B. Haidar

Structural Mechanics (1)

Week No-04

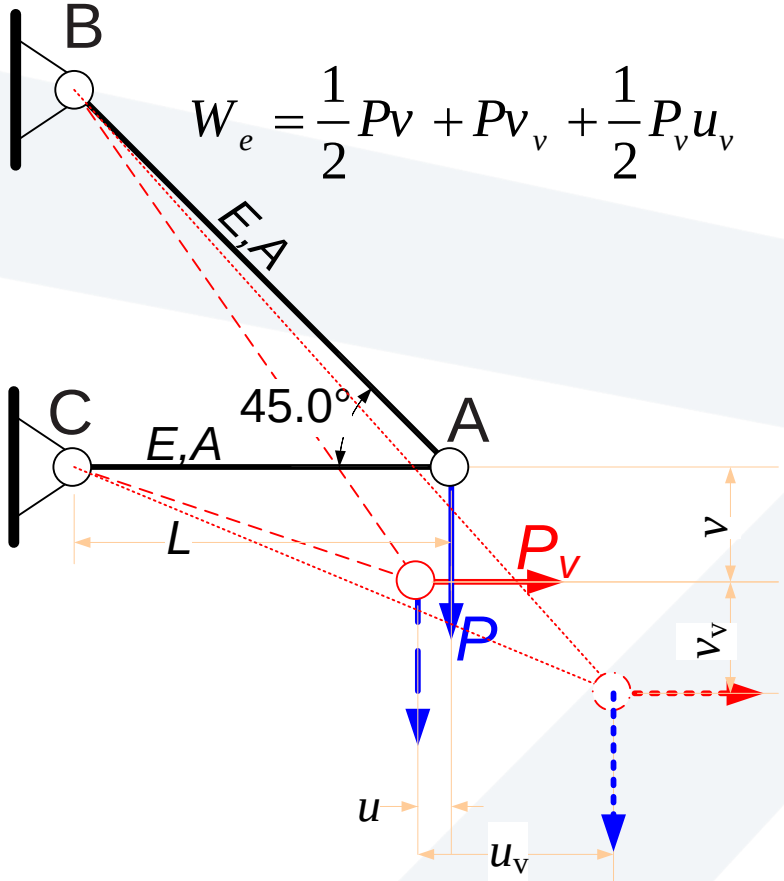
Deflection in Determinate Structures

B. Haidar

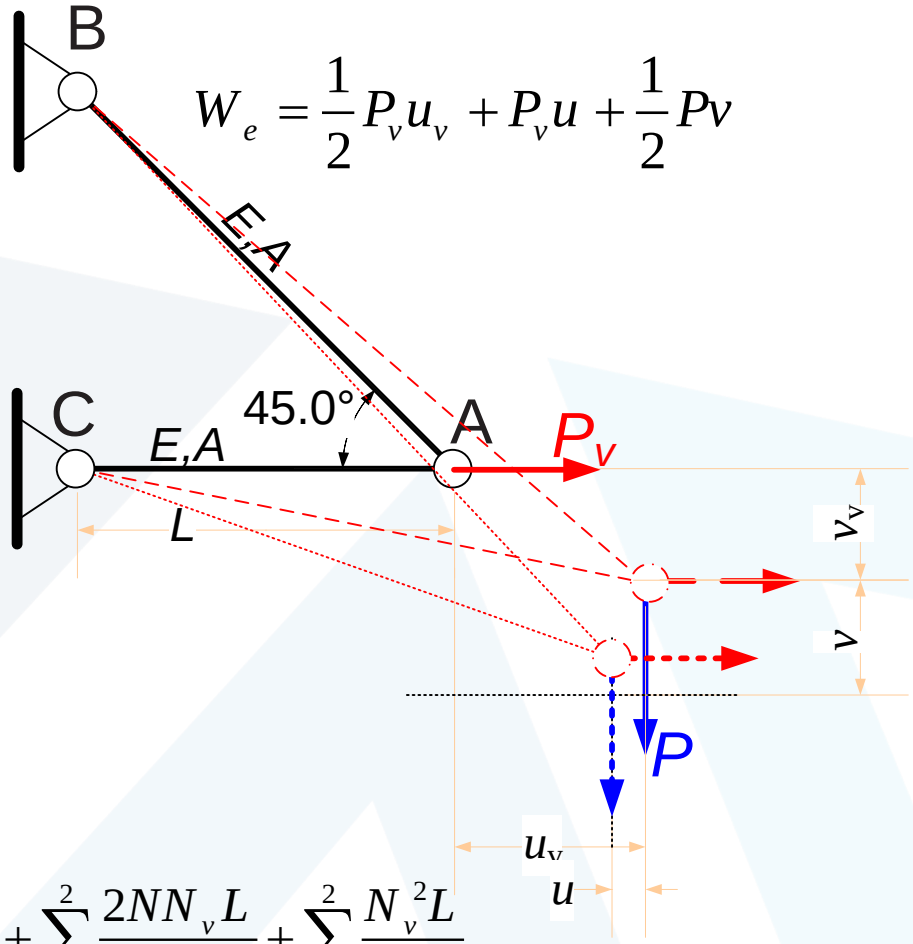
Deflections of Trusses, Beams, & Frames: Work-Energy Methods

- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

Deflections of Trusses by the Virtual Work Method



$$W_e = \frac{1}{2}Pv + Pv_v + \frac{1}{2}P_v u_v$$



$$W_e = \frac{1}{2}P_v u_v + P_v u + \frac{1}{2}Pv$$

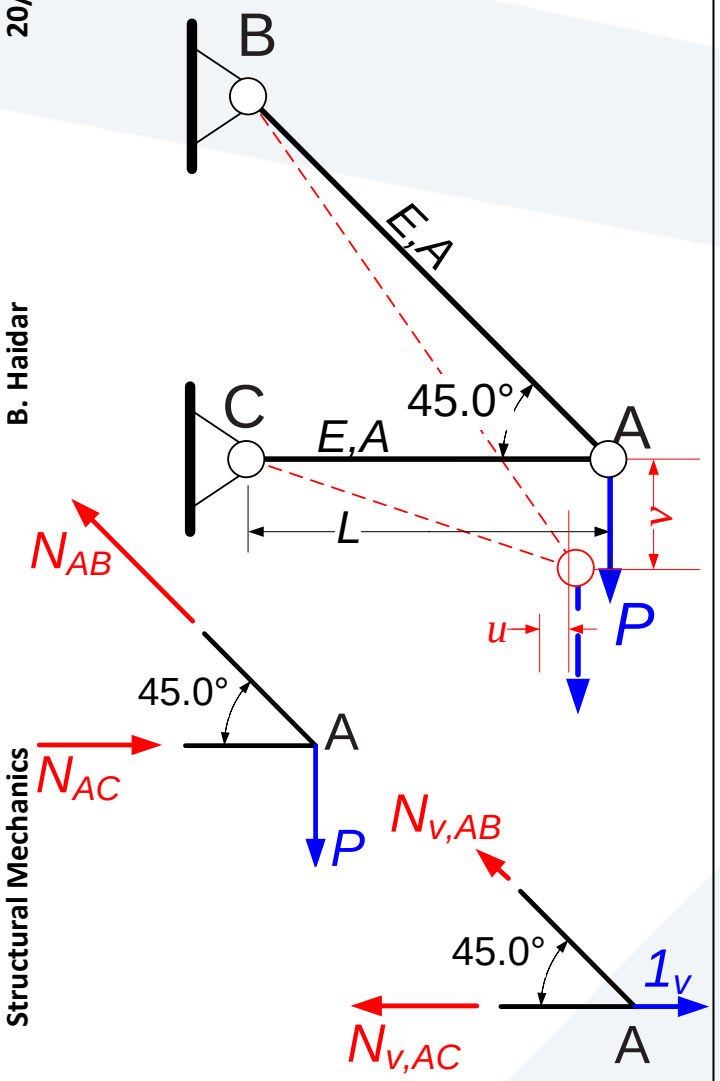
$$U = \sum_{i=1}^{2}\frac{(N + N_v)^2 L}{2EA} = \sum_{i=1}^{2}\frac{N^2 L}{2EA} + \sum_{i=1}^{2}\frac{2NN_v L}{2EA} + \sum_{i=1}^{2}\frac{N_v^2 L}{2EA}$$

$$W_e = U \Rightarrow Pv_v = P_v u = \sum_{i=1}^{2}\frac{NN_v L}{EA}$$

Making P_v=1

$$(1_v)u = u = \sum_{i=1}^{2}\frac{NN_v L}{EA}$$

The truss shown in Fig. carries a gradually applied load P at the joint A. **Compute the horizontal deflection u at A.**



1) Analyzing the truss under the real load, for N in the two members. We found

$$N_{AB}=1.41P\,(T) \qquad N_{AC}=P\,(C)$$

2) Applying a virtual unit load at A in the direction of u. Then Analyzing For N_v

Considering the vertical equil. at A:

$$N_{v,AB}\cos 45^{\circ}=0 \Rightarrow N_{v,AB}=0$$

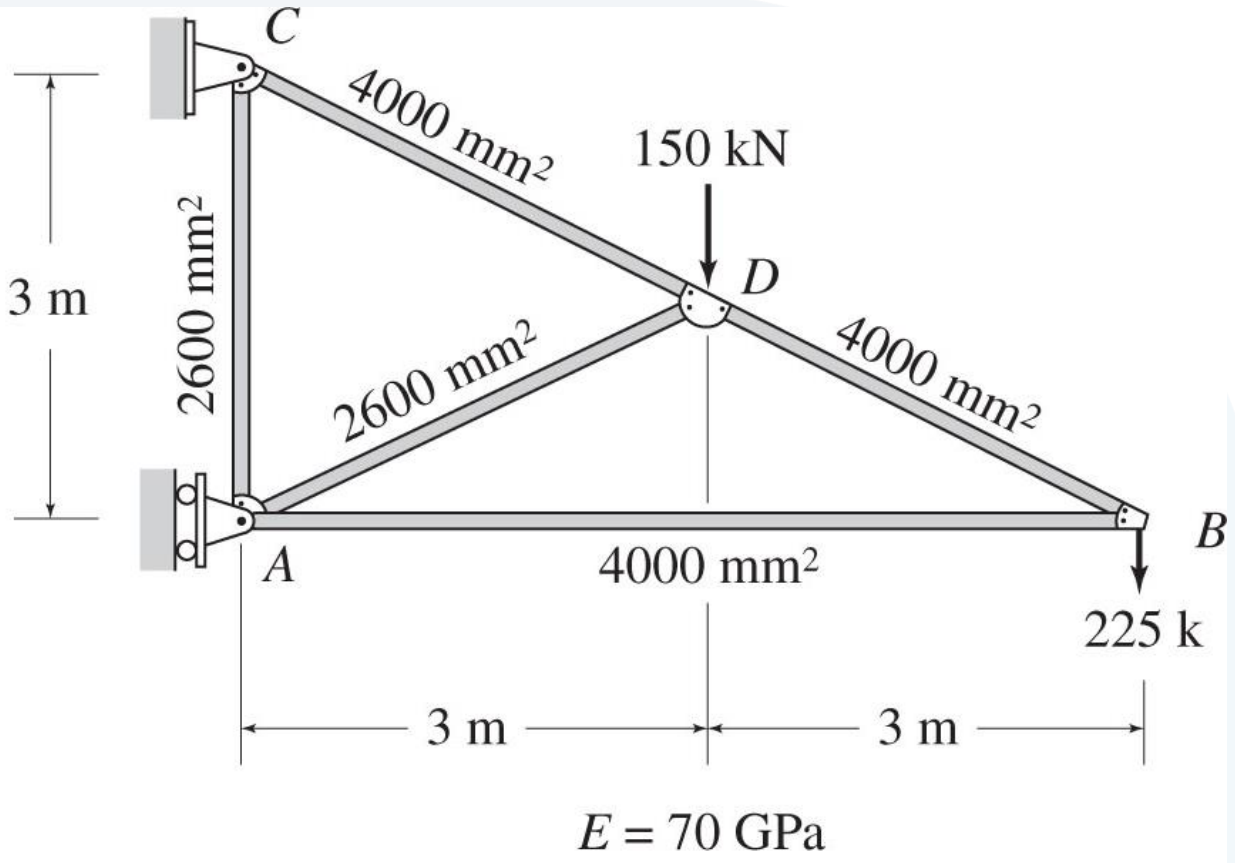
Considering the horizontal equil. at A:

$$-N_{v,AC}+1_v=0 \Rightarrow N_{v,AC}=1\ (T)$$

3) Applying the V.W.M $\quad (1_v)u=u=\sum_{i=1}^{2}\frac{NN_vL}{EA}=0+\frac{(-P)(+1)L}{EA}=\frac{-PL}{EA}$

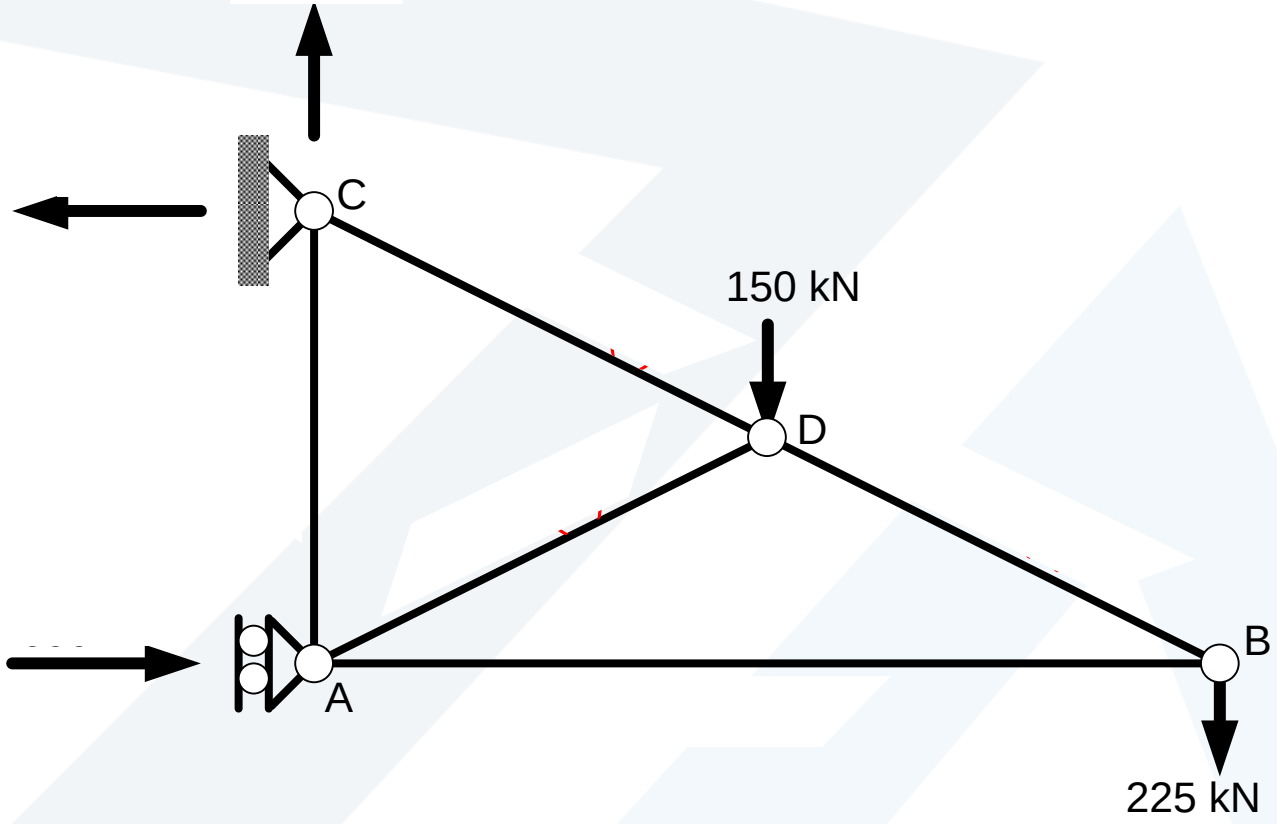
B. Haidar

Solution: Use the virtual work method to determine the horizontal and vertical components of the deflection at joint B of the truss shown in the following figure. **Then find the vertical deflection at D without V.U.L.**

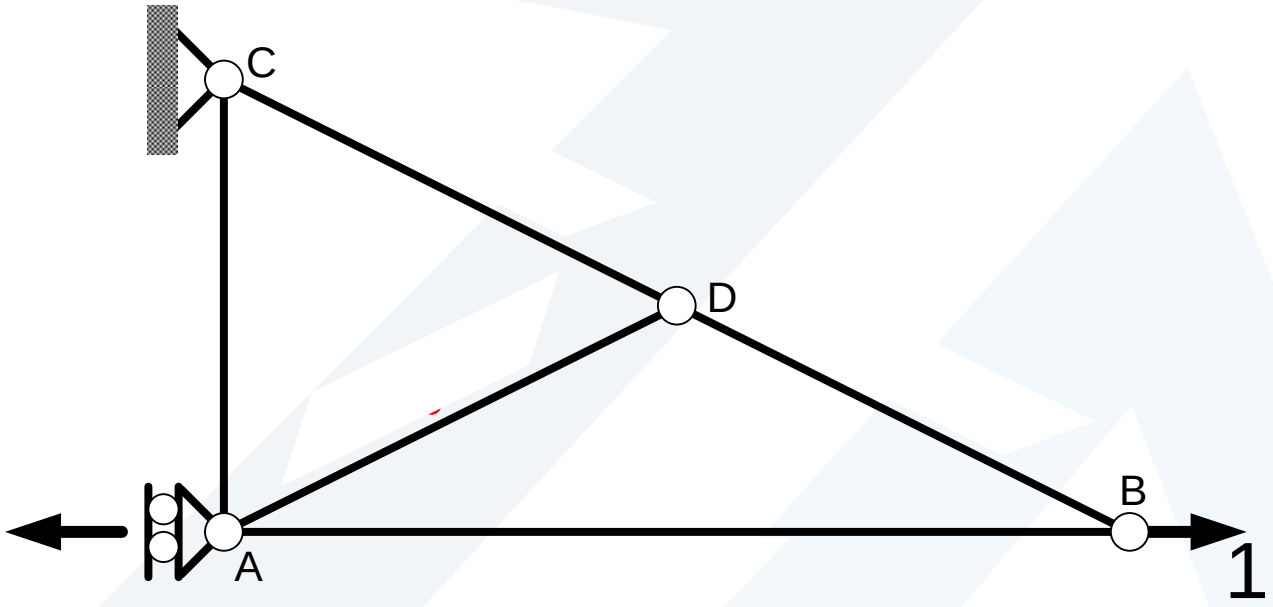


Solution: Horizontal Deflection at B, Δ_{BH}

Real System The real system and the corresponding member axial forces (N) are shown in the following figure.



Solution: Virtual System Horizontal Deflection at B, Δ_{BH} : The virtual system used for determining the horizontal deflection at B consists of a 1-kN load applied in the horizontal direction at joint B, as shown in the following fig. The member axial forces (N_{v1}) due to this virtual load are also shown in this figure.



Solution: The member axial forces due to the real system (N) and this virtual system (N_{v1}) are then tabulated as shown in the following table:

Member	L (m)	A (m^2)	N (kN)	N_{v1} (kN)	$N_{v1}(NL/A)$ (kn^2/m)
AB	6	0.0040	-450	1	-675.000
AC	3	0.0026	75	0	0
AD	3.354	0.0026	-167.7	0	0
CD	3.354	0.0040	670.8	0	0
BD	3.354	0.0040	503.1	0	0
					-675.000

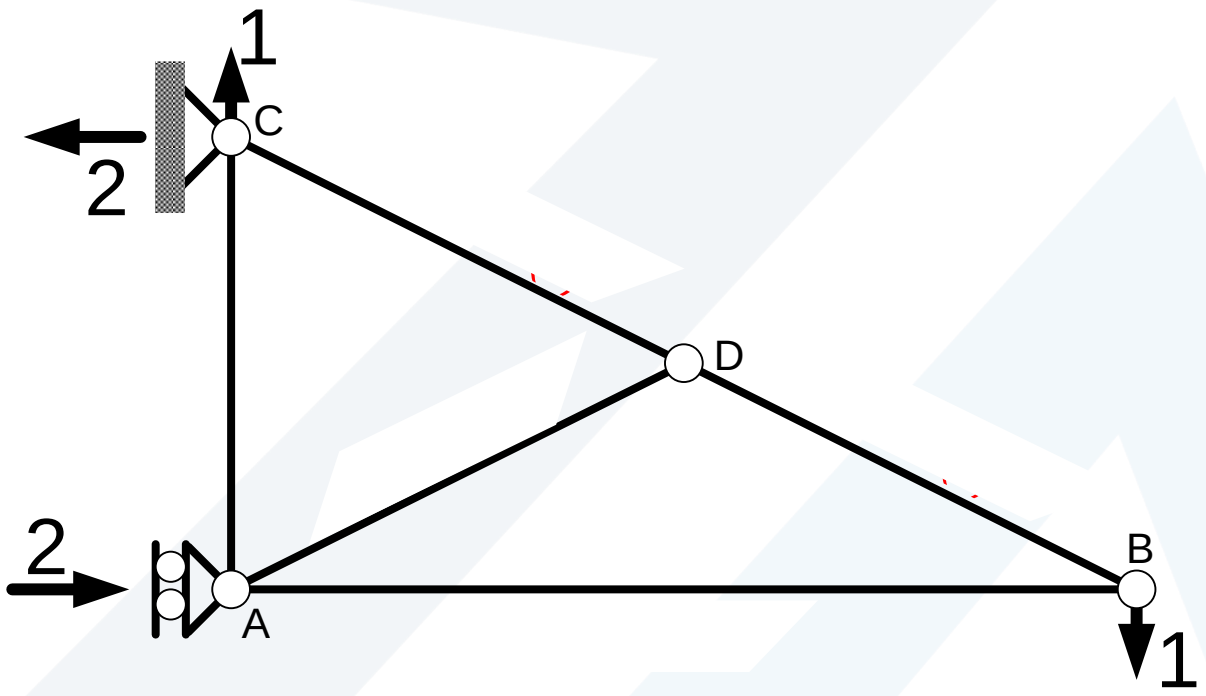
Solution: the virtual work expression is applied to determine Δ_{BH} as shown below:

$$1(\Delta_{BH}) = \frac{1}{E}\sum \frac{N_{v1}(NL)}{A}$$

$$(1)(\Delta_{BH}) = -\frac{675,000\ \text{kN.m}}{70(10^6)\ \text{kN-m}} = -0.00964\,\text{m}$$

$$\Delta_{BH} = 9.64\,\text{mm} \leftarrow$$

Solution: **Virtual System** **Vertical Deflection at B, Δ_{BV}** : The virtual system used for determining the vertical deflection at B consists of a 1-kN load applied in the vertical direction at joint B, as shown in the following fig. The member axial forces (N_{v2}) due to this virtual load are also shown in this figure.



Solution: The member axial forces due to the real system (N) and this virtual system (N_{v2}) are then tabulated as shown in the following table:

Member	L (m)	A (m^2)	N (kN)	N_{v2} (kN)	$N_{v2}(NL/A)$ (kn^2/m)
AB	6	0.0040	-450	-2	1,350,000
AC	3	0.0026	75	0	0
AD	3.354	0.0026	-167.7	0	0
CD	3.354	0.0040	670.8	2.236	1,257,674
BD	3.354	0.0040	503.1	2.236	943,255
				$\sum$	**3,550,929**

Solution: the virtual work expression is applied to determine Δ_{BV} as shown below:

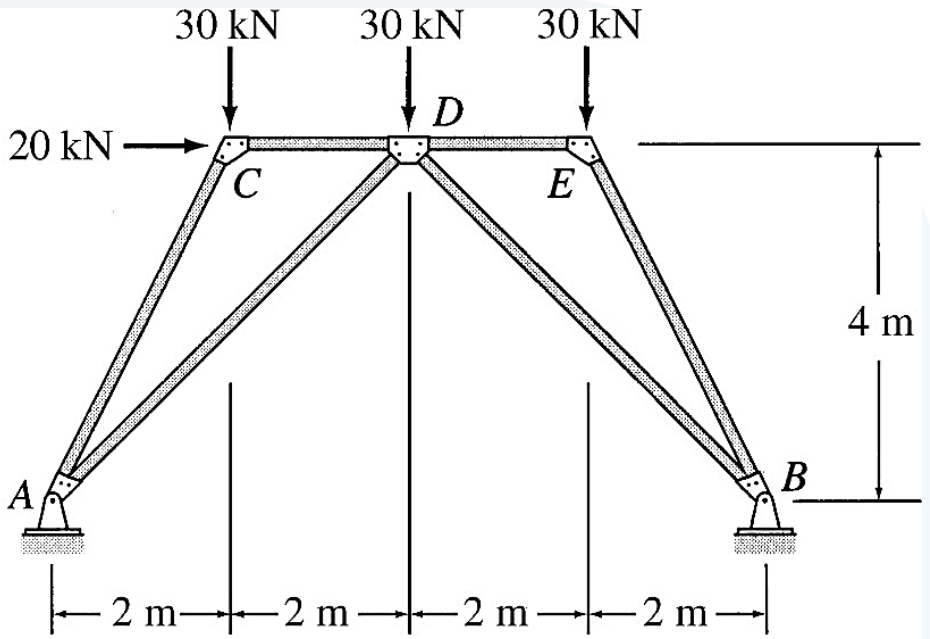
$$1(\Delta_{BV}) = \frac{1}{E}\sum \frac{N_{v2}(NL)}{A}$$

$$(1)(\Delta_{BV}) = \frac{3,550,929}{70(10^6)}\frac{\text{kN.m}}{\text{kN-m}} = 0.05073\,\text{m}$$

$$\Delta_{BV} = 50.73\,\text{mm}\downarrow$$

Homework

Problem.01: Use the virtual work method to determine the horizontal deflection at joint (E) of the truss shown.



EA = constant
E = 200 GPa
A = 5000 mm^2

Homework

Problem.02: Determine the smallest cross-sectional area (A) required for the members of the truss shown, so that the horizontal deflection at joint (D) does not exceed 10 mm. Use the virtual work method.