مقرر الإشارات والنظم

قسم هندسة الروبوت والأنظمة الذكية

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محاضرة العملي الأسبوع ٩

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Discrete signals analysis in the frequency domain :

Discrete-Time Fourier Transform - DTFT

$$X(w) = \sum_{n=-\infty}^{+\infty} x(n)e^{-jwn}$$

$$x(n) = \mathrm{b(a)}^n u(n)$$

حتى يكون $X(w)$ موجوداً يجب أن يكون $a < 1$

$$X(w) = \sum_{n=-\infty}^{+\infty} \mathrm{b(a)}^n u(n)e^{-jwn} = b\sum_{n=0}^{+\infty} (ae^{-jw})^n = b\frac{1}{1-ae^{-jw}}$$

$$|X(w)| = \frac{|b|}{|1-ae^{-jw}|} = \frac{|b|}{|1-acosw+ajsinw|} = \frac{|b|}{\sqrt{(1-acosw)^2+(asinw)^2}} = \frac{|b|}{\sqrt{1+a^2-2\mathrm{acos}(w)}}$$

$$\angle X(w) = \angle b - \angle(1-acosw+ajsinw) = \angle b - arctan\left(\frac{asinw}{1-acosw}\right) =$$

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Discrete signals analysis in the frequency domain :

Discrete-Time Fourier Transform - DTFT

$$X(w) = \sum_{n=-\infty}^{+\infty} x(n)e^{-jwn}$$

$x(n) = \delta(n) + \delta(n-1)$	$X(w) = 1 + e^{-jw} = \underbrace{1}_{\text{Dc Component}} + \cos(w) - j\sin(w)$
$x(n) = \begin{cases} 1, & n = 0, 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$	$X(w) = 1 + e^{-jw} + e^{-j2w} + e^{-j3w}$ $= \left[e^{j3w/2} + e^{jw/2} + e^{-jw/2} + e^{-j3w/2}\right]e^{-j3w/2}$ $= [2\cos(w/2) + 2\cos(3w/2)]e^{-j3w/2}$
$x(n) = (0.7)^n u(n)$	$X(w) = \sum_{n=0}^{\infty} (0.7)^n e^{-jwn} = \frac{1}{1 - 0.7e^{-jw}} = \frac{1}{1 - 0.7\cos(w) + j0.7\sin(w)}$
$y(n) = \frac{1}{3} s(n) * [\delta(n+1) + \delta(n) + \delta(n-1)]$ $h(n) = \frac{1}{3}\,[\delta(n+1) + \delta(n) + \delta(n-1)]$	$H(w) = \frac{1}{3}\left[e^{jw} + 1 + e^{-jw}\right]$

ارسم المطال والطور لكل من الاستجابة
السابقة باستخدام MATLAB

```
w = -pi: 0.01*pi:pi;
%x =1+exp(-j*w);
%x =(2*cos(w/2)+2*cos(3*w/2)).*exp(-j*3*w/2);
%x =1./(1-0.7*cos(w)+j*0.7*sin(w));
%x =1/3*(exp(j*w) + 1+exp(-j*w));
subplot (211)
plot (w  ,abs( x))
grid
subplot (212)
plot (w  ,angle( x))
grid
```

Time Duration		
Finite	Infinite	
Discrete FT (DFT) $X(k)=\sum_{n=0}^{N-1} x(n)e^{-j\omega_k n}$ $k=0,1,\ldots,N-1$	Discrete Time FT (DTFT) $X(\omega)=\sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$ $\omega\in(-\pi,+\pi)$	discr. time n

IDFT	IDTFT
$x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi k}{N}n}$	$x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)e^{jwn}dw$

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Finite Impulse Response(FIR) filter response 1

$s(n) = \{\mathbf{1}, 2,2,1\}$

$h(n) = \{\mathbf{1}, 2,3\}$,

$y(n) = h(n) * s(n)$

$L = 3 + 4 - 1 = 6$

$Y(k) = H(k).S(k)$

$$N - \ point\ DFT\ \{s(n)\} = \sum_{n=0}^{N-1} s(n).e^{-j\frac{2\pi k}{N}n}$$

$$N - \ point\ DFT\ \{h(n)\} = \sum_{n=0}^{N-1} h(n).e^{-j\frac{2\pi k}{N}n}$$

$$w_k = \frac{2\pi k}{N}: \quad k = 0 : N - 1$$

$$N = 2^3 = 8$$

Finite Impulse Response(FIR) filter response 1

$h(n) = \{\mathbf{1}, 2,3\}$,

$s(n) = \{\mathbf{1}, 2,2,1\}$

$y(n) = h(n) * s(n)$ $L = 3 + 4 - 1 = 6$

$$8 - \ point\ DFT\ \{h(n)\} = \sum_{n=0}^{7} h(n).e^{-jwn}$$

$$8 - \ point\ DFT\ \{s(n)\} = \sum_{n=0}^{7} s(n).e^{-jwn}$$

$$w_k = \frac{2\pi k}{N}$$

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$$8 - \ point\ DFT\ \{s(n)\} = \sum_{n=0}^{7} s(n).e^{-j\frac{2\pi k}{N}n}$$

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Finite Impulse Response(FIR) filter response 2

$$8-\ point\ DFT\ \{h(n)\} = \sum_{n=0}^{7} h(n).\,e^{-j\frac{2\pi k}{N}n}$$

$\boldsymbol{H}(0) = 6$
$H(1) = (1+\sqrt{2}) - j(3+\sqrt{2})$
$H(2) = -2-2j$
$H(3) = (1-\sqrt{2}) + j(3-\sqrt{2})$
$H(4) = 2$
$H(5) = (1-\sqrt{2}) + j(3-\sqrt{2})$
$H(6) = -2+2j$
$H(7) = (1+\sqrt{2}) + j(3+\sqrt{2})$

$$8-\ point\ DFT\ \{s(n)\} = \sum_{n=0}^{7} s(n).\,e^{-j\frac{2\pi k}{N}n}$$

$S(0) = 6$
$S(1) = \frac{2+\sqrt{2}}{2} - j\frac{4+3\sqrt{2}}{2}$
$S(2) = -1-j$
$S(3) = \frac{2-\sqrt{2}}{2} + j\frac{4-3\sqrt{2}}{2}$
$S(4) = 0$
$S(5) = \frac{2-\sqrt{2}}{2} - j\frac{4-3\sqrt{2}}{2}$
$S(6) = -1+j$
$S(7) = \frac{2+\sqrt{2}}{2} + j\frac{4+3\sqrt{2}}{2}$

$$Y(k) = H(k).S(k)$$

$\boldsymbol{Y}(0) = \mathbf{36}$
$Y(1) = -14.07 - j17.48$
$Y(2) = 4j$
$Y(3) = 0.07 + j0.515$
$Y(4) = 0$
$Y(5) = 0.07 - j0.515$
$Y(6) = -4j$
$Y(7) = -14.07 + j17.48$

Finite Impulse Response(FIR) filter response 3

$$8-\ point\ IDFT\ \{Y(k)\} = y(n) = \frac{1}{8}\sum_{k=0}^{7} Y(k).e^{j\frac{2\pi k}{N}n}$$

$$y(n) = \{1,4,9,11,8,3,0,0\}$$

```
s = [1  2  2  1];
h = [1  2   3] ;
ss = fft (s, 8);
hh = fft (h , 8);
yy= ss.*hh;
y= ifft (yy);
W= 2*pi*(0:7)/8;  % discrete frequency variable
plot (W, abs(yy))
```

Fast Fourier Transform in MATLAB :

FFT in MATLAB

```
Fs = 25; % Sampling frequency
t = -1:1/Fs:1; % Time vector of 1 second
f = 5; % Create a sine wave of f Hz.
x = sin(2* pi*t*f);% sampled sin
L=length (x);
nfft =512;
% Take fft, padding with zeros so that length(X)
X = fft(x,nfft);

% Take the magnitude of fft of x
mx = abs(X);
% Frequency vector
f=2*pi*((-nfft/2 : nfft/2-1))/nfft
%f = 2*pi*(0:nfft-1)/nfft;
% Generate the plot, title and labels.
figure(1);
stem(t,x);grid
title('sampled Sine Wave Signal');
xlabel('Time (s)');
ylabel('Amplitude');
figure(2);
plot(f,mx);grid
title('Power Spectrum of a Sine Wave');
xlabel('wk');
ylabel('Power');
```

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