Lecture 5

Karnaugh Map

Dr. Bassam Atieh

The idea of Karnaugh Map: (with minterms)

The combining **property 14a** ($x \cdot y + x \cdot \bar{y} = x$) allow us **to replace** any two minterms that **differ** in the value of only one variable **with a** single product term that does not include that variable at all. Also, **theorem 7b** ($x = x + x$) allow us **to use** any minterm **more than once**.

Karnaugh map is an alternative to the truth-table form for representing a function. The map allows easy recognition of minterms that can be combined using **property 14a**.

The more you **practice** using K-maps, the more **intuitive** you become in finding the minimum cost for any circuit.

Example 1: (Two-Variable Map)

Find the minimum-cost SOP form for the following function:

$f=1$ when $x1=0$ regardless of $x2$; $f=1$ when $x2=0$ regardless of $x1$

$$f(x_1, x_2) = \bar{x}_1 + \bar{x}_2$$

Answer

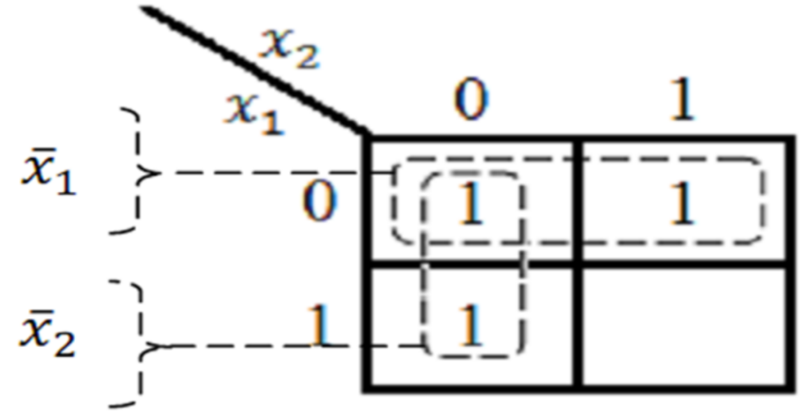
x_1	x_2	$f(x_1, x_2)$	Minterm#
0	0	1	m_0
0	1	1	m_1
1	0	1	m_2
1	1	0	m_3

K-Map:

$$f(x_1, x_2) = \overbrace{\bar{x}_1} + \overbrace{\bar{x}_2}$$

$x_1 \backslash x_2$	0	1
0	m_0	m_1
1	m_2	m_3

$x_1 \backslash x_2$	0	1
0	1	1
1	1	0



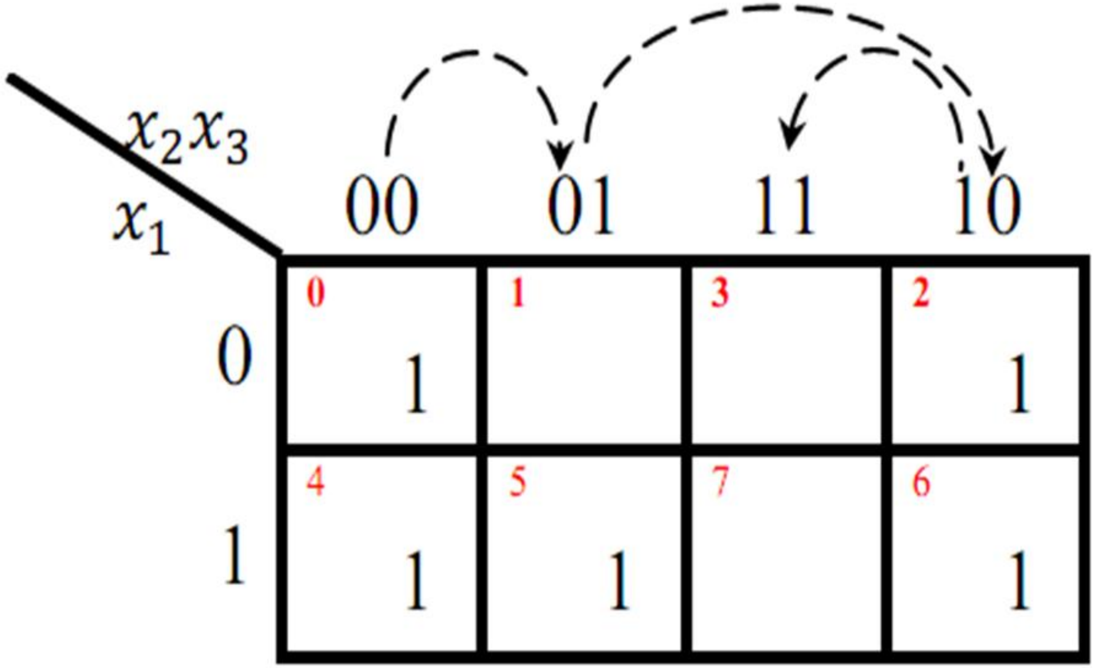
$$f_{SOP} = \bar{x}_1 + \bar{x}_2$$

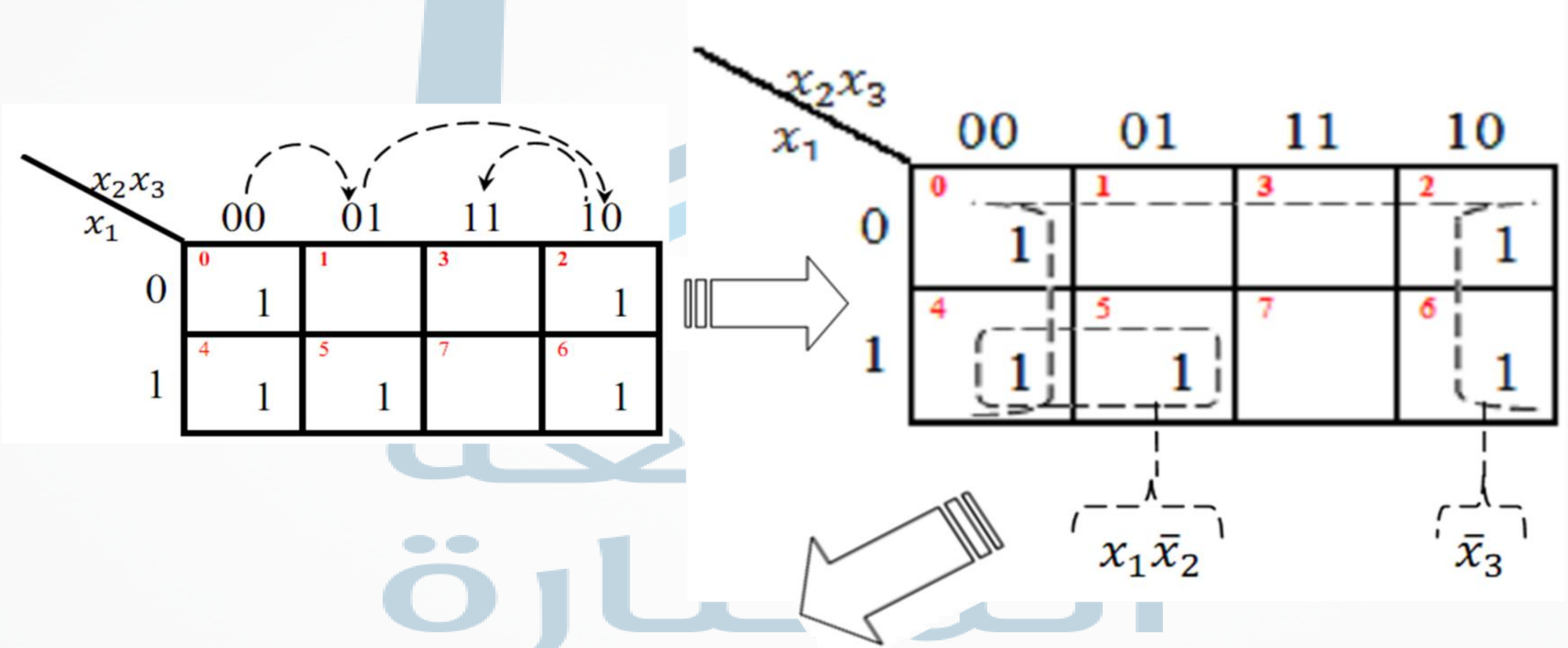
Example 2: (Three-Variable Map)

Find the minimum-cost SOP form for the following function:

x_1	x_2	x_3	$f(x_1, x_2)$	$Minterm\#$	Product#
0	0	0	1	m_0	1^{st}
0	0	1	0	m_1	
0	1	0	1	m_2	1^{st}
0	1	1	0	m_3	
1	0	0	1	m_4	1^{st}, 2^{nd}
1	0	1	1	m_5	2^{nd}
1	1	0	1	m_6	1^{st}
1	1	1	0	m_7	

$$f(x_1, x_2, x_3) = \overbrace{\underbrace{\bar{x}_3}_{**0}}^{1st\ Product} + \overbrace{\underbrace{x_1\bar{x}_2}_{10*}}^{2nd\ Product}$$



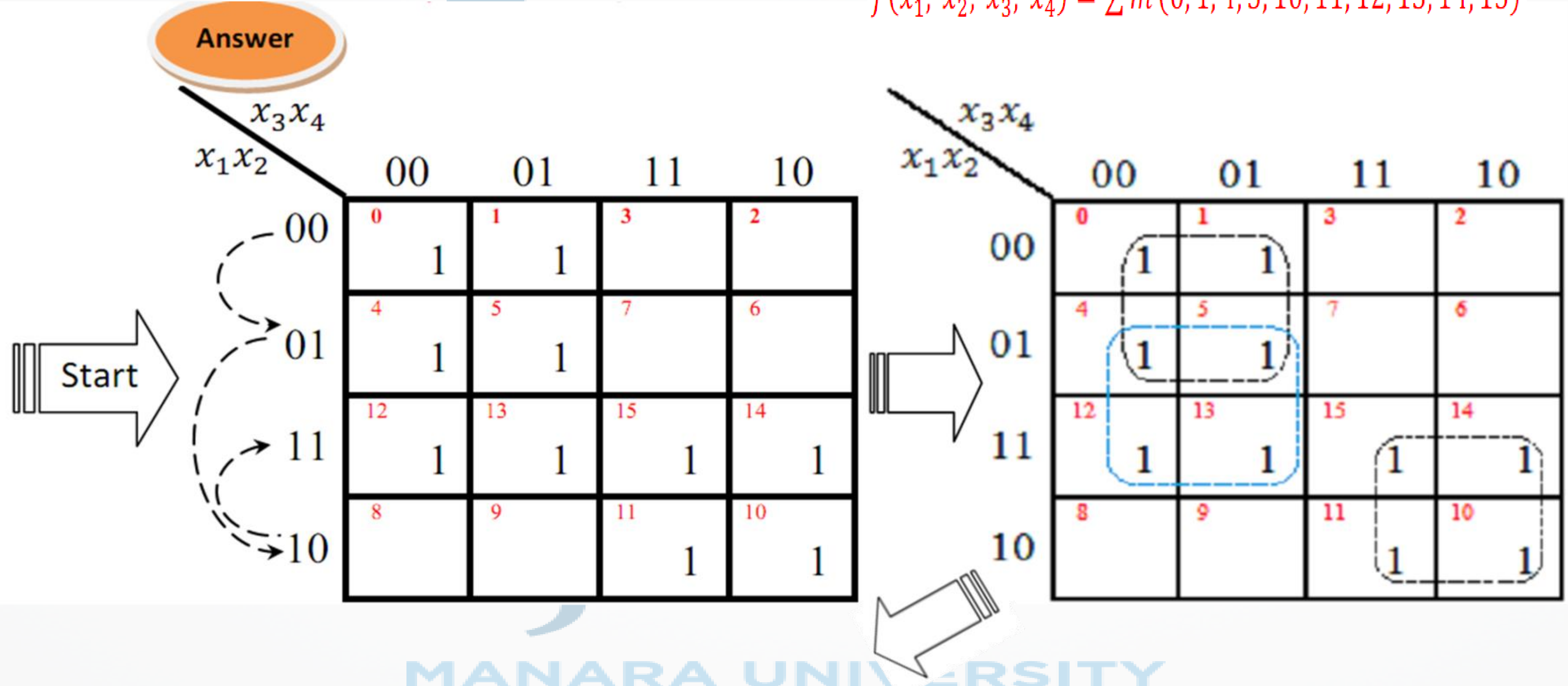


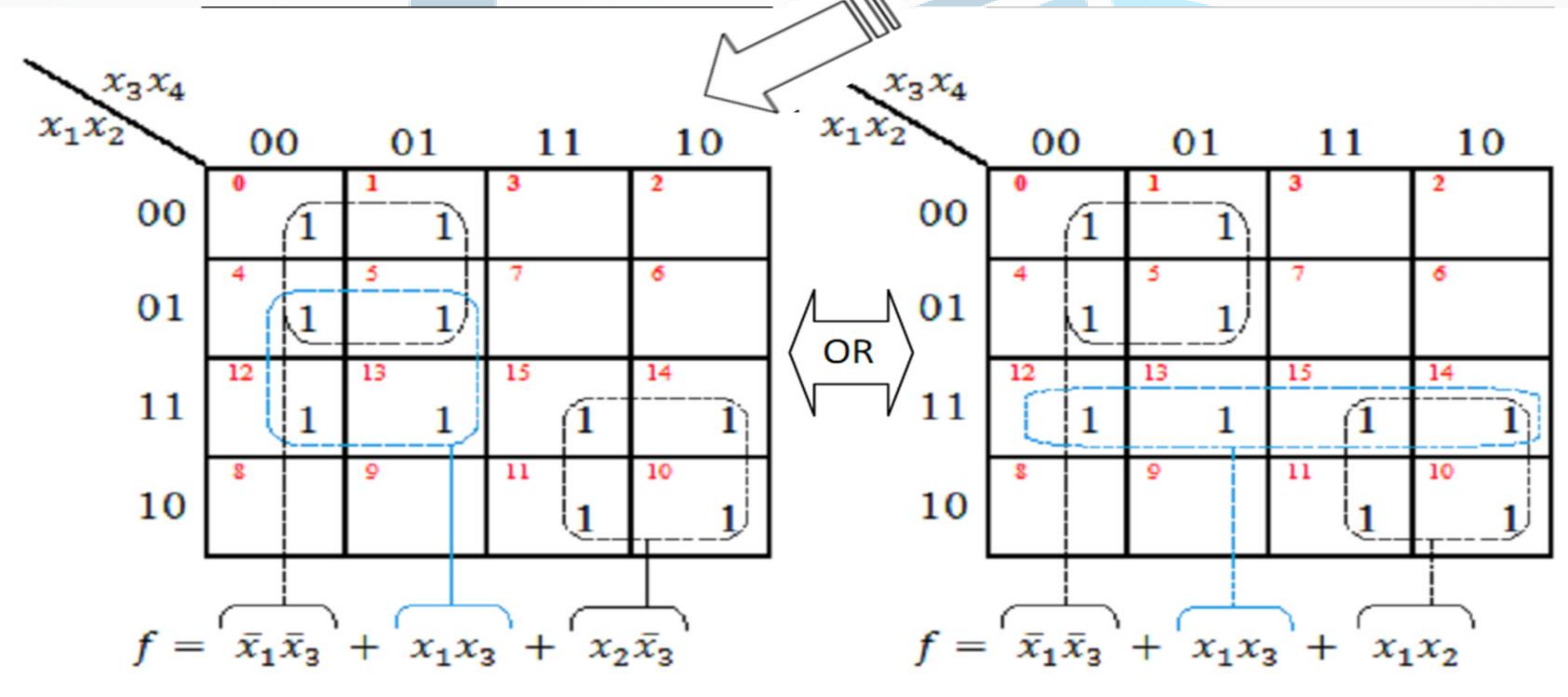
$$f_{SOP} = \bar{x}_3 + x_1\bar{x}_2$$

Example 3: (Four-Variable Map)

Find the minimum-cost SOP form for the following function:

$$f(x_1, x_2, x_3, x_4) = \sum m(0, 1, 4, 5, 10, 11, 12, 13, 14, 15)$$





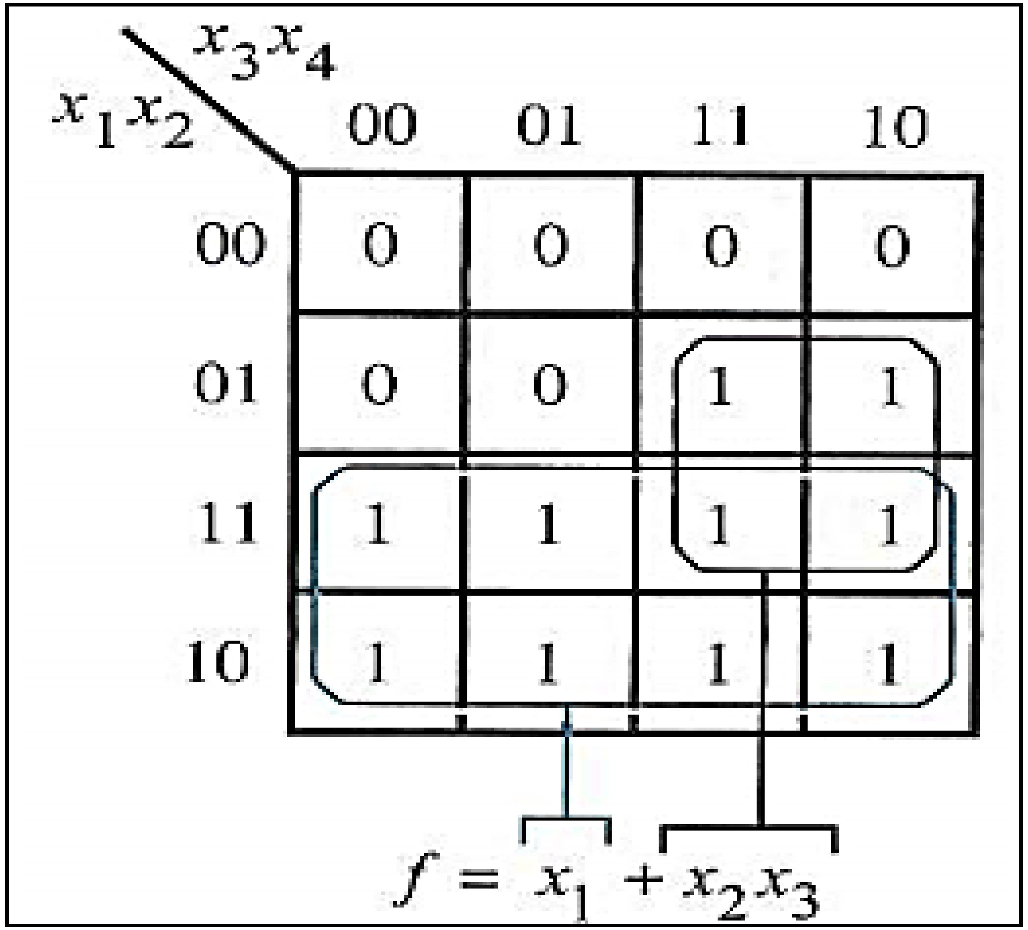
x_3x_4
x_1x_2
00
01
11
10
00
01
11
10
1
1
1
1
1
1
1
1
1
1
$f = \bar{x}_1\bar{x}_3 + x_1x_3 + x_2\bar{x}_3$
OR
x_3x_4
x_1x_2
00
01
11
10
00
01
11
10
1
1
1
1
1
1
1
1
1
1
$f = \bar{x}_1\bar{x}_3 + x_1x_3 + x_1x_2$

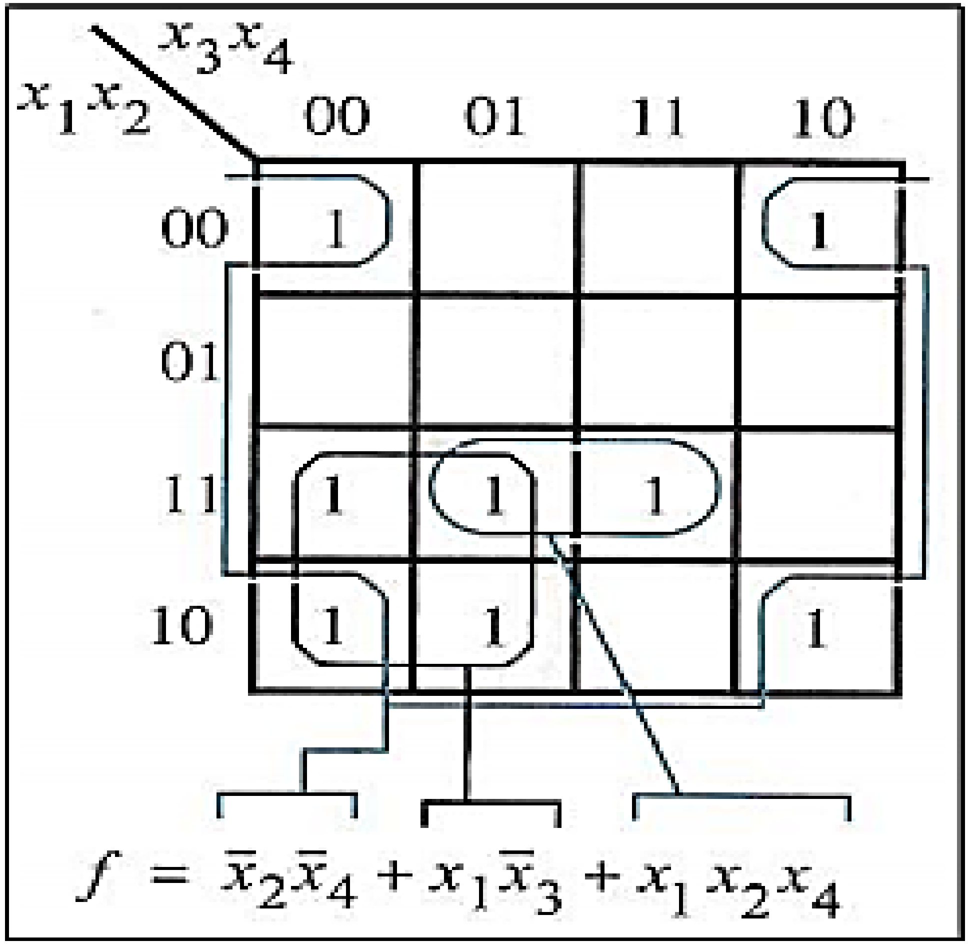
Example 4:

Find the minimum-cost SOP form

(a) $f(x_1, x_2, x_3, x_4) = \sum m(6,7,8,9,10,11,12,13,14,15)$

(b) $f(x_1, x_2, x_3, x_4) = \sum m(0,2,8,9,10,12,13,15)$



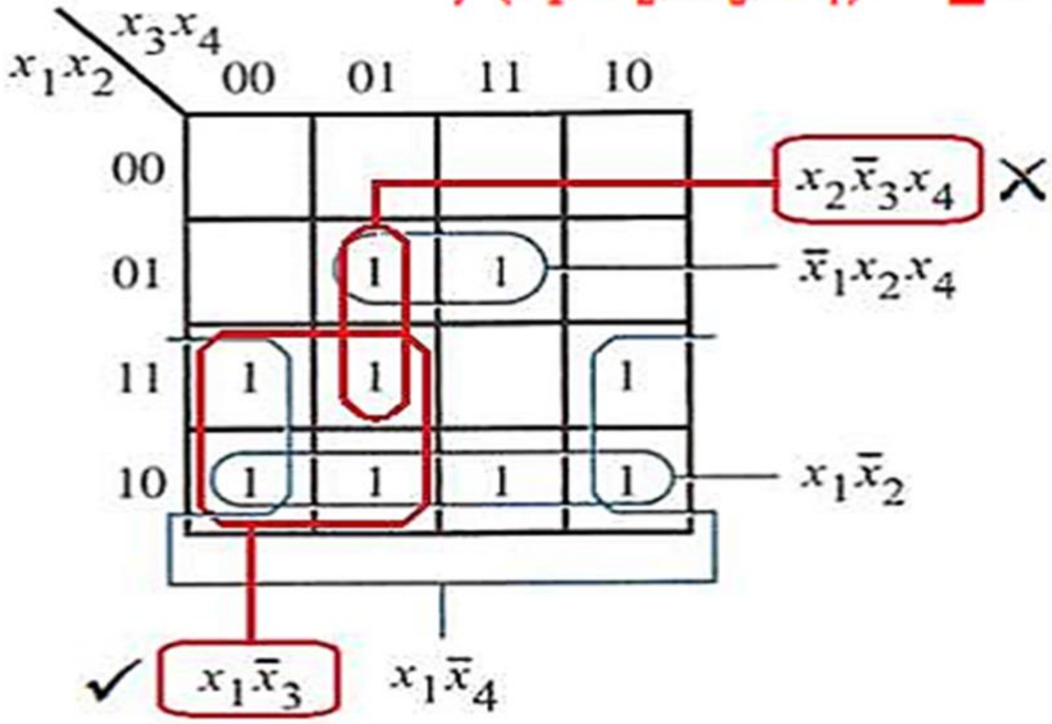


Example 6:

Answer

Find the minimum-cost SOP form for the following function:

$$f(x_1, x_2, x_3, x_4) = \sum m(5, 6, 8, 9, 10, 11, 12, 13, 14)$$



$$f = \bar{x}_1x_2x_4 + x_1\bar{x}_2 + x_1\bar{x}_4 + x_1\bar{x}_3$$

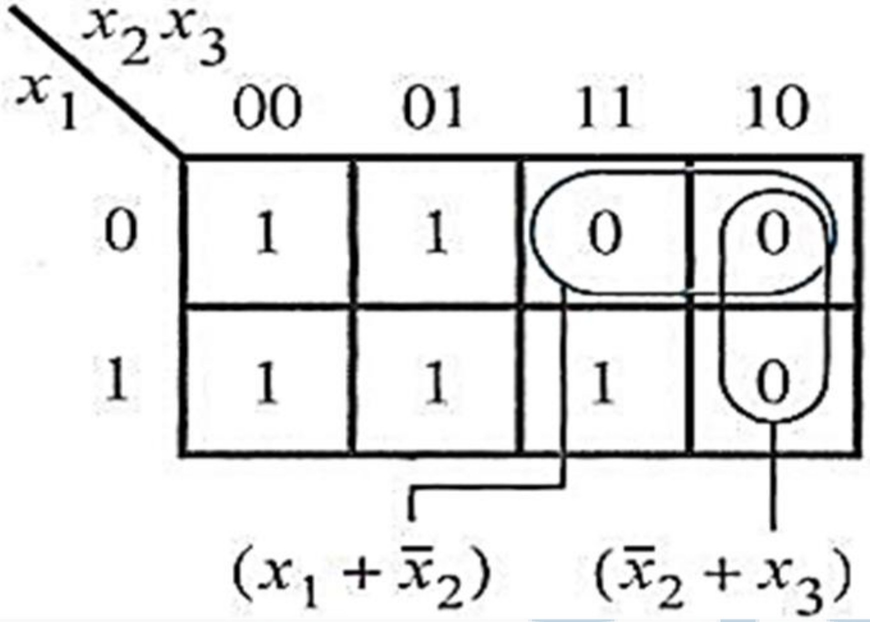
For m_{13}, we could have formed 2 different clusters (the ones in red boxes). But according to strategy #2, a cluster must be as big as possible. Therefore, we chose the bigger cluster $x_1\bar{x}_3$.

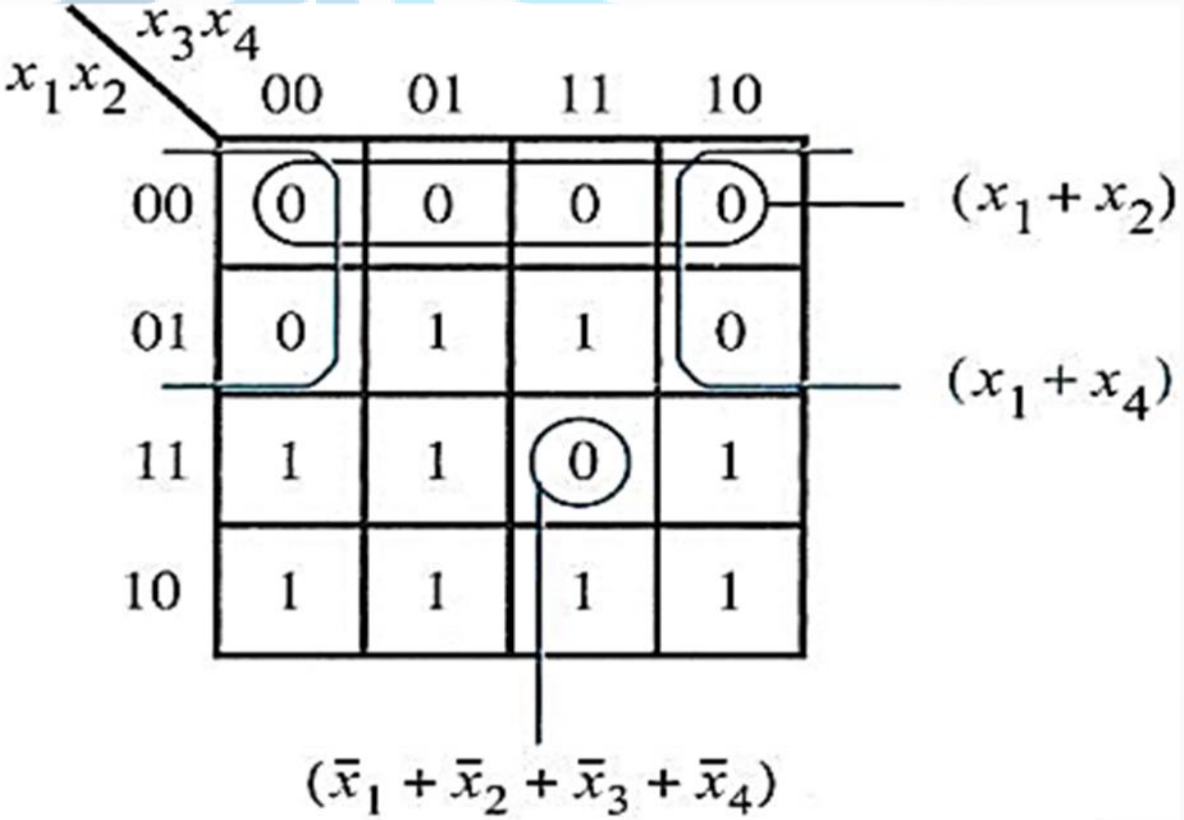
Example 7:

Find the minimum-cost POS form for the following functions:

(a) $f(x_1, x_2, x_3) = \prod M(2,3,6)$

(b) $f(x_1, x_2, x_3, x_4) = \prod M(0,1,2,3,4,6,15)$





$$f = (x_1 + \bar{x}_2)(\bar{x}_2 + x_3)$$

$$f = (x_1 + x_2)(x_1 + x_4)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)$$

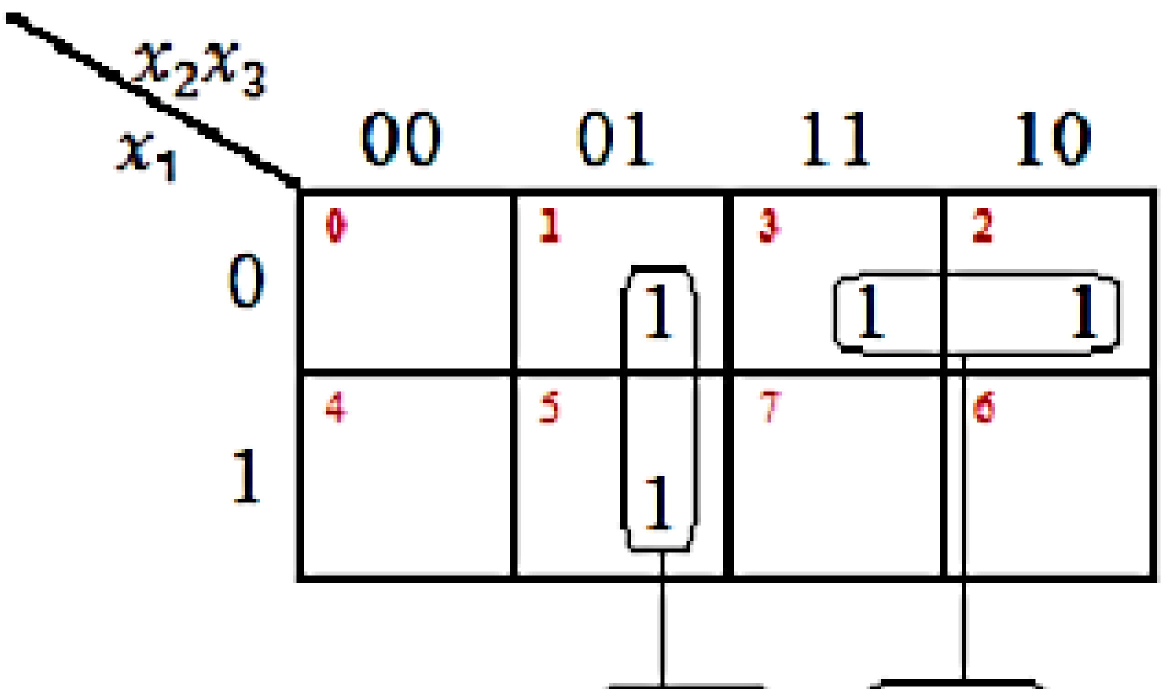
Find the minimum-cost SOP and POS forms for the following functions:

(a) $f(x_1, x_2, x_3) = \sum m(1,2,3,5)$.

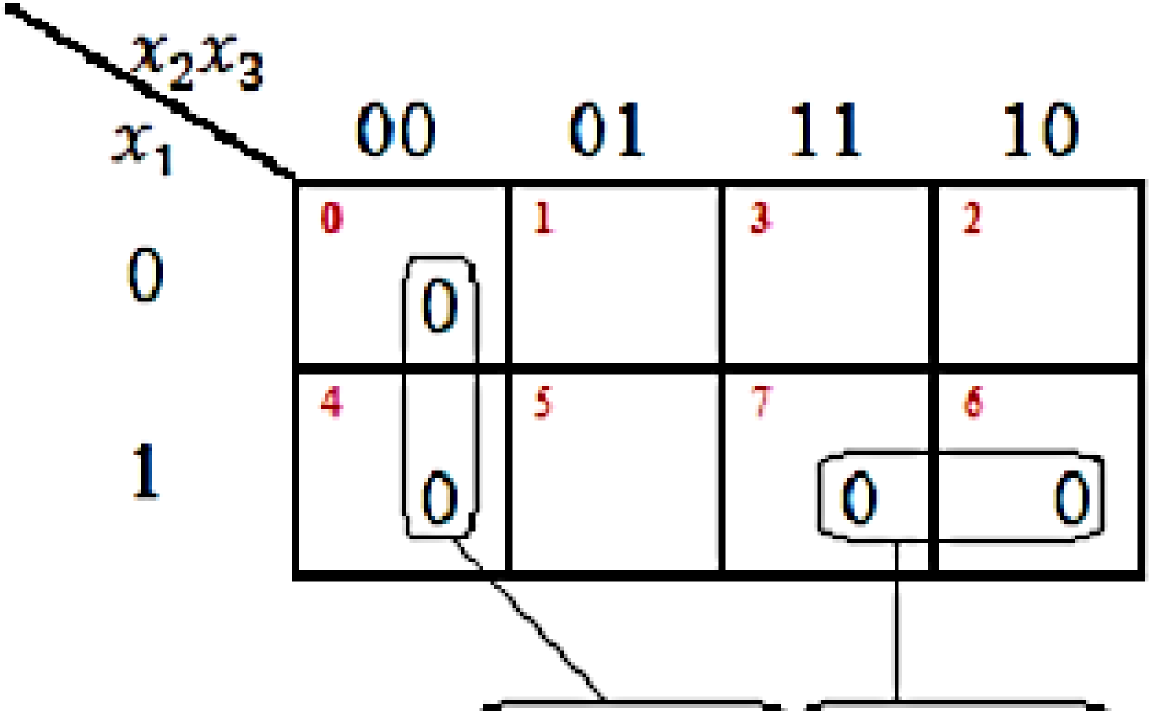
(b) $f(x_1, x_2, x_3, x_4) = \prod M(0,1,2,4,5,7,8,9,10,12,14,15)$.

Answer

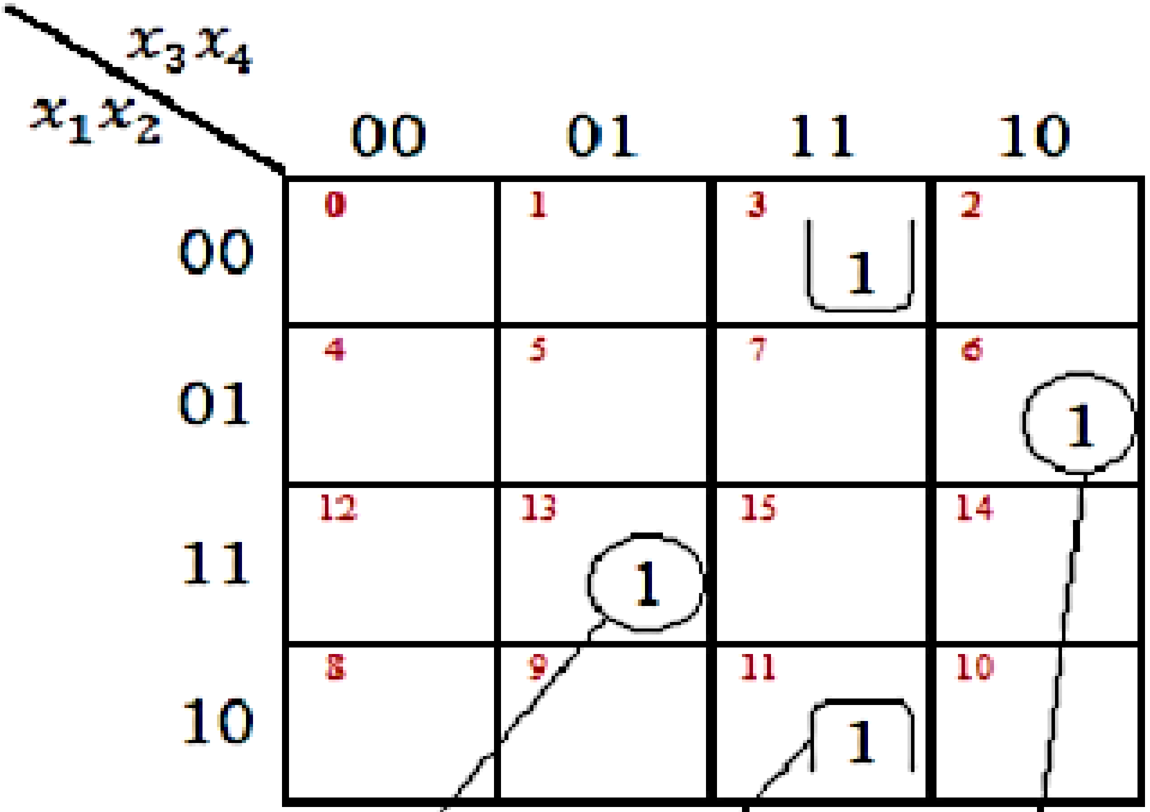
(a)



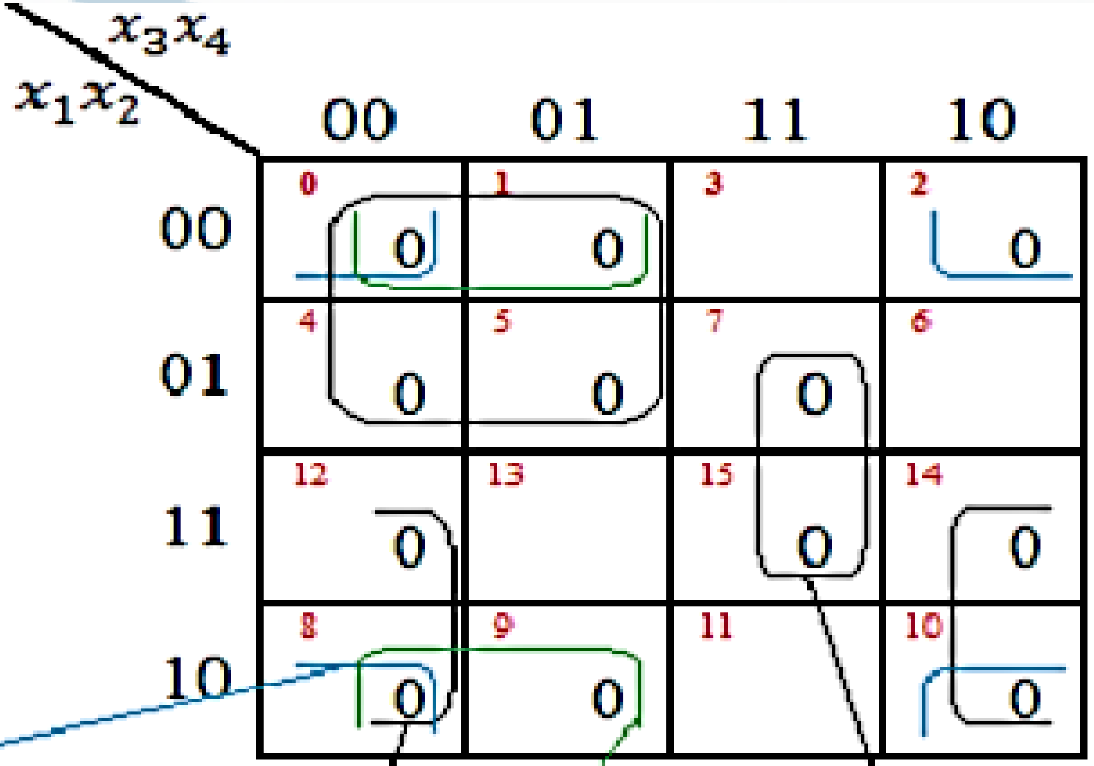
$$f_{SOP} = \bar{x}_2x_3 + \bar{x}_1x_2$$



$$f_{POS} = (x_2 + x_3)(\bar{x}_1 + \bar{x}_2)$$



$$f_{SOP} = x_1 x_2 \bar{x}_3 x_4 + \bar{x}_2 x_3 x_4 + + \bar{x}_1 x_2 x_3 \bar{x}_4$$



$$f_{POS} = (\bar{x}_1 + x_4)(x_2 + x_3)(\bar{x}_2 + \bar{x}_3 + \bar{x}_4)$$
$$(x_2 + x_4)(x_1 + x_3)$$