Red-Black Tree

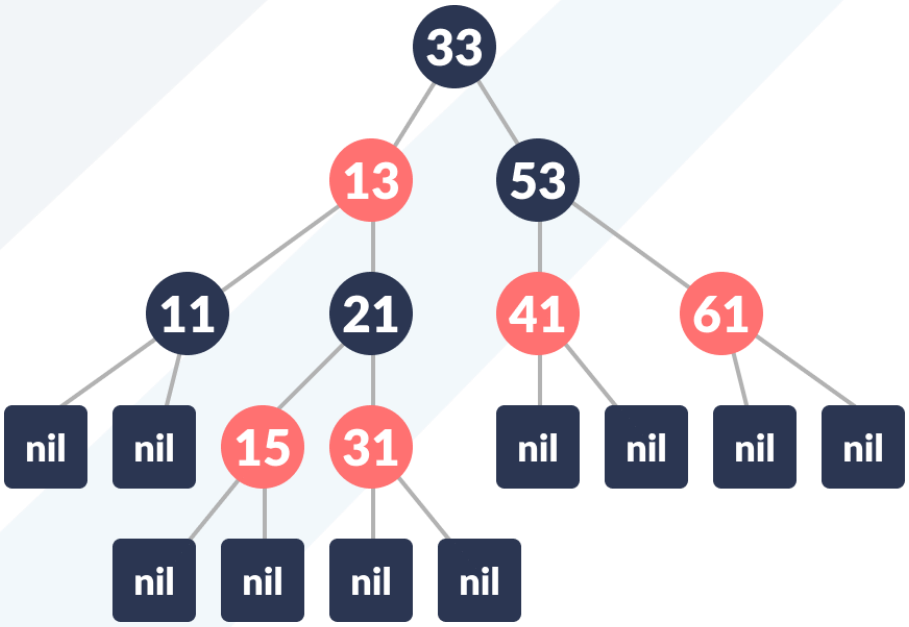
In this tutorial, you will learn what a red-black tree is. Also, you will find working examples of various operations performed on a red-black tree in C, C++, Java and Python.

Red-Black tree is a self-balancing binary search tree in which each node contains an extra bit for denoting the color of the node, either red or black.

A red-black tree satisfies the following properties:

1. **Red/Black Property:** Every node is colored, either red or black.
2. **Root Property:** The root is black.
3. **Leaf Property:** Every leaf (NIL) is black.
4. **Red Property:** If a red node has children then, the children are always black.
5. **Depth Property:** For each node, any simple path from this node to any of its descendant leaf has the same black-depth (the number of black nodes).

An example of a red-black tree is:



Red Black Tree

Each node has the following attributes:

- color
- key
- leftChild
- rightChild
- parent (except root node)

How the red-black tree maintains the property of self-balancing?

The red-black color is meant for balancing the tree.

The limitations put on the node colors ensure that any simple path from the root to a leaf is not more than twice as long as any other such path. It helps in maintaining the self-balancing property of the red-black tree.

Operations on a Red-Black Tree

Various operations that can be performed on a red-black tree are:

Rotating the subtrees in a Red-Black Tree

In rotation operation, the positions of the nodes of a subtree are interchanged.

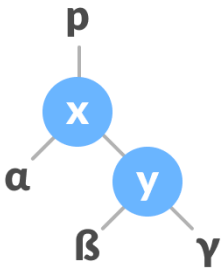
Rotation operation is used for maintaining the properties of a red-black tree when they are violated by other operations such as insertion and deletion.

There are two types of rotations:

Left Rotate

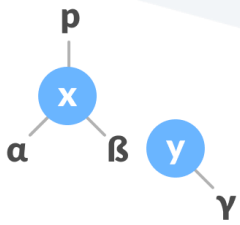
In left-rotation, the arrangement of the nodes on the right is transformed into the arrangements on the left node.

Algorithm



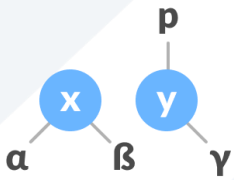
1. Let the initial tree be: Initial tree

2. If *y* has a left subtree, assign *x* as the parent of the left subtree of *y*.



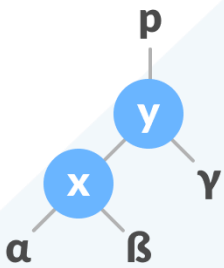
Assign x as the parent of the left subtree of y

3. If the parent of *x* is `NULL`, make *y* as the root of the tree.
4. Else if *x* is the left child of *p*, make *y* as the left child of *p*.



5. Else assign *y* as the right child of *p*.

Change the parent of x to that of y

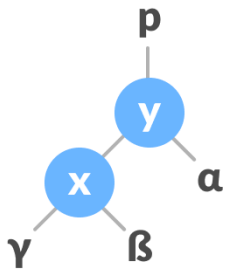


6. Make *y* as the parent of *x*.

Assign y as the parent of x.

Right Rotate

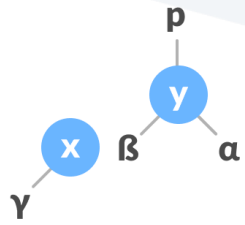
In right-rotation, the arrangement of the nodes on the left is transformed into the arrangements on the right node.



1. Let the initial tree be:

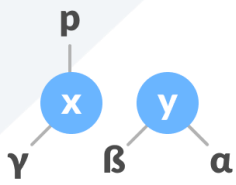
Initial Tree

2. If *x* has a right subtree, assign y as the parent of the right subtree of *x*.



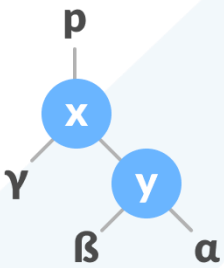
Assign y as the parent of the right subtree of x

3. If the parent of *y* is `NULL`, make *x* as the root of the tree.
4. Else if *y* is the right child of its parent *p*, make *x* as the right child of *p*.



5. Else assign *x* as the left child of *p*.

Assign the parent of y as the parent of x

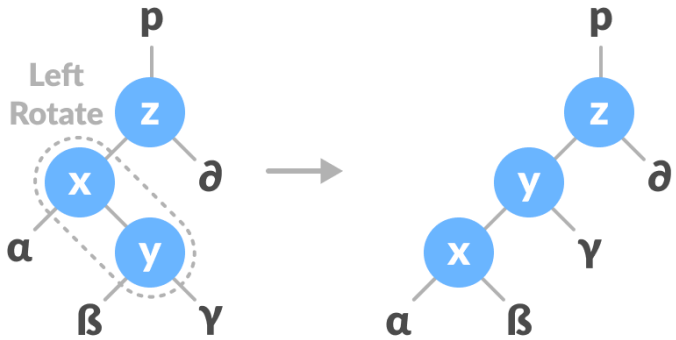


6. Make *x* as the parent of *y*.

Assign x as the parent of y

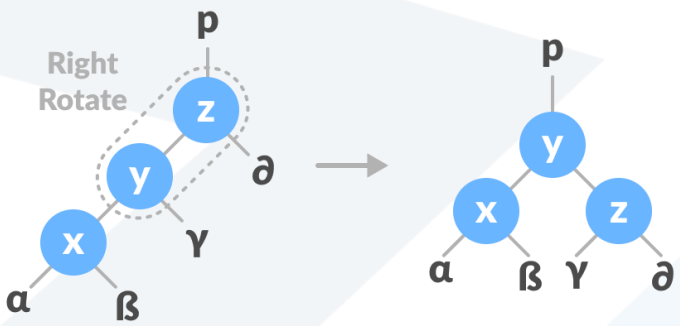
Left-Right and Right-Left Rotate

In left-right rotation, the arrangements are first shifted to the left and then to the right.



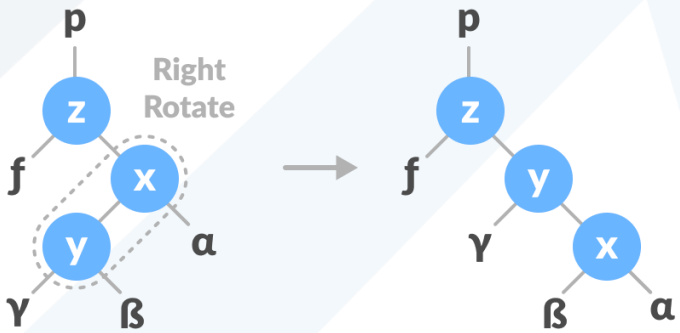
1. Do left rotation on x-y.

Left rotate x-y

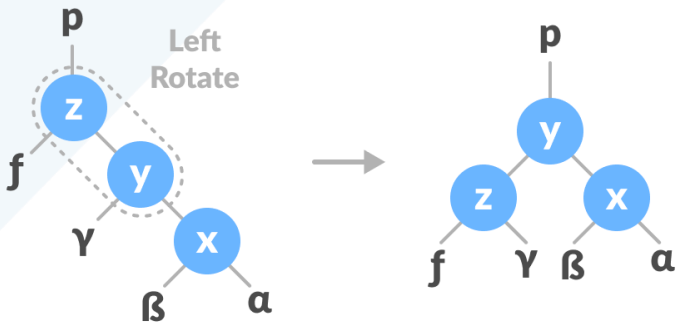


2. Do right rotation on y-z. Right rotate z-y

In right-left rotation, the arrangements are first shifted to the right and then to the left.



1. Do right rotation on x-y. Right rotate x-y



2. Do left rotation on z-y. Left rotate z-y

Inserting an element into a Red-Black Tree

While inserting a new node, the new node is always inserted as a RED node. After insertion of a new node, if the tree is violating the properties of the red-black tree then, we do the following operations.

1. Recolor
2. Rotation

Algorithm to insert a node

Following steps are followed for inserting a new element into a red-black tree:

1. Let y be the leaf (ie. `NIL`) and x be the root of the tree.
2. Check if the tree is empty (ie. whether *x* is `NIL`). If yes, insert *newNode* as a root node and color it black.
3. Else, repeat steps following steps until leaf (`NIL`) is reached.
    a. Compare *newKey* with *rootKey*.
    b. If *newKey* is greater than rootKey, traverse through the right subtree.
    c. Else traverse through the left subtree.
4. Assign the parent of the leaf as a parent of *newNode*.
5. If *leafKey* is greater than *newKey*, make *newNode* as *rightChild*.
6. Else, make *newNode* as *leftChild*.
7. Assign `NULL` to the left and *rightChild* of *newNode*.
8. Assign RED color to *newNode*.
9. Call InsertFix-algorithm to maintain the property of red-black tree if violated.

Why newly inserted nodes are always red in a red-black tree?

This is because inserting a red node does not violate the depth property of a red-black tree.

If you attach a red node to a red node, then the rule is violated but it is easier to fix this problem than the problem introduced by violating the depth property.

Algorithm to maintain red-black property after insertion

This algorithm is used for maintaining the property of a red-black tree if the insertion of a newNode violates this property.

1. Do the following while the parent of *newNode p* is RED.
2. If *p* is the left child of *grandParent gP* of *z*, do the following.
   Case-I:
    a. If the color of the right child of *gP* of *z* is RED, set the color of both the children of *gP* as BLACK and the color of *gP* as RED.
    b. Assign *gP* to *newNode*.
       Case-II:
    c. Else if *newNode* is the right child of *p* then, assign *p* to *newNode*.
    d. Left-Rotate *newNode*.
       Case-III:

e. Set color of *p* as BLACK and color of *gP* as RED.
f. Right-Rotate *gP*.

3. Else, do the following.
   a. If the color of the left child of *gP* of *z* is RED, set the color of both the children of *gP* as BLACK and the color of *gP* as RED.
   b. Assign *gP* to *newNode*.
   c. Else if newNode is the left child of *p* then, assign *p* to *newNode* and Right-Rotate *newNode*.
   d. Set color of *p* as BLACK and color of *gP* as RED.
   e. Left-Rotate *gP*.
4. Set the root of the tree as BLACK.

Deleting an element from a Red-Black Tree

This operation removes a node from the tree. After deleting a node, the red-black property is maintained again.

Algorithm to delete a node

1. Save the color of *nodeToBeDeleted* in *origrinalColor*.
2. If the left child of *nodeToBeDeleted* is `NULL`
   a. Assign the right child of *nodeToBeDeleted* to x.
   b. Transplant *nodeToBeDeleted* with *x*.
3. Else if the right child of *nodeToBeDeleted* is `NULL`
   a. Assign the left child of *nodeToBeDeleted* into *x*.
   b. Transplant *nodeToBeDeleted* with *x*.
4. Else
   a. Assign the minimum of right subtree of *noteToBeDeleted* into *y*.
   b. Save the color of *y* in *originalColor*.
   c. Assign the *rightChild* of *y* into *x*.
   d. If *y* is a child of *nodeToBeDeleted*, then set the parent of *x* as *y*.
   e. Else, transplant *y* with *rightChild* of *y*.
   f. Transplant *nodeToBeDeleted* with *y*.
   g. Set the color of y with originalColor.
5. If the *originalColor* is BLACK, call DeleteFix(x).

Algorithm to maintain Red-Black property after deletion

This algorithm is implemented when a black node is deleted because it violates the black depth property of the red-black tree.

This violation is corrected by assuming that node *x* (which is occupying *y*'s original position) has an extra black. This makes node *x* neither red nor black. It is either doubly black or black-and-red. This violates the red-black properties.

However, the color attribute of *x* is not changed rather the extra black is represented in *x*'s pointing to the node.

The extra black can be removed if

1. It reaches the root node.
2. If *x* points to a red-black node. In this case, *x* is colored black.
3. Suitable rotations and recoloring are performed.

The following algorithm retains the properties of a red-black tree.

1. Do the following until the *x* is not the root of the tree and the color of *x* is BLACK
2. If *x* is the left child of its parent then,
    a. Assign w to the sibling of x.
    b. If the right child of parent of *x* is RED,
    Case-I:
        i. Set the color of the right child of the parent of *x* as BLACK.
        ii. Set the color of the parent of *x* as RED.
        iii. Left-Rotate the parent of *x*.
        iv. Assign the *rightChild* of the parent of *x* to *w*.
    c. If the color of both the right and the *leftChild* of *w* is BLACK,
    Case-II:
        i. Set the color of *w* as RED
        ii. Assign the parent of *x* to *x*.
    d. Else if the color of the *rightChild* of *w* is BLACK
    Case-III:
        i. Set the color of the *leftChild* of *w* as BLACK
        ii. Set the color of *w* as RED
        iii. Right-Rotate *w*.
        iv. Assign the *rightChild* of the parent of *x* to *w.*
    e. If any of the above cases do not occur, then do the following.
    Case-IV:
        i. Set the color of *w* as the color of the parent of *x*.
        ii. Set the color of the parent of *x* as BLACK.
        iii. Set the color of the right child of *w* as BLACK.
        iv. Left-Rotate the parent of *x*.
        v. Set *x* as the root of the tree.
3. Else the same as above with right changed to left and vice versa.
4. Set the color of *x* as BLACK.