B. Haidar

Structural Mechanics (1)

Week No-05

Deflection in Determinate Structures

B. Haidar

Deflections of Trusses, Beams, & Frames: Work-Energy Methods

- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

Deflections of Trusses Under External Loads and Other Effects

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The virtual work method is used to determine the deflections of trusses under the action of external load, *and temperature change or fabrication errors*.

Let us assume that we want to determine the vertical deflection Δ, at joint B of the truss due to the given external Loads P_1 & P_2.

If N represents the internal axial force in an arbitrary member j of the truss then from the axial deformation, δ, of this member is given by: $\delta = NL/EA$, where L, A & E, denote respectively, the length, cross-section and elastic modulus of member j. $W_{ve} = 1_v(\Delta_r)$ $U_{vi} = \Sigma N_v(NL/EA) = \Sigma N_v(\delta_r)$

1) Under the action of external loads:

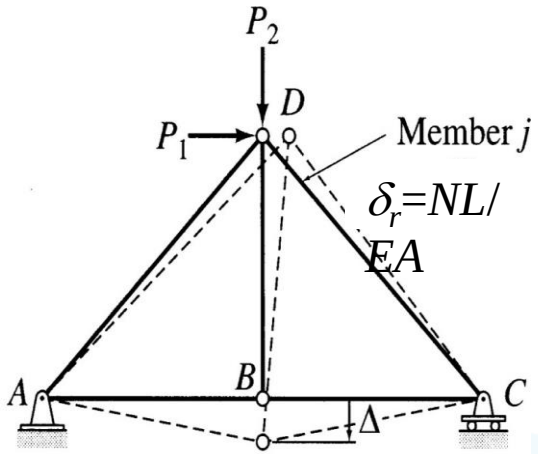
$$1_v(\Delta_r) = \Sigma N_v(N_r L/EA)$$

2) Under the action of a temperature change ΔT:

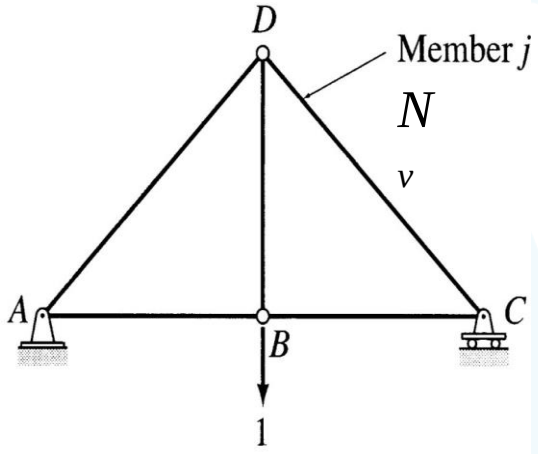
$$1_v(\Delta_r) = \Sigma N_v[\alpha(\Delta T)_r L]$$

3) Under the action of a fabrication error δ_r:

$$1_v(\Delta) = \Sigma N_v(\delta_r)$$



(a) Real System



(b) Virtual System

Deflections of Trusses Under External Loads and Fabrication Errors

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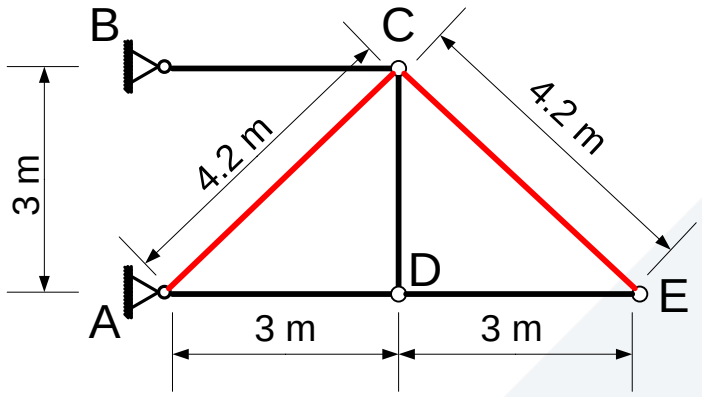
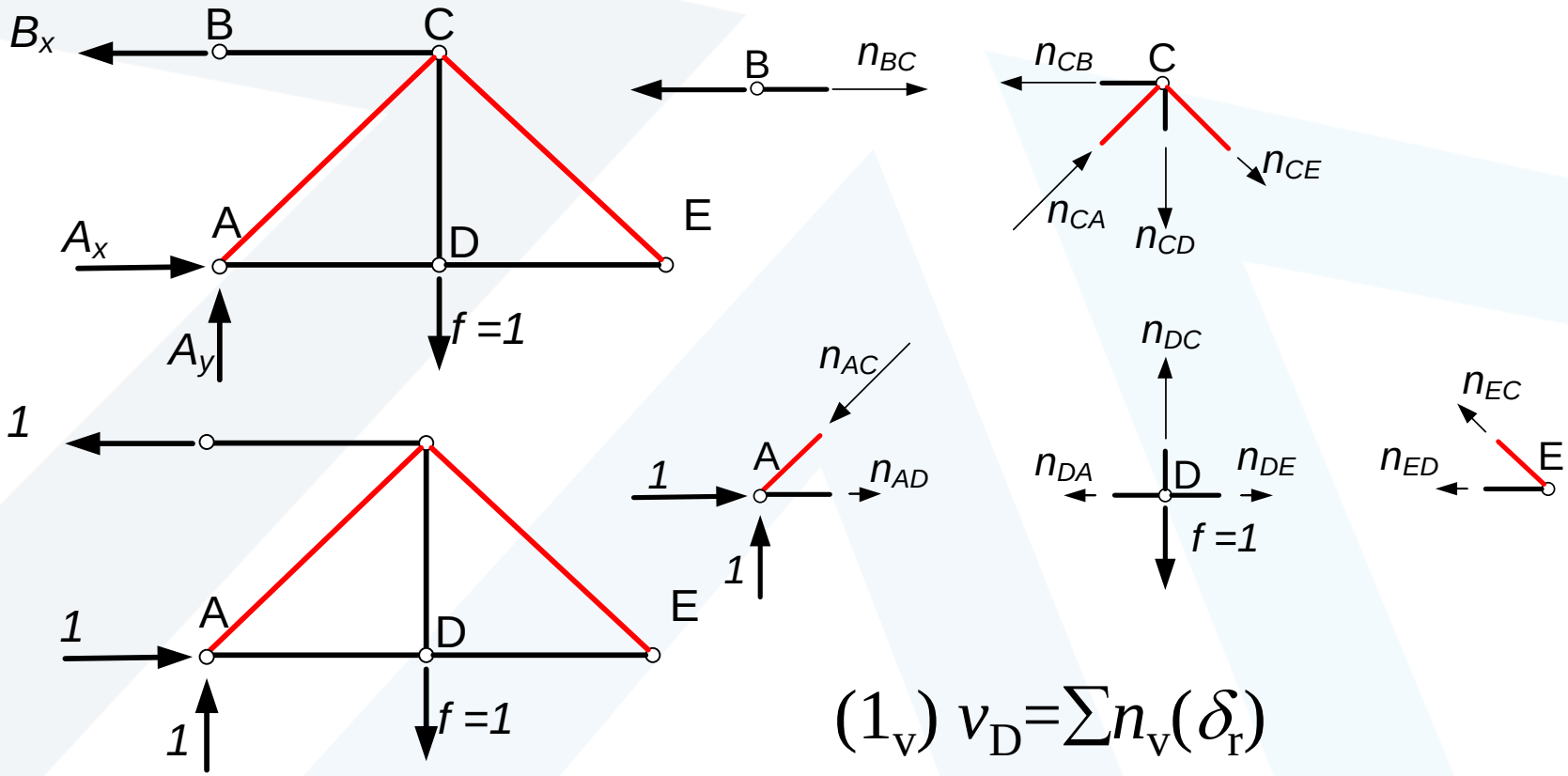
Example: Compute the vertical displacement at joint D in the shown truss resulting from a fabrication error in the length of the two diagonal bars AC & CE. Suppose that A=0.01 m^2 & E=200 GPa.



Solution:

The geometry shows that $L_{AC}=L_{CE}=3\sqrt{2}=4.24264$ m

$\delta_{AC}=\delta_{CE}=-0.04264$ m



$$(1_v)\ v_D=\sum n_v(\delta_r)$$

Deflections of Trusses Under External Loads and Fabrication Errors

Memb.	ΔL	n_v	$(n_v)(\Delta L)$
AC	-0.04264	$-\sqrt{2}$	$0.04264\sqrt{2}$
AD	0	0	0
BC	0	1	0
CD	0	1	0
CE	-0.04264	0	0
DE	0	0	0
Σ			$0.04264\sqrt{2}$

$$v_D = 0.04246\sqrt{2} = 0.0603\text{m}$$

Deflections of Trusses Under External Loads and Temperature Changes

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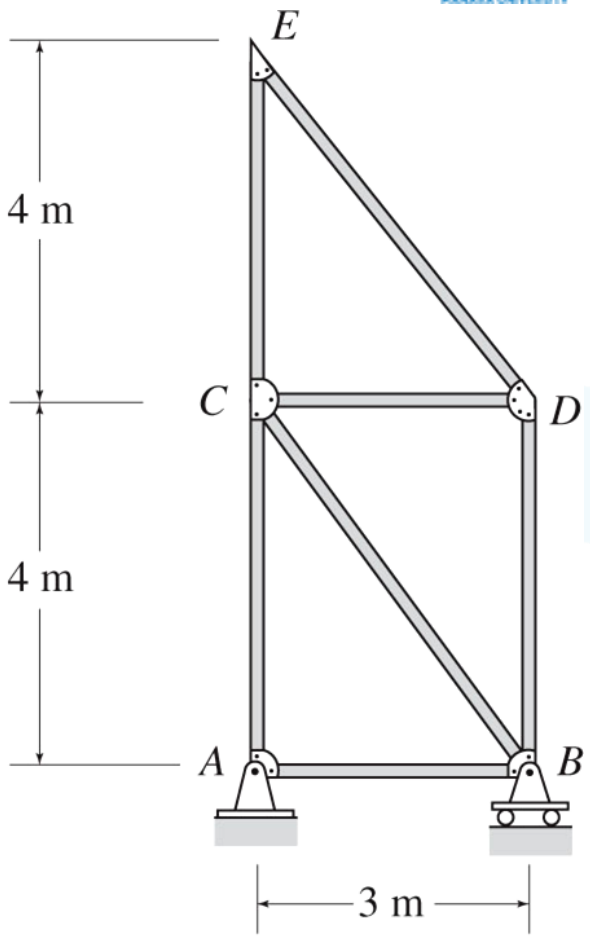
Ex. Determine the horizontal deflection at joint *E* of the truss shown in the following figure due to a temperature increase of 50°C in members *AC* and *CE*. Use the method of virtual work.



Solution.

Real System: The real system consists of the temperature changes (ΔT) given in the problem, as shown in fig.

Virtual System: The virtual system consists of a unit load applied in the horizontal direction at joint *E*, as shown in the following fig. Note that the virtual axial forces (n_v) are computed for only those members that are subjected to temperature changes. Because the temperature changes in the remaining members in the truss are zero, their axial deformations are zero; therefore, no internal virtual work is done on those members.

Deflections of Trusses Under External Loads and Temperature Changes

Horizontal Deflection at E, Δ_E: The temperature changes (ΔT) and the virtual member forces (N_v) are tabulated along with the lengths (L) of the members, in the following table. The coefficient of thermal expansion, α, is the same for all the members, so its value is not included in the table. The desired deflection Δ_E is determined by applying the virtual work expression, as shown in the table

Member	L (m)	ΔT (°C)	n_v (..)	$n_v (\Delta T) L$ (..°C-m)
AC	4	50	2.67	534
CE	4	50	1.33	266
			Σ	**800**

$$1_v(\Delta_E) = \alpha \sum N_v(\Delta T) L$$

$$(1_v)(\Delta_E) = 1.2(10^{-5})(800) = 0.0096\,\text{m} \qquad \Delta_E = 9.6\,\text{mm} \rightarrow$$

Deflections of Trusses Under External Loads and Fabrication Errors

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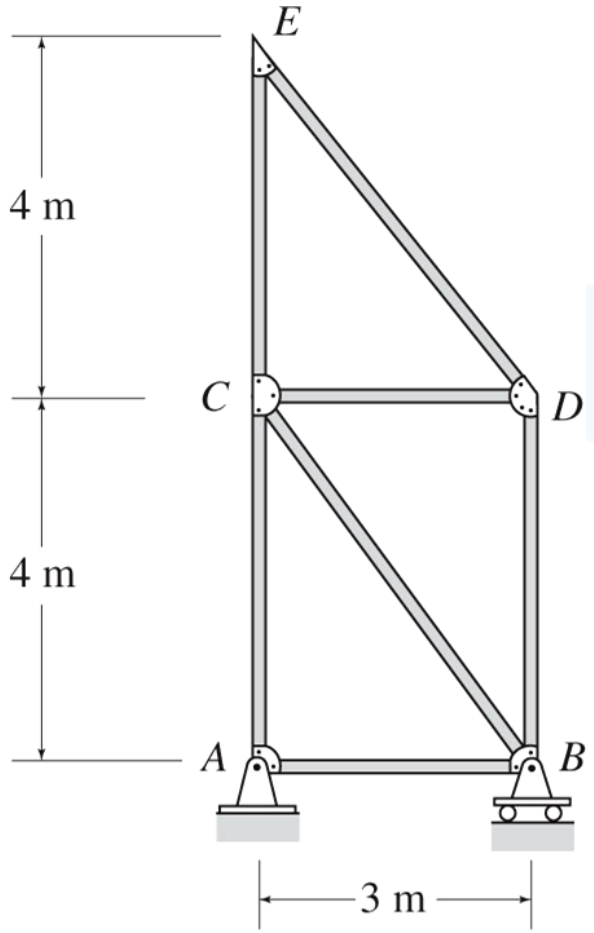
Ex. Determine the horizontal deflection at joint E of the truss shown in the previous problem if member BC is 18 mm too long and member CE is 15 mm too short. Use the method of virtual work.



Solution.

Real System: The real system consists of the changes in the lengths (δ) of members *BC* and *CE* of the truss, as shown in fig.

Virtual System: The virtual system consists of a 1-kN load applied in the horizontal direction at joint *E*, as shown in the following fig.

Horizontal Deflection at E, Δ_E: The desired deflection is determined by applying the virtual work expression as shown in the following table.

Deflections of Trusses Under External Loads and Fabrication Errors

Member	L (m)	δ (mm)	n_v (..)	n_v (δ) (..-mm)
BC	5	18	-1.67	-30
CE	4	-15	1.33	-20
			Σ	**-50**

$$1(\Delta_E) = \sum n_v(\delta)$$

$$(1_v)(\Delta_E) = -50 \text{ kN.mm}$$

$$\Delta_E = 50 \text{ mm} \leftarrow$$