Financial Risk Management

Manara University

Department of Banking and Finance

Second Term

2021-2022

Dr Hayan Omran

Calculation of duration

- Example:

For the bond in Table 3, the bond price, *B*, is 94.213 and the duration, *D*, is 2.653, so that equation $\Delta B = -DB\Delta y$ gives:

$$\Delta B = -94.213 \times 2.653\Delta y$$

Or: $\Delta B = -249.95\Delta y$

- When the yield on the bond increases by 10 basis points (= 0.1%),

$\Delta y = +0.001$. The duration relationship predicts that:

$$\Delta B = -249.95 \times 0.001 = -0.250$$

So that the bond price goes down to $94.213 - 0.250 = 93.963$.

How accurate is this? When the bond yield increases by 10 basis points to 12.1%, the bond price is:

$$5e^{-0.121\times 0.5} + 5e^{-0.121\times 1.0} + 5e^{-0.121\times 1.5} + 5e^{-0.121\times 2.0} + 5e^{-0.121\times 2.5} + 105e^{-0.121\times 3.0} = 93.963$$

Dollar Duration

- The dollar duration of a bond is defined as the product of its duration and its price. If $D_{\$}$ is dollar duration, it follows from equation (1) that

$$\Delta B = -D_{\$}\Delta y$$

- or using calculus notation:

$$D_{\$} = -\frac{dB}{dy}$$

- Whereas duration relates proportional changes in a bond's price to its yield, dollar duration relates actual changes in the bond's price to its yield.

Convexity

- The duration relationship measures exposure to small changes in yields. This is illustrated in Figure 9.2, which shows the relationship between the percentage change in value and change in yield for two bonds with the same duration. The gradients of the two curves are the same at the origin.
- This means that both portfolios change in value by the same percentage for small yield changes, as predicted by equation (1).
- For large yield changes, the bonds behave differently. Bond *X* has more curvature in its relationship with yields than bond *Y*. A factor known as *convexity* measures this curvature and can be used to improve the relationship between bond prices and yields.

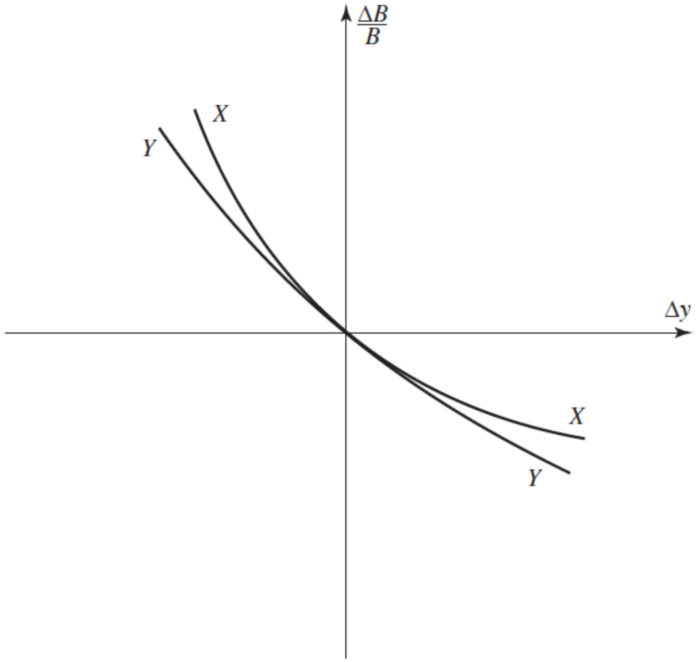

Figure 9.2 Two Bonds with the Same Duration and Different Convexities

- The convexity for a bond is:

$$C = \frac{1}{B}\frac{d^2B}{dy^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-yt_i}}{B}$$

where y is the bond's yield measured with continuous compounding. This is the weighted average of the square of the time to the receipt of cash flows.

- A second order approximation to the change in the bond price leads to:

$$\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2} C(\Delta y)^2 \qquad (4)$$

- **Example:**
- Consider again the bond in Table 3.The bond price, *B*, is 94.213, and the duration, *D*, is 2.653. The convexity is:

$$0.05 \times 0.5^2 + 0.047 \times 1.0^2 + 0.044 \times 1.5^2 + 0.042 \times 2.0^2 + 0.039 \times 2.5^2 + 0.779 \times 3.0^2 = 7.570$$

- The convexity relationship in equation (4) is therefore

$$\frac{\Delta B}{B} = -2.653\Delta y + \frac{1}{2} \times 7.570 \times (\Delta y)^2$$

- Consider a 2% change in the bond yield from 12% to 14%. The duration relationship predicts that the dollar change in the value of the bond will be $-94.213 \times 2.653 \times 0.02 = -4.999$.

- The convexity relationship predicts that it will be:

$$-94.213 \times 2.653 \times 0.02 + 0.5 \times 94.213 \times 7.570 \times 0.02^2 = -4.856$$

- The actual change in the value of the bond is −4.859. This shows that the convexity relationship gives much more accurate results than duration for a large change in the bond yield.