Lecture 2

Number Systems

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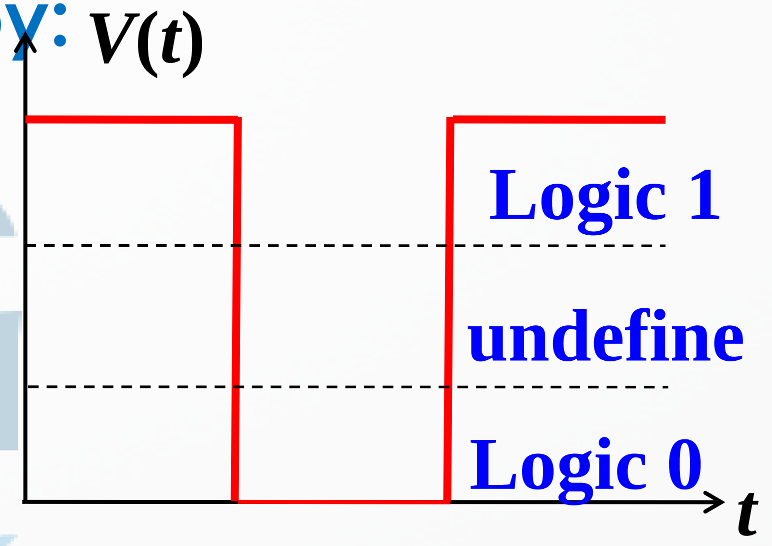
Outline

1. Digital Systems
2. Binary Numbers
3. Number-base Conversions
4. Octal and Hexadecimal Numbers
5. Complements
6. Signed Binary Numbers
7. Binary Codes
8. Binary Storage and Registers
9. Binary Logic

BINARY DIGITAL SIGNAL

- An information variable represented by physical quantity.
- For digital systems, the variable takes on discrete values.
  - ➢Two level, or binary values are the most prevalent values.
- Binary values are represented abstractly by:
  - ➢ Digits 0 and 1
  - ➢ Words (symbols) False (F) and True (T)
  - ➢ Words (symbols) Low (L) and High (H)
  - ➢ And words On and Off
- Binary values are represented by values or ranges of values of physical quantities.



Binary digital signal

DECIMAL NUMBER SYSTEM

- **Base** (also called radix) = 10
  - 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
- **Digit Position**
  - Integer & fraction
- **Digit Weight**
  - Weight = (*Base*) $^{\textit{Position}}$
- **Magnitude**
  - Sum of "*Digit* x *Weight*"
- **Formal Notation**

Position	2	1	0	-1	-2
Digit	5	1	2	7	4
Weight	*100*	*10*	*1*	*0.1*	*0.01*
Magnitude	500	10	2	0.7	0.04

$$d_2{*}B^2+d_1{*}B^1+d_0{*}B^0+d_{-1}{*}B^{-1}+d_{-2}{*}B^{-2}$$

$$(512.74)_{10}$$

OCTAL NUMBER SYSTEM

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- Base = 8
  - 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }
- Weights
  - Weight = $(Base)^{Position}$
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

$(512.74)_8$

64	8	1		1/8	1/64
5	1	2	.	7	4
2	1	0		-1	-2

$5 *8^2+1 *8^1+2 *8^0+7 *8^{-1}+4 *8^{-2}$

$=(330.9375)_{10}$

HEXADECIMAL NUMBER SYSTEM

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- Base = 16
  - 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }
- Weights

$(1E5.7A)_{16}$

256	16	1	1/16	1/256
1	E	5	7	A
2	1	0	-1	-2

  - Weight = (Base) Position
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

$1 *16^{2}+14 *16^{1}+5 *16^{0}+7 *16^{-1}+10 *16^{-2}$

$=(485.4765625)_{10}$

The Power of 2

n	2^n
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
4	$2^4=16$
5	$2^5=32$
6	$2^6=64$
7	$2^7=128$

n	2^n	
8	$2^8=256$	
9	$2^9=512$	
10	$2^{10}=1024$	Kilo
11	$2^{11}=2048$	
12	$2^{12}=4096$	
20	$2^{20}=1M$	Mega
30	$2^{30}=1G$	Giga
40	$2^{40}=1T$	Tera

Addition

- **Decimal Addition**

	1	1	
		5	5
+		5	5
	1	1	0

← **Carry**

= ***Ten ≥ Base***

➔ Subtract a Base

Binary Addition

- Column Addition

	1	1	1	1	1	1		
		1	1	1	1	0	1	$= 61$
+			1	0	1	1	1	$= 23$
	1	0	1	0	1	0	0	$= 84$

$\geq (2)_{10}$

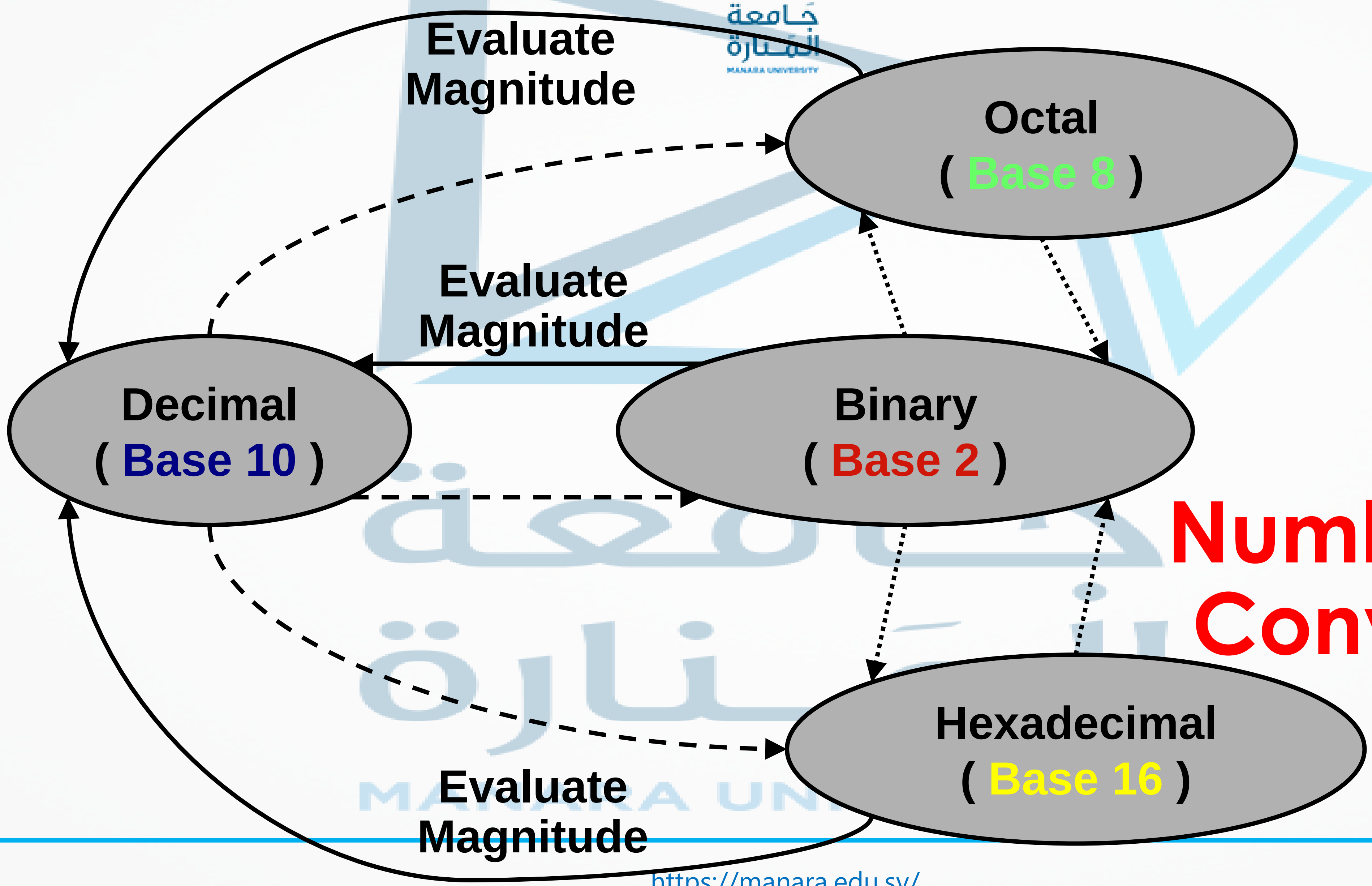
Binary Subtraction

- **Borrow a "Base" when needed**

$= (10)_2$

		1			2			
	0	~~2~~	2	0	0	2		
	~~1~~	0	0	~~1~~	~~1~~	0	1	77
−			1	0	1	1	1	- 23
	0	1	1	0	1	1	0	= 54

Number Base Conversions



Binary Multiplication

- Bit by bit

				1	0	1	1	1	23
x					1	0	1	0	10
				0	0	0	0	0	
			1	0	1	1	1		
		0	0	0	0	0			
	1	0	1	1	1				
	1	1	1	0	0	1	1	0	230

Decimal (Fraction) to Binary Conversion

- Multiply the number by the ‘Base’ (=2)
- Take the integer (either 0 or 1) as a coefficient
- Take the resultant fraction and repeat the division

Example: $(0.625)_{10}$

	Integer	Fraction	Coefficient
0.625 * 2 =	1	. 25	$a_{-1} = 1$
0.25 * 2 =	0	. 5	$a_{-2} = 0$
0.5 * 2 =	1	. 0	$a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_2 = (0.101)_2$

MSB LSB

Decimal (Integer) to Binary Conversion

- Divide the number by the ‘Base’ (=2)
- Take the remainder (either 0 or 1) as a coefficient
- Take the quotient and repeat the division

Example: $(13)_{10}$

	Quotient	Remainder	Coefficient
13 / 2 =	6	1	$a_0 = 1$
6 / 2 =	3	0	$a_1 = 0$
3 / 2 =	1	1	$a_2 = 1$
1 / 2 =	0	1	$a_3 = 1$

Answer: $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

MSB LSB

Decimal to Octal Conversion

Example: $(175)_{10}$

	Quotient	Remainder	Coefficient
175/ 8 =	21	7	$a_0 = 7$
21 / 8 =	2	5	$a_1 = 5$
2 / 8 =	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_2\ a_1\ a_0)_8 = (257)_8$

Example: $(0.3125)_{10}$

	Integer	Fraction	Coefficient
0.3125 * 8 =	2	. 5	$a_{-1} = 2$
0.5 * 8 =	4	. 0	$a_{-2} = 4$

Answer: $(0.3125)_{10} = (0.a_{-1}\ a_{-2}\ a_{-3})_8 = (0.24)_8$

Binary – Octal Conversion

- $8 = 2^3$
- Each group of 3 bits represents an octal digit

Example:

Assume Zeros

$$(10110.01)_2$$

$$(2\quad 6\ .\ 2)_8$$

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Works both ways (*Binary* to *Octal* & *Octal* to *Binary*)

Binary – Hexadecimal Conversion

- $16 = 2^4$
- Each group of 4 bits represents a hexadecimal digit

Example:

Assume Zeros

$(1\ 0\ 1\ 1\ 0\ .\ 0\ 1\ )_2$

$(1\quad 6\quad .\quad 4\ )_{16}$

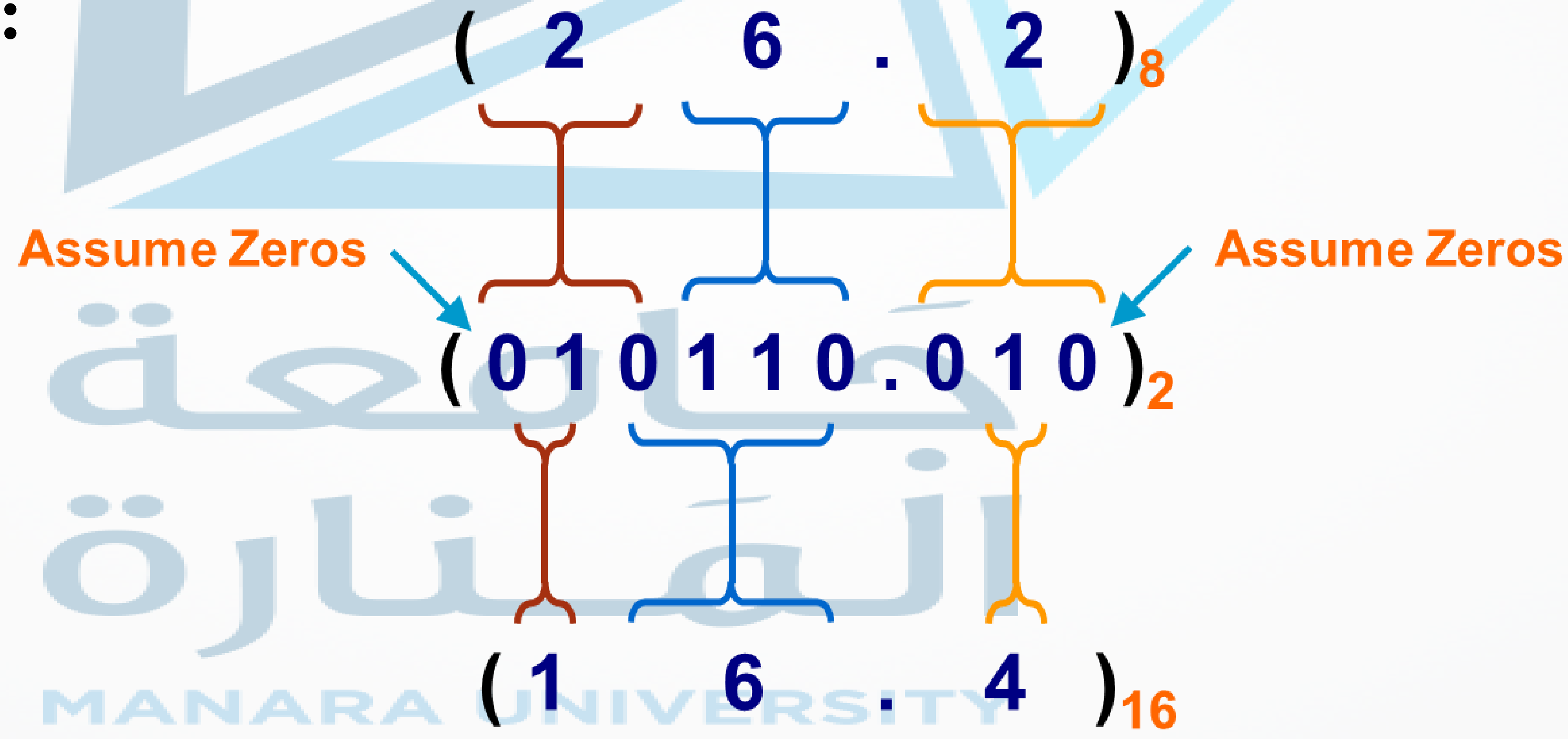
Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

Works both ways (*Binary* to *Hex* & *Hex* to *Binary*)

Octal – Hexadecimal Conversion

- Convert to **Binary** as an intermediate step

Example:



Works both ways (*Octal* to *Hex* & *Hex* to *Octal*)

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A

Decimal	Binary	Octal	Hex
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

NEXT LECTURE

البوابات المنطقية

Logic Gates