Portfolio Management

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Simple techniques for determining the efficient frontier

- In this chapter we describe the methods for selecting optimal portfolios that are appropriate when the single-index model is accepted as description of the covariance structure between securities.

- In the text of this chapter we present the rules for optimal portfolio selection and show how to use them.

The single-index model

- In this section we present and demonstrate the optimum procedure for selecting portfolios when the single-index model is accepted as the best way to forecast the covariance structure of returns.

- First we present the ranking criteria that can be used to order stocks for selection for the optimal portfolio. We next present the technique for employing this ranking device to form an optimum portfolio, along with a logical explanation for why it works.

The formation of optimal portfolios

- The calculation of optimal portfolios would be greatly facilitated, and the ability of practicing security analysts and portfolio managers to relate to the construction of optimal portfolios greatly enhanced, if there were a single number that measured the desirability of including a stock in the optimal portfolio.

- If one is willing to accept the standard form of the single-index model as describing the co-movement between securities, such a number exists. In this case, the desirability of any stock is directly related to its excess return to beta ratio.

- Excess return is the difference between the expected return on the stock and the riskless rate of interest such as the rate on a Treasury bill. The excess return to beta ratio measures the additional return on a security (beyond that offered by a riskless asset) per unit of non-diversifiable risk.

- The form of this ratio should lead to its easy interpretation and acceptance by security analysts and portfolio managers because they are used to thinking in terms of the relationship between potential rewards and risk.

- The index we use to rank stocks is *excess return to beta*, or:

$$\frac{\bar{R}_i - R_F}{\beta_i}$$

- Where:

$\bar{R}_i$ the expected return on stock i.

R_F the return on a riskless asset.

β_i the expected change in the rate of return on stock i associated with a 1% change in the market return.

- If stocks are ranked by excess return to beta (from highest to lowest), the ranking represents the desirability of any stock's inclusion in a portfolio. In other words, if a stock with a particular ratio of $(\bar{R}_i - R_F)/\beta_i$ is included in an optimal portfolio, all stocks with a higher ratio will also be included. Conversely, if a stock with a particular $(\bar{R}_i - R_F)/\beta_i$ is excluded from an optimal portfolio, all stocks with lower ratios will be excluded.

- When the single-index model is assumed to represent the covariance structure of security returns, then a stock is included or excluded depending only on the size of its excess return to beta ratio. How many stocks are selected depends on a unique cut-off rate such that all stocks with higher ratios of $(\bar{R}_i - R_F)/\beta_i$ will be included and all stocks with lower ratios will be excluded. We call this cut-off ratio C^*.

- The rules for determining which stocks are included in the optimum portfolio are as follows:

1. Find the excess return to beta ratio for each stock under consideration and rank from highest to lowest.

2. The optimum portfolio consists of investing in all stocks for which $(\bar{R}_i - R_F)/\beta_i$ is greater than a particular cut-off point C^*.

- In Tables 9.1 and 9.2 we present an example that illustrates this procedure. Table 9.1 contains the data necessary to apply our simple ranking device to determine an optimal portfolio. There are 10 securities in the tables. We have already ranked the securities according to $(\bar{R}_i - R_F)/\beta_i$ and have used numbers that make the calculations easy to follow.

- The application of rule 2 involves the comparison of $(\bar{R}_i - R_F)/\beta_i$ with C^*, supposing that $C^* = 5.45$. Examining Table 9.1 shows that for securities 1 to 5, $(\bar{R}_i - R_F)/\beta_i$ is greater than C^*, while for security 6, it is less than C^*. Hence, an optimal portfolio consists of securities 1 to 5.

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Table 9.1 Data Required to Determine Optimal Portfolio $R_F = 5\%$

1 Security No. i	2 Mean Return $\bar{R}_i$	3 Excess Return $\bar{R}_i - R_F$	4 Beta β_i	5 Unsystematic Risk σ^2_{ei}	6 Excess Return over Beta $\frac{(\bar{R}_i - R_F)}{\beta_i}$
1	15	10	1	50	10
2	17	12	1.5	40	8
3	12	7	1	20	7
4	17	12	2	10	6
5	11	6	1	40	6
6	11	6	1.5	30	4
7	11	6	2	40	3
8	7	2	0.8	16	2.5
9	7	2	1	20	2
10	5.6	0.6	0.6	6	1.0

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Table 9.2 Calculations for Determining Cutoff Rate with $\sigma_m^2 = 10$

1 Security No. i	2 $\dfrac{(\bar{R}_i - R_F)}{\beta_i}$	3 $\dfrac{(\bar{R}_i - R_F)\beta_i}{\sigma_{ei}^2}$	4 $\dfrac{\beta_i^2}{\sigma_{ei}^2}$	5 $\sum_{j=1}^{i} \dfrac{(\bar{R}_j - R_F)\beta_j}{\sigma_{ej}^2}$	6 $\sum_{j=1}^{i} \dfrac{\beta_j^2}{\sigma_{ej}^2}$	7 C_i
1	10	2/10	2/100	2/10	2/100	1.67
2	8	4.5/10	5.625/100	6.5/10	7.625/100	3.69
3	7	3.5/10	5/100	10/10	12.625/100	4.42
4	6	24/10	40/100	34/10	52.625/100	5.43
5	6	1.5/10	2.5/100	35.5/10	55.125/100	5.45
6	4	3/10	7.5/100	38.5/10	62.625/100	5.30
7	3	3/10	10/100	41.5/10	72.625/100	5.02
8	2.5	1/10	4/100	42.5/10	76.625/100	4.91
9	2.0	1/10	5/100	43.5/10	81.625/100	4.75
10	1.0	0.6/10	6/100	44.1/10	87.625/100	4.52

Calculating the cut-off rate C*

- Recall that stocks are ranked by excess return to risk from highest to lowest. For a portfolio of i stocks, C_i is given by:

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \frac{(\bar{R}_i - R_F)\beta_j}{\sigma_{ej}^2}}{1 + \sigma_m^2 \sum_{j=1}^{i} \left(\frac{\beta_j^2}{\sigma_{ej}^2}\right)}$$

- Where:

σ_m^2 is the variance in the market index.

σ_{ej}^2 is the variance of a stock's movement that is not associated with the movement of the market index; this is usually referred to as a stock's unsystematic risk.