi f\frac{T}{2}} = \frac{TX(F)}{j2\pi f}\left(e^{j\pi fT} - e^{-j\pi fT}\right)$$

$$H(F) = \frac{Y(F)}{X(F)} = T^2\frac{\left(e^{j\pi fT} - e^{-j\pi fT}\right)}{j2\pi fT} = T^2\,Sinc\,(\pi f\mathrm{T}) \quad \Rightarrow \quad h(t) = T rect\left(\frac{t}{T}\right)$$

Fourier Transform



The End