Portfolio Management

Manara University

Department of Banking and Finance

Second Term

2021-2022

Dr Hayan Omran

The Standard Capital Asset Pricing Model

The assumptions underlying the standard capital asset pricing model (CAPM):

The real world is sufficiently complex that to understand it and construct models of how it works, one must assume away those complexities that are thought to have only a small (or no) effect on its behaviour. As the physicist builds models of the movement of matter in a frictionless environment, the economist builds models where there are no institutional frictions to the movement of stock prices.

- The first assumption is that there are no transaction costs. There is no cost (friction) of buying or selling any asset. If transaction costs were present, the return from any asset would be a function of whether the investor owned it before the decision period.
- The second assumption behind the CAPM is that assets are infinitely divisible. This means that investors could take any position in an investment, regardless of the size of their wealth. For example, they can buy one dollar's worth of IBM stock.
- The third assumption is the absence of personal income tax. This means, for example, that the individual is indifferent to the form (dividends or capital gains) in which the return on the investment is received.
- The fourth assumption is that an individual cannot affect the price of a stock by his buying or selling action. This is analogous to the assumption of perfect competition. Although no single investor can affect prices by an individual action, investors in total determine prices by their actions.

- The fifth assumption is that investors are expected to make decisions solely in terms of expected values and standard deviations of the returns on their portfolios.
- The sixth assumption is that unlimited short sales are allowed. The individual investor can sell short any number of any shares.
- The seventh assumption is unlimited lending and borrowing at the riskless rate. The investor can lend or borrow any amount of funds desired at a rate of interest equal to the rate for riskless securities.
- The eighth and ninth assumptions deal with the homogeneity of expectations. First, investors are assumed to be concerned with the mean and variance of returns (or prices over a single period), and all investors are assumed to define the relevant period in exactly the same manner. Second, all investors are assumed to have identical expectations with respect to the necessary inputs to the portfolio decision. These inputs are expected returns, the variance of returns, and the correlation matrix representing the correlation structure between all pairs of stocks.
- The tenth assumption is that all assets are marketable. All assets can be sold and bought on the market.

Deriving the CAPM-a simple approach:

In the presence of short sales, but without riskless lending and borrowing, each investor faced an efficient frontier such as that shown in Figure 9.1. In this figure, *BC* represents the efficient frontier, while *ABC* represents the set of minimum-variance portfolios. In general, the efficient frontier will differ among investors because of differences in expectations.

In terms of riskless lending and borrowing, the portfolio of risky assets that any investor would hold could be identified without regard to the investor's risk

preferences. This portfolio lies at the tangency point between the original efficient frontier of risky assets and a ray passing through the riskless return (on the vertical axis). This is depicted in Figure 9.2, where P_i denotes investor i's portfolio of risky assets. The investors satisfy their risk preferences by combining portfolio P_i with lending or borrowing.

If all investors have homogeneous expectations and they all face the same lending and borrowing rate, then they will each face a diagram such as in Figure 9.2 and, furthermore, all of the diagrams will be identical. The portfolio of risky assets P_i held by any investor will be identical to the portfolio of risky assets held by any other investor.

If all investors hold the same risky portfolio, then, in equilibrium, it must be the market portfolio. The market portfolio is a portfolio comprising all risky assets. Each asset is held in the proportion that the market value of that asset represents of the total market value of all risky assets. For example, if IBM stock represents 3% of all risky assets, then the market portfolio contains 3% IBM stock, and each investor will take 3% of the money that will be invested in all risky assets and place it in IBM stock.

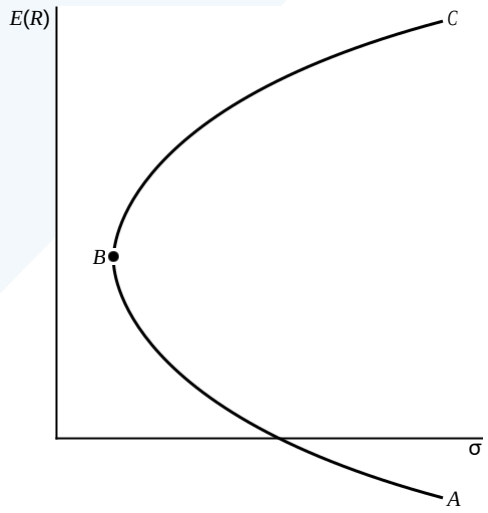

Figure 9.1: The efficient frontier—no lending and borrowing.

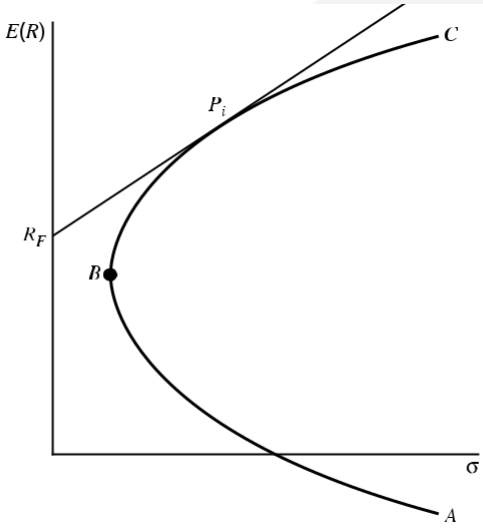

Figure 9.2: The efficient frontier with lending and borrowing.

Notice that we have already learned something important. All investors will hold combinations of only two portfolios: the market portfolio (*M*) and a riskless security. This is sometimes referred to as the *two mutual fund theorem* because all investors would be satisfied with a market fund, plus the ability to lend or borrow a riskless security.

The straight line depicted in Figure 9.2 is usually referred to as the *capital market line*. All investors will end up with portfolios somewhere along the capital market line, and all *efficient portfolios* would lie along the capital market line.

However, not all securities or portfolios lie along the capital market line. In fact, from the derivation of the efficient frontier, we know that all portfolios of risky and riskless assets, except those that are efficient, lie below the capital market line. The equation of a line connecting a riskless asset and a risky portfolio (the line we call the capital market line) is:

$$\overline{R_e} = R_f + \frac{\overline{R_M} - R_f}{\sigma_M}\sigma_e$$

where the subscript e denotes an efficient portfolio.

The term $[\frac{\overline{R_M} - R_f}{\sigma_M}]$ can be thought of as the market price of risk for all efficient portfolios. It is the extra return that can be gained by increasing the level of risk (standard deviation) on an efficient portfolio by one unit.

The second term on the right-hand side of this equation is simply the market price of risk times the amount of risk in a portfolio. The second term represents that element of required return that is due to risk. The first term is simply the price of time or the return that is required for delaying potential consumption, one period given perfect certainty about the future cash flow. Thus, the expected return on an efficient portfolio is:

$$(Expected\ return) = (Price\ of\ time) + (Price\ of\ risk) \times (Amount\ of\ risk)$$

Although this equation establishes the return on an efficient portfolio, it does not describe equilibrium returns on nonefficient portfolios or on individual securities.