Financial Risk Management

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Duration

- Duration is a widely used measure of a portfolio's exposure to yield curve movements. Suppose y is a bond's yield and B is its market price. The duration D of the bond is defined as:

$$D = -\frac{1}{B}\frac{\Delta B}{\Delta y} \quad (1)$$

- So that,

$$\Delta B = -DB\Delta y$$

- where Δy is a small change in the bond's yield and ΔB is the corresponding change in its price. Duration measures the sensitivity of percentage changes in the bond's price to changes in its yield. Using calculus notation, we can write:

$$D = -\frac{1}{B}\frac{dB}{dy} \quad (2)$$

- Consider a bond that provides cash flows $c_1, c_2, \ldots, c_n$ at times $t_1, t_2, \ldots, tn$. (The cash flows consist of the coupon and principal payments on the bond.) The bond yield, y, is defined as the discount rate that equates the bond's theoretical price to its market price. We denote the present value of the cash flow c_i, discounted from time t_i to today at rate y, by v_i so that the price of the bond is:

$$B = \sum_{i=1}^{n} v_i$$

- An alternative definition of duration is:

$$D = \sum_{i=1}^{n} t_i \left(\frac{v_i}{B}\right) \qquad (3)$$

- The term in parentheses in equation (3) is the ratio of the present value of the cash flow at time t_i to the bond price. Equation (3) therefore defines duration as a weighted average of the times when payments are made, with the weight applied to time t_i being equal to the proportion of the bond's total present value provided by the cash flow at time t_i. (The sum of the weights is 1.0.) This definition explains where the term duration comes from.

- Duration is a measure of how long the bondholder has to wait for cash flows. A zero-coupon bond that lasts n years has a duration of n years. However, a coupon-bearing bond lasting n years has a duration of less than n years, because the holder receives some of the cash payments prior to year n.
- If the bond's yield, y, in equation (1) is measured with continuous compounding, it turns out that the definitions of duration in equations (1) and (3) are the same.

- Consider a three-year 10% coupon bond with a face value of $\$100$. Suppose that the yield on the bond is 12% per annum with continuous compounding. This means that $y = 0.12$. Coupon payments of $\$5$ are made every six months.
- Table (3) shows the calculations necessary to determine the bond's duration. The present values of the bond's cash flows, using the yield as the discount rate, are shown in column 3. (For example, the present value of the first cash flow is $5e^{-0.12\times 0.5} = 4.709$.)
- The sum of the numbers in column 3 is the bond's market price, 94.213. The weights are calculated by dividing the numbers in column (3) by 94.213. The sum of the numbers in column 5 gives the duration as 2.653 years.

- Small changes in interest rates are often measured in *basis points*. A basis point is 0.01% per annum. The following example shows that equation (1) is correct when duration is defined as in equation (3) and yields are measured with continuous compounding.

- **Table 3** Calculation of Duration:

Time (years)	Cash Flow ($)	Present Value	Weight	Time × Weight
0.5	5	4.709	0.050	0.025
1.0	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2.0	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3.0	105	73.256	0.778	2.333
Total	130	94.213	1.000	2.653