1. Stress–Strain Curves for Ductile Materials

2. Stress–Strain Curves for Brittle Materials

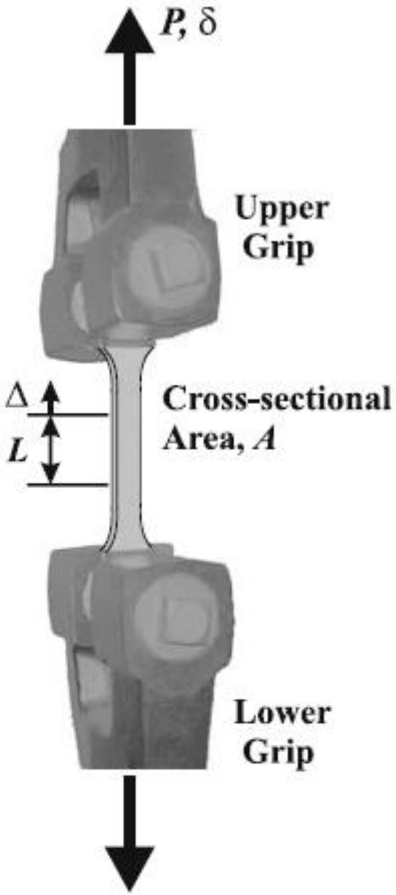
Stress–Strain Curves for Ductile Materials

Stress–strain curves are determined from experiments on axial bars (*Fig.*).

A bar with cross-sectional area A is clamped into a set of grips of a testing machine. One grip is fixed and the other moves by known displacement δ ; the force P required to cause the displacement is measured. Such an experiment is known as a *displacement* or *strain controlled* experiment.

Measurements are taken of the change in length Δ and the associated force P.

By dividing the applied force P by area A and the elongation Δ by gage length L, the stress required to cause a certain strain is found: $\sigma = P/A\ ;\ \varepsilon = \Delta/L$.



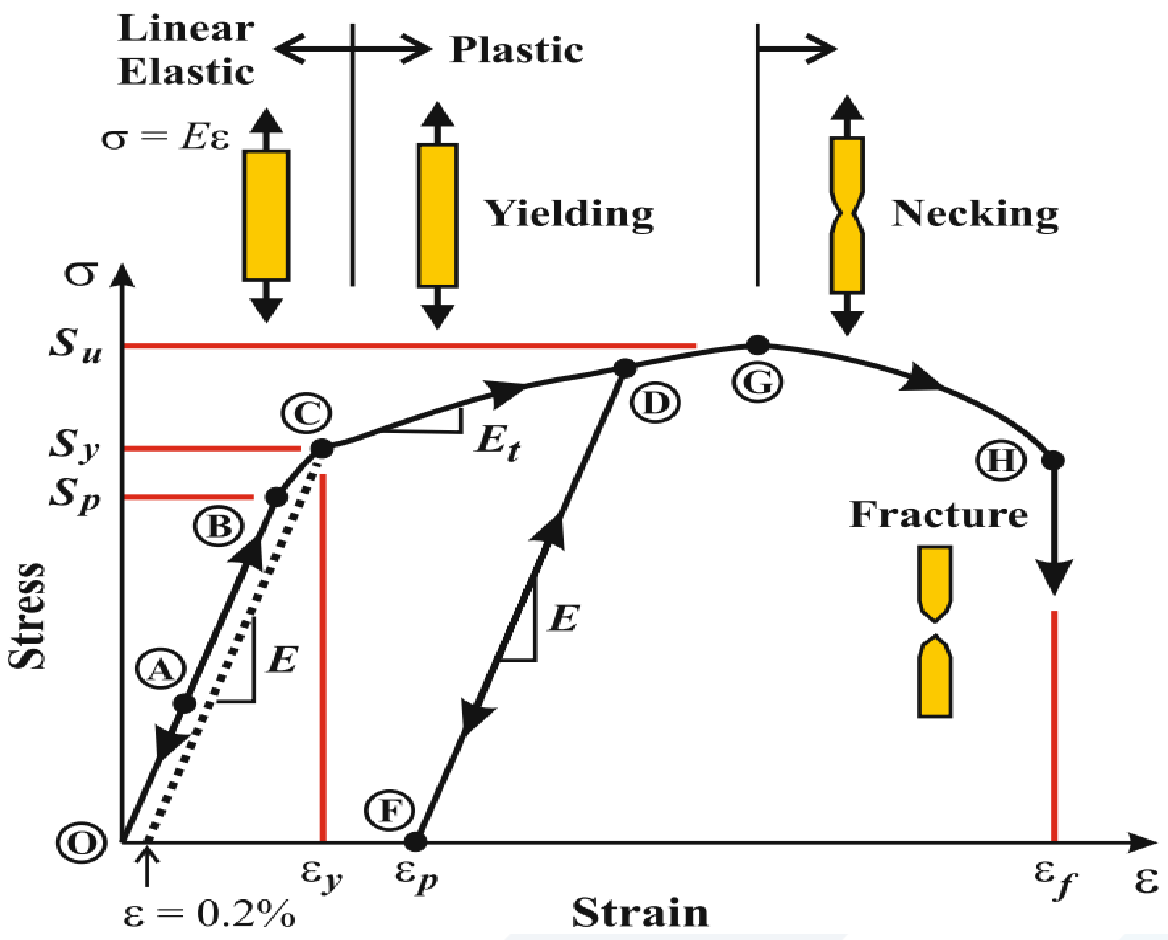
Strain can also be measured with *strain gages,* which are about 10 mm in length. A strain gage is epoxied to the specimen, and is part of an electronic circuit. As the specimen – and thus the strain gage – changes length, the resistance of the strain gage changes, providing an electric signal that indicates the strain of the specimen.

Plotting stress against strain gives the *stress–strain* curve. A representative *stress–strain* curve for a ductile material (most metals) is given in next Fig.

Linear–Elastic Loading and Unloading

For initial loading (line OAB), the stress is linear with slope E : $\sigma = E\varepsilon$.

Removing load along BAO (Unloading), the strain returns to zero. ⟶ Elasticity.



Key to plot:

⟶Arrows indicate direction of $\sigma - \varepsilon$ Plot. B-D & D-H are one-way.
Unloading from a point on CG is linear with slope E.

O. Origin.

A. Point on Linear-Elastic Curve.

B. Proportional Limit.

C. Yield Point ("0.2% Yield").

D. Point on Plastic-Curve.

F. Permanent Strain due to Yielding.

G. Ultimate Strength, onset of Necking.

H. Failure Strain.

E, Et: Young's Modulus, Tangent Modulus.

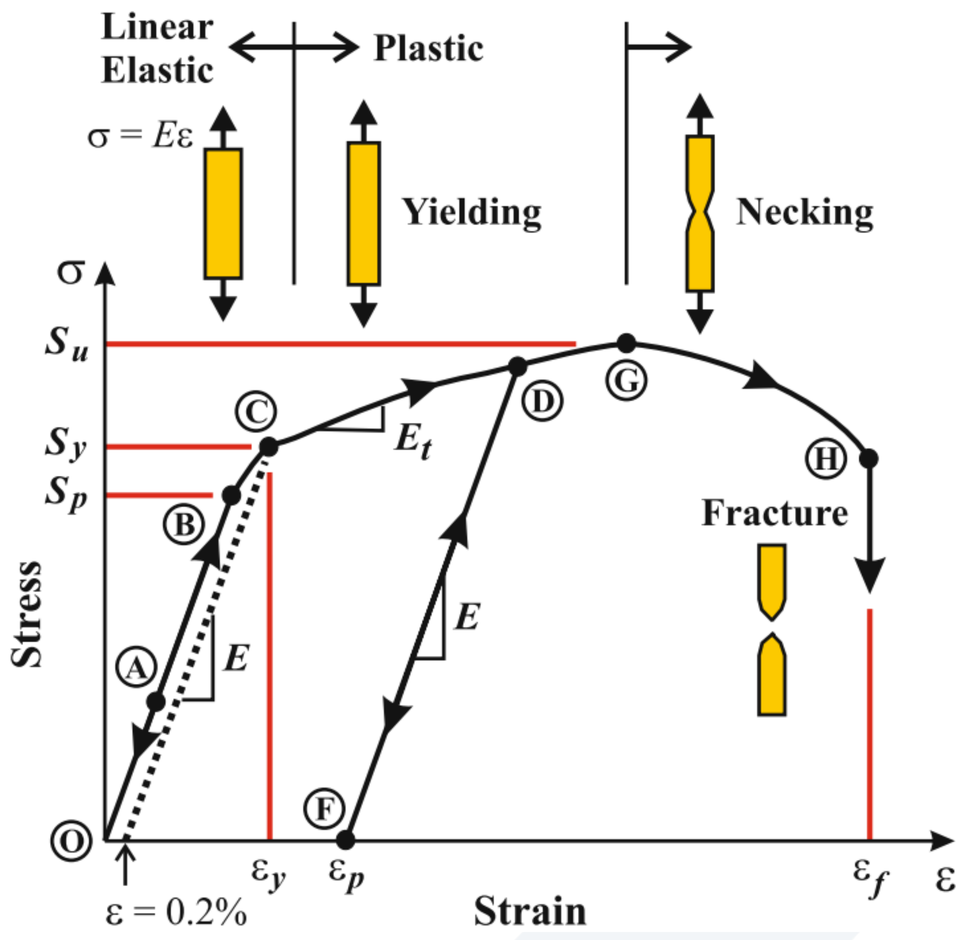
S_p, S_y, S_u: Proportional Limit, Yield Limit, Ultimate Limit.

$\varepsilon_y, \varepsilon_p, \varepsilon_f$: Yield strain,Permanent strain, Failure Strain.

Yielding

Linear–elastic behavior ends at a value of stress called: *proportional limit* S_p (point B), difficult to be determined from experiments.

An engineering convention called *yield strength* S_y (σ_y), replaces S_p by intersecting the curve with the dotted parallel to *OAB*, displaced by 0.2% at the strain axis (point *C*). $\varepsilon_y = Sy/E$, is the *yield strain*.



<u>Typical values of S_y & ε_y are given in Tables</u>

<u>Key to plot:</u>

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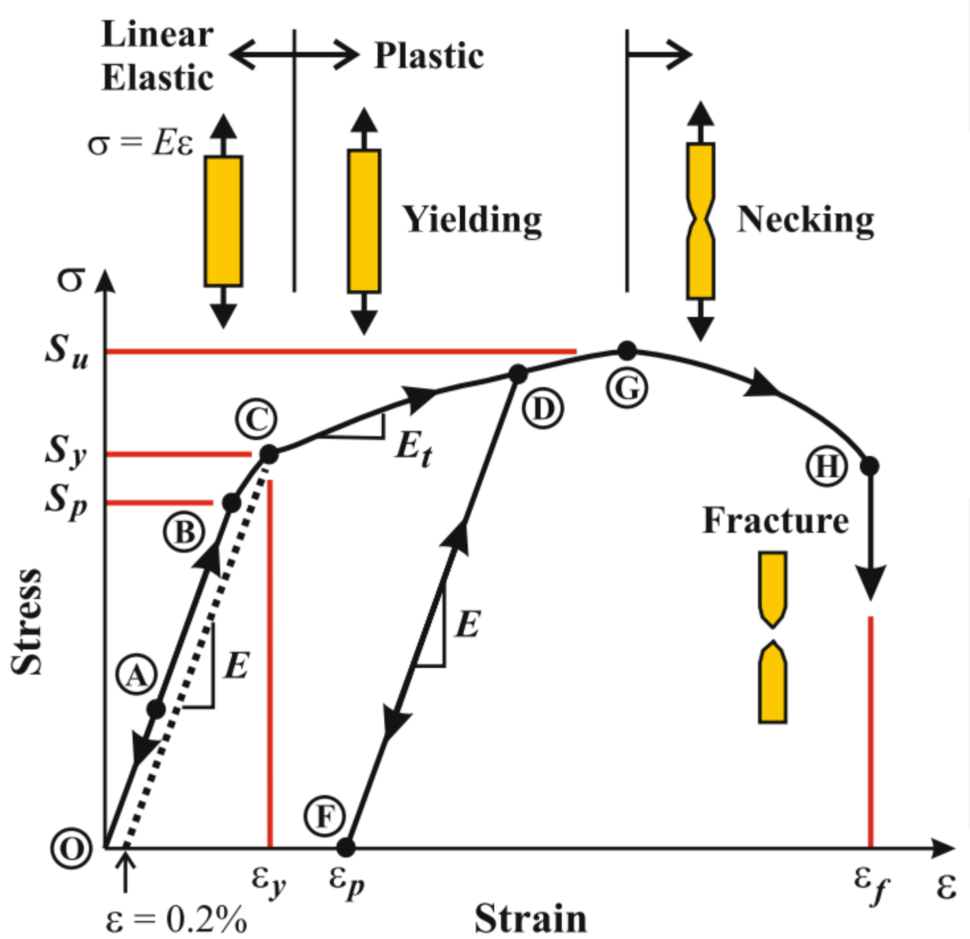
Plastic Deformation, Necking, and Failure

As strain increases from C towards G, the slope of the $\sigma - \varepsilon$ curve, the *tangent modulus* E_t, decreases eventually to zero when the stress reaches a maximum: The *ultimate tensile strength* S_u.



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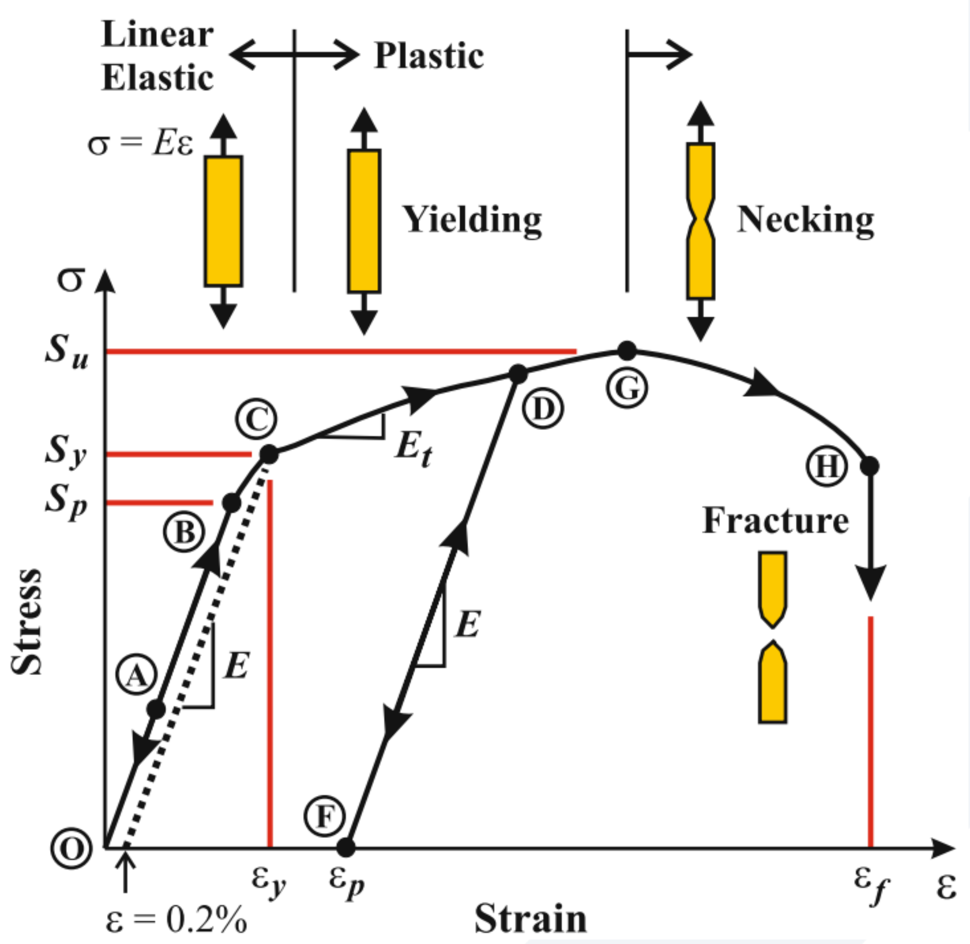
Plastic Deformation, Necking, and Failure

After reaching S_u, stress decreases with increasing strain (from G to H). Force $P = \sigma A$ decreases because somewhere along the bar, its cross-sectional area begins to decrease significantly.



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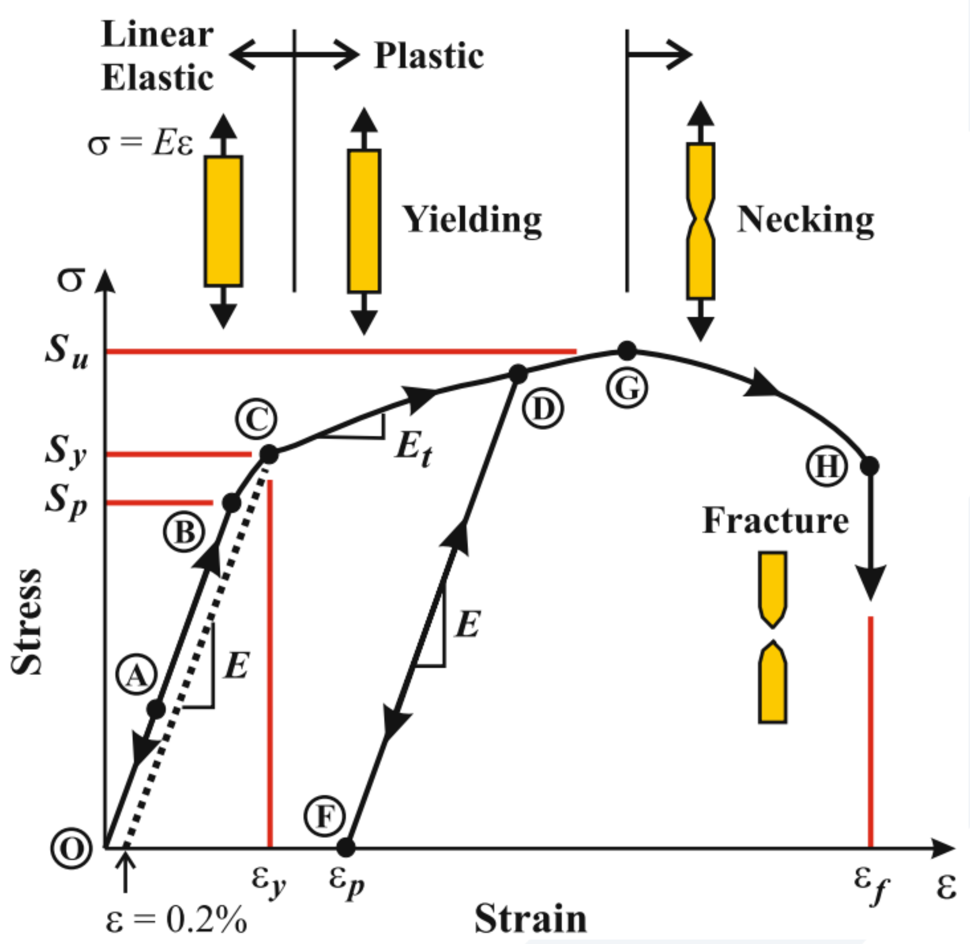
Plastic Deformation, Necking, and Failure

At this localized reduction in area, *necking*, the stress is higher than the nominal stress, so the strain and elongation are concentrated there.

Finally fracture into two pieces occurs in the *neck* at the *failure strain* ε_f.



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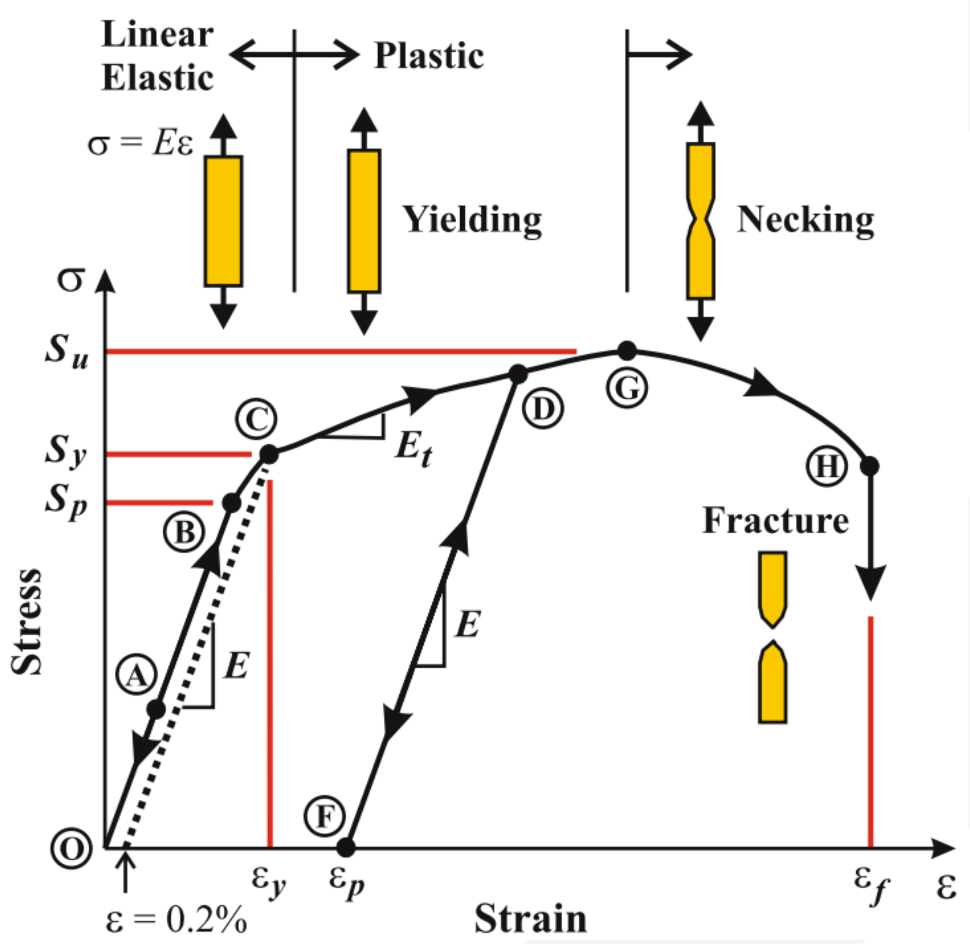
Plastic Deformation, Necking, and Failure

A material is generally classified as *ductile* if the strain to failure $\varepsilon_f \gg \varepsilon_y$ (by an order of magnitude).

Ductile materials typically have failure strains on the order of 15% or more.

The ductility of metals allows them to be bent into various shapes without breaking.



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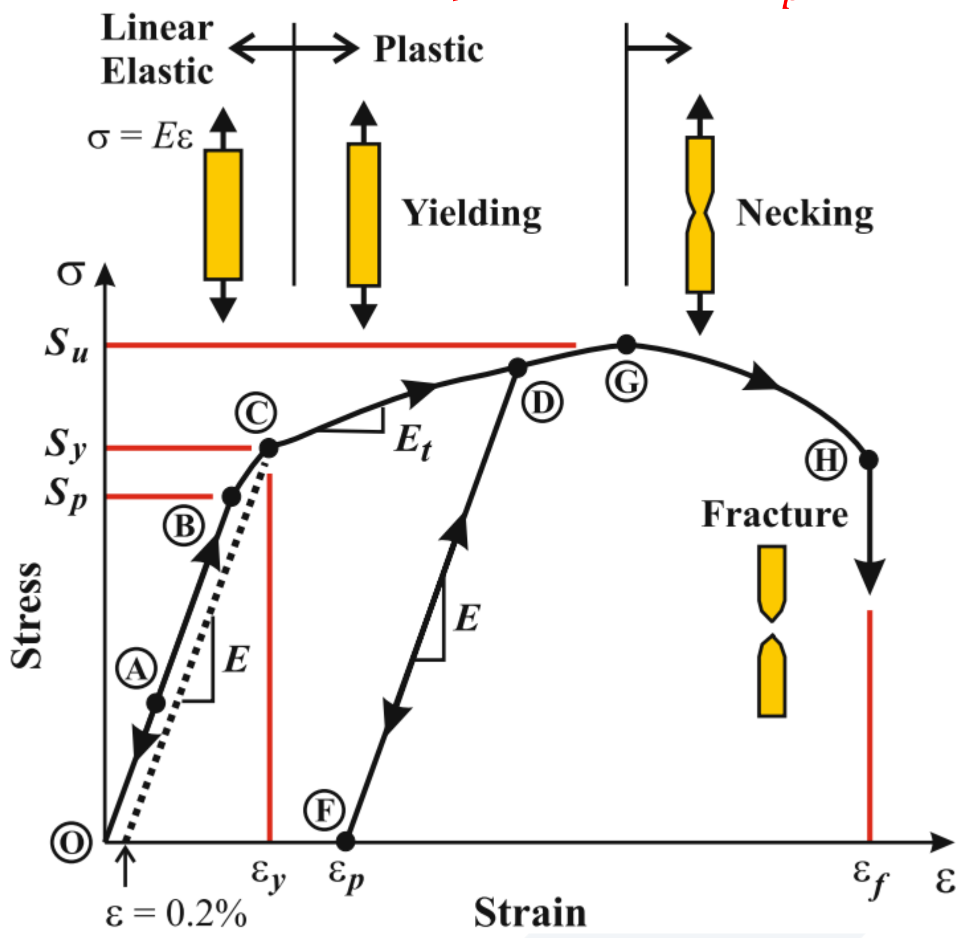
Unloading After Plastic Deformation and Reloading

Removing load after yielding but before necking (between C & G), the $\sigma - \varepsilon$ response follows an *elastic unloading line DF,* having the same slope as the linear–elastic loading line *OAB*.

When the stress is completely removed, the bar does not return to its original length, but suffers a *permanent strain or plastic strain* ε_p.



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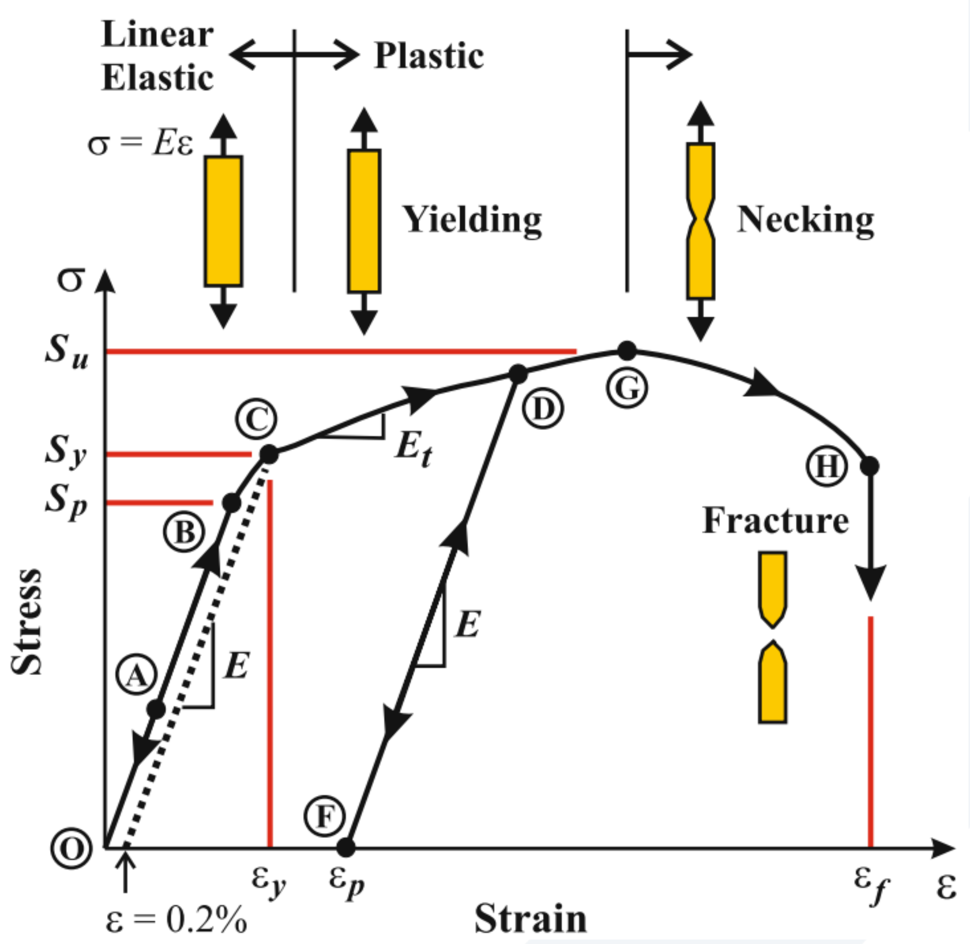
ε_y, ε_p, ε_f: Yield strain, Permanent strain, Failure Strain.

Unloading After Plastic Deformation and Reloading

Reapplying load, the response begins at point *F* and is linear up to a greater stress than the original yield strength S_y at point *D*, where it rejoins the overall curve. This phenomenon is known as *strain-hardening*.



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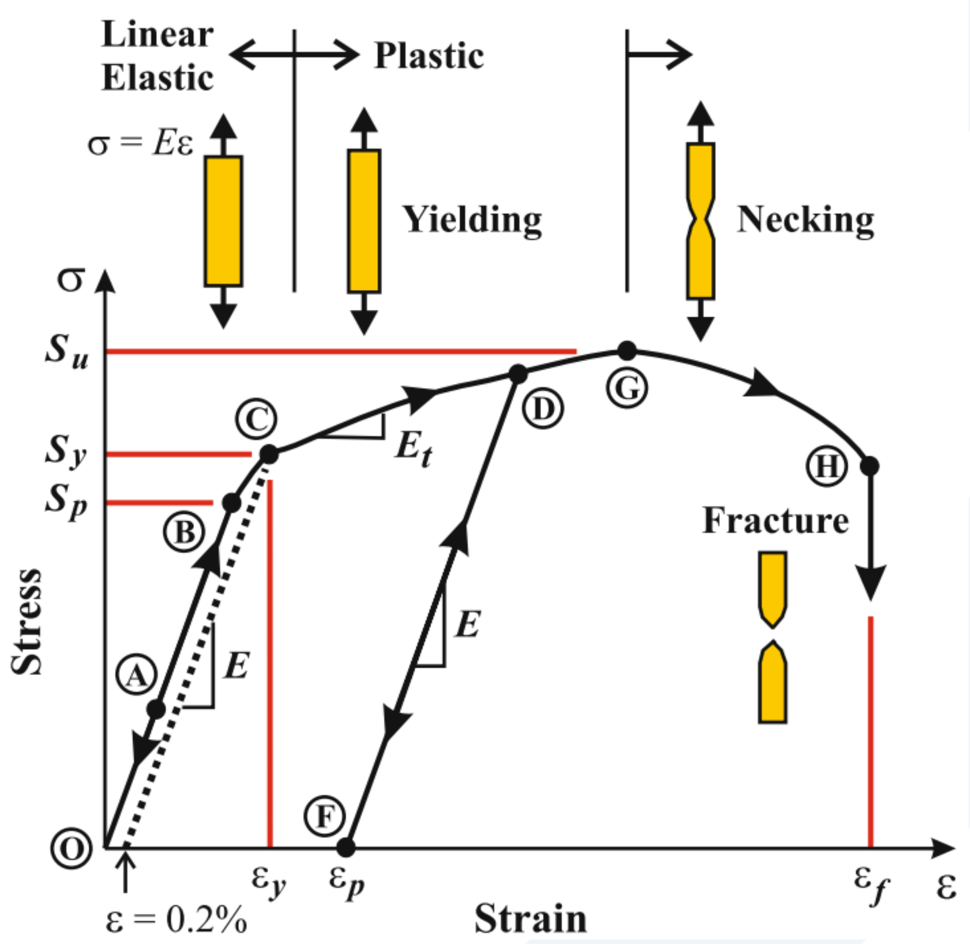
Unloading After Plastic Deformation and Reloading

By mechanical processing a material yield strength can be increased. However, a bar that has been strain-hardened is less ductile, the failure strain is reduced from ε_f to $\varepsilon_f - \varepsilon_p$ (although in most cases, there is still sufficient strain to failure).



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$\varepsilon_y, \varepsilon_p, \varepsilon_f$: Yield strain,Permanent strain, Failure Strain.

Material	E (Gpa)	S_y (Gpa)	ε_y(%)	ε_f(%)
Steels	207	250-1900	0.12-0.95	25+
Titanium alloys	115	200-1300	0.17-1.2	20
Aluminum alloys	70	100-600	0.14-0.86	15
Nickel Alloys	215	200-1600	0.1-0.74	30
Cast irons	180	220-1000	0.12-0.55	0(gray) 15(ductile)
Douglas fir (parallel to grain)	12.4	100 (S_u)	N/A	-
Glass	70	N/A	Very small	~0
Rubbers	0.01-0.1	30 (S_u)	>10	500+
Polymers	0.1-5	20-30	0.5-2	-
Engineering ceramics	300-450	N/A	Very small	~0
Carbon fiber/polymer matrix composite	70-200	1800 (S_u) In fiber direction	N/A	N/A

Values of yield strength S_y in metals depend on chemical composition, mechanical processing, thermal processing, etc.

Ceramics & glasses are brittle and exhibit little, if any, plastic strain.

General Comments

In $\sigma - \varepsilon$ experiments, it is standard practice to apply a displacement/strain and measure the required force/stress. The *displacement-controlled* or *strain-controlled* is test just discussed.

The alternative is to apply a force/stress and measure the resulting strain (a *force-controlled test*).

Data collection in the force-controlled test is difficult because beyond the proportional limit, the slope of the stress–strain curve decreases; small increments of stress cause large changes in strain.

Better results during yielding are achieved using the strain-controlled test; small increments of strain require very small changes in stress. Additionally, since force continuously increases in the force-controlled test, the decrease in stress at necking is not captured.

Unlike the modulus, the yield strength of a metal can be significantly increased by the addition of atoms of another element (*alloying*), by mechanical processing, or by heat treatment. Metals can, therefore, have a wide range of yield strengths, as shown in the previous Table.

By understanding processing techniques (metallurgy course), a metal alloy can be engineered to have a specific yield strength S_y .

Increasing the yield strength does not generally influence the value of Young's modulus E .

Of course, the more complex the processing route, the more expensive the material.

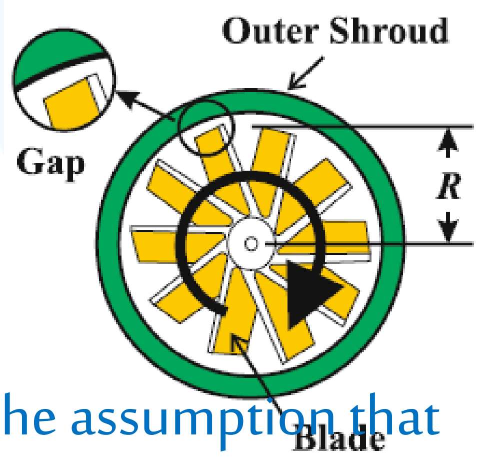
High strength metals are expensive, so only used in special applications where the cost is justified.

Yield strength S_y of steel used in buildings & bridges is down the range with a value of 250 MPa.

By contrast, modern pressure vessel is possible using high-strength steels with $S_y \approx 1900$ MPa .

High-strength, high-temperature nickel-based alloys are used in jet engines. Where the acting forces are large and a design with compact dimensions is only possible using strong nickel alloys. If only yield strength were considered in the design, the system could be made strong enough. However, larger allowable stress levels mean larger elastic deflections. An elastic extension is not necessarily negligible. Ex, a strain of ε =0.6% is within the elastic region for high-strength nickel alloys ($\varepsilon_y = 0.75\%, Sy = 1600\ MPa, E = 215\ GPa$).

If the radius of the engine's compressor disc R is1.0 m and the dynamic loading causes a strain of $\varepsilon = \Delta R\ /R = 0.006$, then the increase in radius is $\Delta R = 6$ mm.



The gap between the blades and the outer shroud (which itself may deform) must be large enough to accommodate this expansion (*Fig.*). Elastic deformation must be considered.

The majority of materials used in practice are ductile. Design methods are often based on the assumption that the stress in a material is limited by its yield strength S_y so that the material remains elastic.

The ability of a metal to yield before it breaks is a useful property, since plastic deformation provides a visible warning of impending failure.

Stress–Strain Curves for Brittle Materials

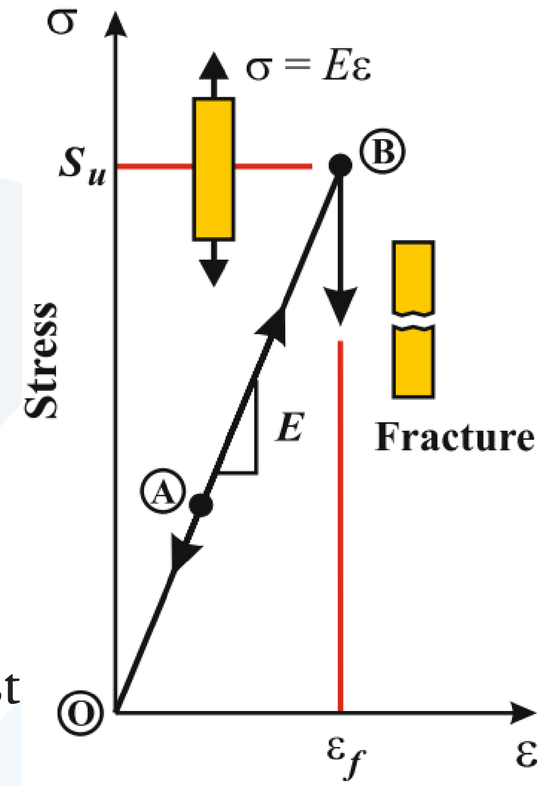
The stress–strain curve of a brittle material such as ceramic and glass is essentially limited to the elastic region, as shown in *Figure*. The strength is usually defined by the *ultimate strength* S_u, i.e., the stress at failure (fracture into two pieces).

It is generally difficult to specify a single value of S_u for a brittle material because there is so much scatter in tests.

The measured strength depends on the specimen's size and on the random distribution of pre-existing flaws or cracks in it.

S_u for brittle materials are often not tabulated, and given with conservative values, or are given with a broad range.

Gray cast iron is used extensively in castings of engine blocks for automobiles and diesel engines. When tested in tension, gray cast iron breaks into two with little warning.



Key to plot:

O. Origin.

A. Point on Linear-Elastic Curve.

B. Ultimate Strength, Failure Strain, Fracture.

E: Young's Modulus.

S_u: Ultimate Strength.

E_f: Failure Strain.

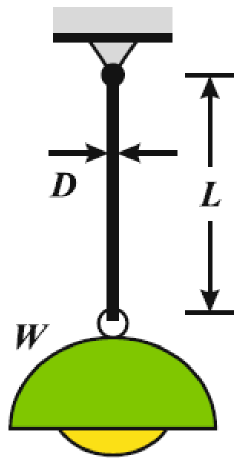
Gray cast iron exhibits some of the characteristics of ductile materials with a failure strain ε_f typically between $[2Sy/E]$ and $[5Sy/E]$. Nevertheless, because the failure strain is small compared with ductile materials, gray cast iron is sometimes described as *semi-brittle*, sending a signal to the designer to proceed with caution.

Ex. 1. Hanging Lamp

A lamp weighing $W = 50$ N hangs from the ceiling by a steel wire of diameter $D = 2.5$ mm.

The wire has a yield strength of $S_y = 400$ MPa. The factor of safety against yielding is to be 2.5 (just in case someone pulls down on it, etc).

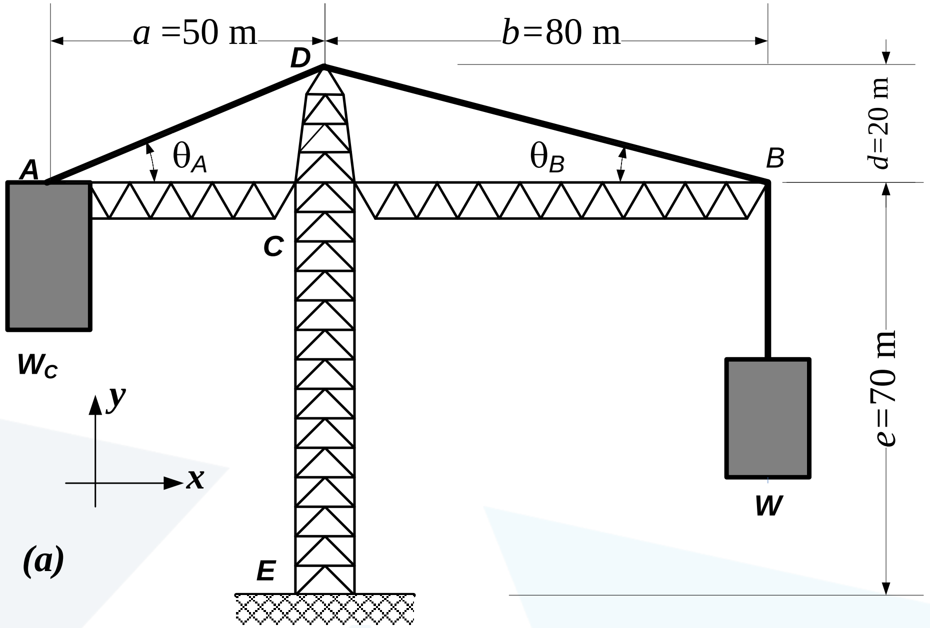
Determine the allowable (design) load P_{allow}.



Ex. 2. Tower Crane

The tower crane shown in (*Figure a*) consists of tower ***DCE*** fixed at the ground, and two jibs ***AC*** and ***CB***. The jibs are supported by tie bars ***AD*** and ***DB***, and are assumed to be attached to the tower by pinned connections.

The counterweight W_C weighs 1750 kN and the crane has a lifting capacity of $W_{max} = 1200$ kN. Neglect the weight of the crane itself. Determine:

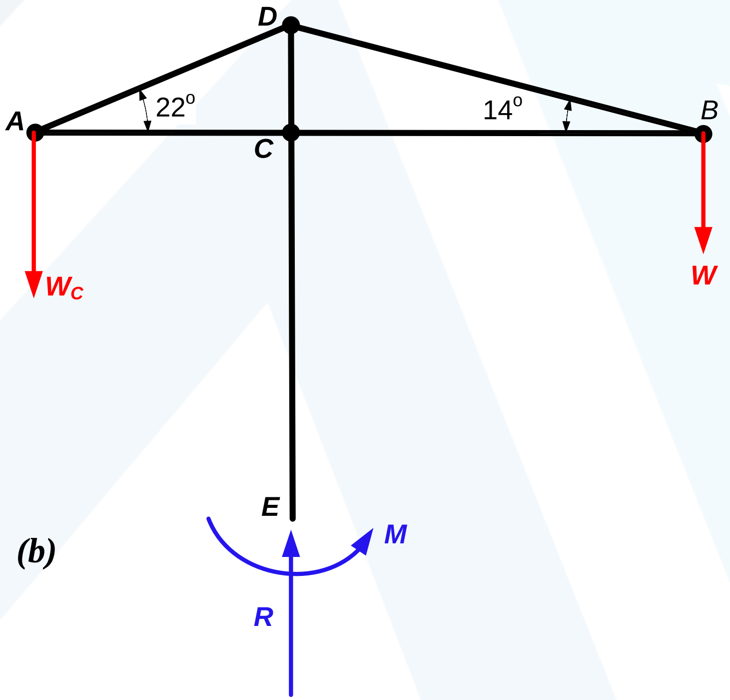


(a) the reactions at the base of the tower when the crane is lifting its capacity.

(b) the axial forces in tie bars ***AD*** and ***DB***, and jibs ***AC*** and ***CB***, and

(c) If the factor of safety against yielding is 2.0, determine the minimum cross-sectional area of tie bar *DB*.

(d) Using the area calculated in Part (c), determine the change in length Δ of *DB*.

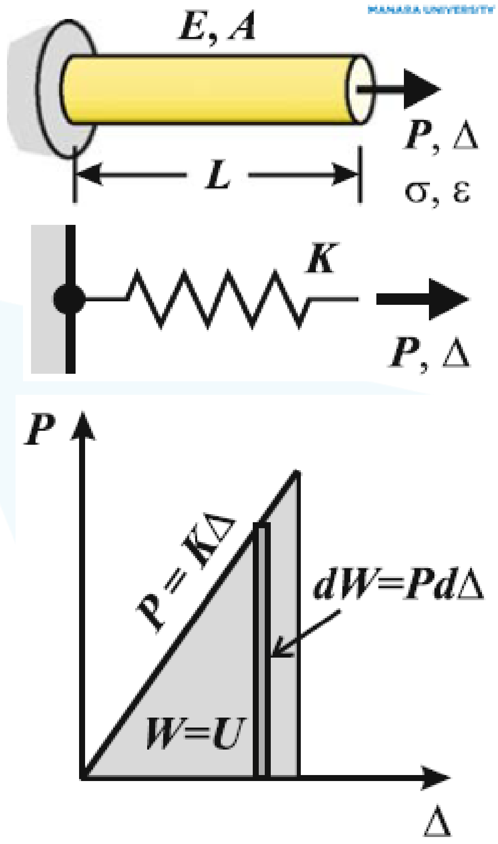


Elastic Strain Energy of a Tensile Bar

The relationship between axial force P and elongation Δ for an elastic bar is: $P = (EA/L)\Delta = K\Delta$

The bar behaves like a spring of stiffness K. The increment of work dW done by force P in deflecting the spring by an additional increment of displacement $d\Delta$, is:

$$dW = Pd\Delta = (K\Delta)d\Delta$$



The total work done to elastically deform the bar is determined by performing the integral:

$$W = \int_0^{\Delta} Pd\Delta = \int_0^{\Delta} (K\Delta)d\Delta = \frac{1}{2}K\Delta^2 = \frac{1}{2}\frac{EA}{L}\Delta^2$$

Using $P = K\Delta$, the total work done can also be: $W = \frac{1}{2}P\Delta = \frac{1}{2}\frac{P^2}{K} = \frac{1}{2}\frac{L}{EA}P^2$

The work done W is stored internally as *elastic strain energy* U. So $W = U$.

The work expression can be transformed to show the *internal energy* expression as:

$$U = W = \frac{1}{2}\frac{L}{EA}P^2 = \frac{1}{2}\frac{L}{EA}(\sigma^2 A^2) = \frac{1}{2}\frac{LA}{E}\sigma^2 = \frac{1}{2}LA\frac{\sigma}{E}\sigma = V\left(\frac{1}{2}\sigma\varepsilon\right) = VU_D$$

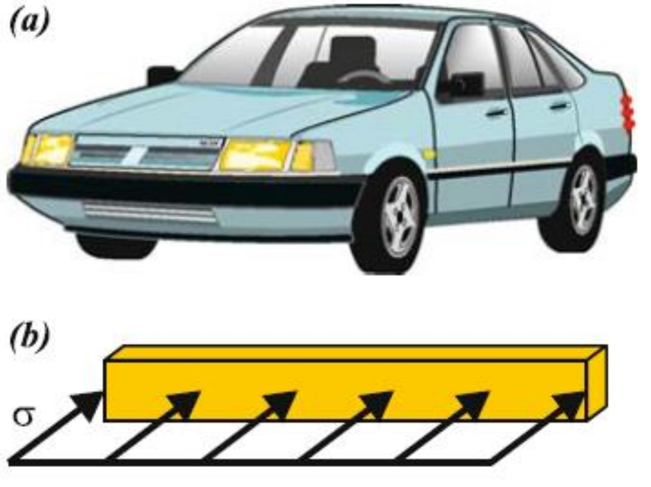
U_D, is the *elastic strain energy density.* $U_D = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}E\varepsilon^2 = \frac{1}{2}\frac{\sigma^2}{E}$

The maximum value of the *elastic strain energy density* is when the stress reaches the *yield strength*. The maximum elastic strain energy density is known as the *modulus of resilience* U_R : (معامل المانعة الميكانيكية)

$$U_D = \frac{1}{2}\frac{\sigma^2}{E} \Rightarrow U_R = \frac{1}{2}\frac{S_y^2}{E}$$

The *resilience* (المانعة الميكانيكية) is the maximum energy per unit volume that can be absorbed by the material without plastic deformation occurring.

Ex: Car Bumper Design



Car bumpers are often protected with a strip of rubber approximately 2.0 m long, 5 mm thick, and 100 mm high. The practical purpose of the strip is to absorb energy in low-speed accidental crashes such as in parking lots or when parallel parking. Assume the strip supports the entire load uniformly.

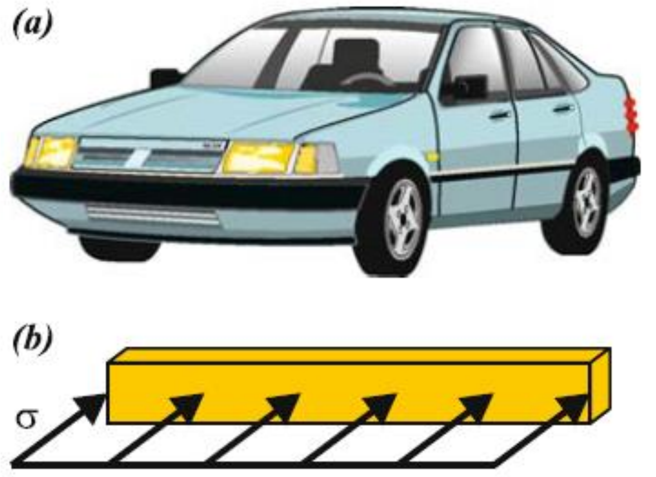
(a) From an elastic energy standpoint, why might rubber be a good choice of material compared to steel and aluminum?

(b) (b) If a 1100 kg car traveling at 2.2 m/s in a parking lot hits a wall, and comes to a complete stop, can the rubber pad absorb the energy without exceeding the elastic limit?

Solution:

(a) From tables giving S_y and E, $U_R = \frac{1}{2}\frac{S_y^2}{E}$ is computed to allow comparison

Material	S_y (MPa)	E (GPa)	U_R (MN·m/m^3)
Steel	250	200	
Aluminum	240	70	
Rubber	20	0.05	



(b) Kinematic Energy of the car…